Kitcher, Ideal Agents, and Fictionalism

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1. Introduction
The notion of idealization has received less attention in philosophical discussions of mathematics than it has in the philosophy of science. An exception to this is Kitcher’s naturalist account [1984] of the nature of mathematics and mathematical objects, according to which mathematics is a set of idealizing theories. Kitcher aims to replace aprioristic accounts of mathematics by developing a naturalistic alternative, arguing that the source of our knowledge of, for instance, set-theoretic or geometric axioms is our perceptual encounters with the physical world and its structures. Kitcher urges us to think of mathematics as an idealized science of human operations rather than a theory describing mathematical objects existing independently in a platonic realm. Mathematics tells us what an ideal mathematical agent does.

One of the main attractions of Kitcher’s view is that it does not commit us to an ontology of abstract mathematical objects. Moreover, it is a naturalizing account, treating mathematics as continuous with science and mathematical knowledge as on par with knowledge in other domains. I will argue, however, that Kitcher’s invocation of idealization cannot do all of the work he clearly intends it to do. It cannot both save all mathematical truth and avoid a bloated platonist ontology. Several commentators have objected that Kitcher merely replaces abstract mathematical objects with another kind of abstract object, the ideal agent. Kitcher’s reply to these objections appeals to an analogy with idealizing theories in science. It is far from clear, however, that this analogy works in the way that he needs it to. This becomes apparent when we take a look at the nature of idealization in empirical science. Nevertheless, what is left of Kitcher’s view is worth holding onto, even if it fails to save the whole of mathematical truth.

In order to provide a better reply to Kitcher’s critics and retain most of the basic motivations of his theory, I propose that his account be fictionalized. In particular, I argue that the ideal agent should be understood to

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be a fictional character. Discourse about her should be understood to be
in important respects the same as discourse about a character like Sherlock
Holmes. This reformulation of Kitcher’s theory suggests a different ana-
logy between mathematics and science, one that highlights the empiricist
aspect of Kitcher’s view. Instead of truth, what we get from mathematics
is empirical adequacy. By understanding mathematics as a set of stories
about a fictional character who resembles us in some ways and acts in a
world somewhat like ours, we lose mathematical truth (beyond the truth of
empirical adequacy) but gain an understanding both of how mathematics
relates to the world and of mathematics as a theory about a human activity.

The second section of the paper briefly outlines Kitcher’s view and his
introduction of idealization into his account of mathematics. In the third
section I argue that the way that idealization functions in science shows
that it cannot be used to save mathematical truth. The fourth section in-
troduces a fictionalist version of Kitcher’s account drawing on Kendall Wal-
ton’s and Gregory Currie’s make-believe accounts of fiction. The resulting
fictionalism retains Kitcher’s empiricism and his focus on mathematics as a
theory of operations. Fictionalizing Kitcher’s theory in this way, however,
undermines what could be seen as one main motivation for introducing the
ideal agent, namely hanging onto mathematical truth without ontological
commitment to mathematical objects. My retention of the notion of an
ideal mathematical agent thus needs to be defended. In the fifth section
of the paper I defend this aspect of the account and also present an argu-
ment that fictionalists should adopt the make-believe account of fictional
discourse independently of their attitude towards the ideal agent.

2. Kitcher’s Account
According to Kitcher, ‘arithmetic describes those structural features of the
world in virtue of which we are able to segregate and recombine objects:
the operations of segregation and recombination bring about the manifes-
tation of underlying dispositional traits’ ([1984], p. 108).\(^1\) Kitcher rewrites
first-order arithmetic in the language of Mill Arithmetic and generates a
set of axioms that quantify over concrete operations rather than abstract
objects like numbers or sets. Their truth, however, requires an infinite do-
main and this entails realism about abstract mathematical objects of some
kind or other. One possible solution to this difficulty could be to say that
the theory is about abstract idealized operations, but it should be clear
that this strategy is not available to Kitcher, since it is a form of platon-
isim. Moreover, since Kitcher asserts that ‘the existence of an operation

\(^1\) For simplicity I will restrict my remarks to Kitcher’s account of the ontology of
arithmetic, though my argument should be understood to apply to his entire account.
The reals are developed along the same lines, as operations rather than objects. Kitcher
further suggests that the more advanced parts of mathematics can be understood as
systematizations of operations made possible by mathematical notations themselves.
consists in its performance’ ([1984], p. 114), we know there are only a very small finite number of concrete operations. Clearly this will not suffice. So Kitcher’s account has a serious problem. If mathematics is true, then it is about something other than concrete operations and if it is about concrete operations, then it is not true.

Kitcher appears to think that the difficulty can be resolved and truth retained. Rather than thinking of mathematical propositions as straightforwardly true, we should think of mathematics as ‘an idealized science of human operations’ ([1988a], p. 313). To explain how talk of an ideal agent helps with the existence problem, Kitcher draws attention to idealizing theories in science. Like ideal gases, ideal agents do not exist. However, ‘[s]tatements of arithmetic, like statements of ideal-gas theory, turn out to be vacuously true’ ([1984], p. 117). And thus we get truth without troublesome ontological commitments.

Or do we? Closer attention to this proposal shows that it is not sufficient as it stands. It does not actually make Kitcher’s Mill Arithmetic true. Consider ideal-gas theory. The ideal-gas law describes the relationships between temperature, pressure, and volume in an ideal gas as follows:

\[ P \cdot V = nR \cdot T. \]

Since there are no ideal gases this law is false. The very non-existence of ideal gases, however, allows us to formulate a true ideal-gas law, one with a universally generalized conditional form:

\[ (x)(Gx \rightarrow P(x) \cdot V(x) = nR \cdot T(x)), \]

where \( Gx \) means ‘\( x \) is an ideal gas’. The antecedent of the embedded conditional is never true, thus the conditional itself is always true which makes it, when universally quantified, also true. This is the vacuous truth to which Kitcher refers.

For the axioms of Mill Arithmetic all to come out vacuously true they will have to be similarly recast in conditional form. For example, the axiom which tells us that each operation has a successor operation:

\[ (x)(\exists y)Syx, \]

will turn out to really have the form:

\[ (w)[Iw \rightarrow (x)(\exists y)Syx], \]

where \( Iw \) means ‘\( w \) is an ideal agent.’ As the antecedent of (2a) is never satisfied, it is vacuously true. This makes Kitcher’s reconstruction of arithmetic truth and knowledge rather unnatural, however. We are not going to
get the truths we want out of it. Indeed, the statements of any conditional theory will also be vacuously true. We can put whatever we like in as the consequent of a material conditional with a false antecedent and generate vacuous truth, but this does not seem an attractive way to reconstruct mathematical truth.\footnote{Resnik ([1997], p. 65) makes the same point. Kitcher actually formulates the conditionals such that the antecedent refers (or fails to refer) to particular operations: \textit{i.e.}, If \( x \) is a one-operation then there is a \( y \) such that \( y \) is \ldots But this will not work either, since there will be a point at which there is a false axiom, namely the one in which the antecedent is the ‘largest’ existing operation, which by hypothesis will not have a successor operation. All after this one will be vacuously true.}

What Kitcher needs is a way to pick out the right conditionalized theory, and this is where idealization comes in. Kitcher proposes that what differentiates Mill Arithmetic from other vacuously true conditional theories is that it corresponds to the stipulations that we make on the ideal that it is intended to capture. That is, the theory is not only vacuously true, but would also \textit{still be true} if the ideal agent it refers to actually existed. It is ‘distinguished from the host of thoroughly uninteresting and pointless vacuously true statements \ldots [by] the fact that the stipulations on the ideal agent abstract from accidental limitations of human agents’ ([1984], note, p. 117). Other vacuously true theories will not bear this relationship to the ideal agent, or ideal gases, since they will not just abstract away accidental limiting features. The same holds of statements of conditioned Mill Arithmetic and variants of it. If the ideal agent existed, then Mill Arithmetic is the theory she would make true. How do we know this? Because the ideal agent is \textit{ideal}. Just as ideal gases have a special relationship to real gases, the ideal agent has a special relationship to us actual agents in the world. Her identity is fixed by stipulation, and this process is guided by idealization from actual agents in the world. Though only vacuously true, Mill Arithmetic has a point and is interesting.

The comparison of arithmetic to idealizing theories in science is interesting and certainly in line with the attractive naturalizing aspect of Kitcher’s approach to mathematics. Nevertheless, it bears emphasizing that without securing the existence of the ideal agent we can not get \textit{truth} for Mill Arithmetic. Invoking idealization does not alter this. At best, it can get us an explanation of why arithmetic applies to the world, even though it is mainly false. To see why this is the case, we need to consider more closely how idealization works in science and the consequences this has for the cases of mathematical theories.

\section{Idealization in Science}

One way to get a grip on the explanatory power of idealizations is to treat idealization as approximation. According to such a view, what happens in ideal circumstances helps explain what happens in real circumstances
because accurate descriptions of ideal circumstances give us approximate descriptions of real circumstances (cf. Hilpinen [1976]; Lui [1999]; Niniluoto [1987]). Others have argued, however, that idealization in science is not essentially about approximation but about representing the essences or natures of the real systems that are idealized (cf. Cartwright [1989]; Ellis [1992]; Nowak [1980]). Taking up the debate between these approaches in detail here would take me too far afield. A very brief discussion is all that is necessary for my purpose here, since it is clear that neither kind of account will get Kitcher the mathematical truth that he wants.

Consider first the idea that the ideal approximates the real and hence can be used to explain it. This account of idealization aims to show why idealizations can play the role they do in science even though they misrepresent the world. Some features left out of the ideal are irrelevant to what is being described and explained by the model. Yet this falsity need not interfere with the approximate truth of the ideal. Other idealizations, however, simplify or leave out relevant features of the real circumstances. Simplifications of this kind do have an impact on the accuracy of the idealization. These, it is argued, are also unproblematic, as long as the complexities can in principle be added back to the model. A process of addition and correction can be used to concretize the idealization so that it properly approximates the real system in question (see Nowak [1980]).

These sorts of simplification do not interfere with the usefulness of idealization. In fact, not only are they not a hindrance to good modelling, they are its essence. Nevertheless, on this account idealization achieves only approximation and not truth. More specifically, it will not get us truth for mathematics unless we accept the existence of the ideal agent of Mill Arithmetic. A true description of an ideal agent is only an approximate description of a real agent. If this is how we are to understand the role of idealization in mathematics then the price for mathematical truth will be precisely the realism that Kitcher sets out to avoid.

On the alternative approach to idealization, ideal circumstances give a picture of the causal factors at work in real circumstances. What matters in idealization according to this sort of view is not really approximation—though we may certainly get that in some central cases—but capturing the right and relevant causal factors at play in concrete circumstances. In this way, idealization gives us truth at the level of general description, but falsity at the level of particular concrete circumstances. Idealizations are explanatory because they tell us some of what is causally relevant in a situation. They tell part of the truth about what is happening in a concrete situation, even though they may not give us even approximately true predictions of the behaviour of a real system.

If this account of idealization is correct, however, the analogy between mathematical idealizations like Mill Arithmetic and scientific idealizations
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like the ideal-gas law breaks down. It is not merely our limitations that prevent the existence of the infinity of concrete operations needed to make Mill Arithmetic true. If the world is spatially and temporally finite then there is a largest possible operation of segregation or collection. Moreover, on this account idealizations pick out factors that are really at work in a situation and specify their tendencies or powers. Even though the tendencies picked out may not end up expressing themselves concretely because they are outweighed by some interfering factors’ tendencies, this account of idealization posits that they really are there and working. This is what gives idealizations their explanatory power and truth. But this does not look like the right way to describe the relation between the ideal agent and finite mathematical agents. Such an account really best fits idealizations of the behaviour of causal systems. For example, consider a simple idealizing theory which asserts that a pendulum moves in accord with a certain equation of motion. This theory idealizes and ignores the interference of any friction at the pivot, air resistance, and the rotation of the earth, etc. These forces impede or change the motion of the actual pendulum, interfering with its ‘natural’ motion. All the forces add together with those present in the ideal model and produce the actual motion of the pendulum. But this is not what is going on with Mill Arithmetic as an idealizing theory.

4. A Fictionalist Alternative

I have been assuming that the goal of Kitcher’s talk of an ideal agent is to hold on to the truth of mathematics. There are indications in Kitcher [1988a] that he may not be entirely committed to this aim. Here Kitcher distances himself from the truth of mathematical theories in two ways: first, by sketching a non-standard account of mathematical truth and, second, by invoking the notion of storytelling. The first is not a direction in which I think we ought to go; so I shall take it up briefly first, and then argue that the notion of storytelling provides the clue to the better way of developing Kitcher’s ideal-agent account of mathematics.

In a section of the paper discussing the epistemic ends of mathematics, Kitcher proposes what looks like a pragmatic theory of mathematical truth. Here he says that mathematical truth ‘is what rational inquiry will produce, in the long run’ and also that ‘there is no independent notion of mathematical truth’. True mathematical statements are those that ‘in the limit of the development of rational mathematical inquiry, our mathematical practice contains’ ([1988a], p. 314). Now this view certainly permits a solution to the current problem. If being true merely means being contained in the mathematical practice at the limit of the development of inquiry, then the objects mathematical theories quantify over do not actually have to exist in order for those theories to be true. But this will not satisfy anyone who accepts standard semantics, nor, I think, anyone who takes seriously the
warning that stipulation does not guarantee truth. Such a view produces
an account of mathematical truth that does not define mathematical truth
in terms of the subject matter of mathematical propositions. Moreover, if
this move is made, it is unclear why talk of an ideal agent is necessary. If
truth can be achieved without the existence of that to which mathematics
refers, then why not just leave it at numbers, sets, groups, and so forth?
Moreover, accepting this account of mathematical truth raises troubling
questions about truth in other kinds of discourse, especially science. An
attractive aspect of Kitcher’s view for many people is its naturalizing of
mathematics. But unless a similarly pragmatized notion of truth is also
adopted for scientific discourse, accepting it for mathematics puts a large
obstacle in the way of seeing mathematics and science as continuous.

We can refuse to follow Kitcher’s rejection of an independent notion of
mathematical truth, however, and still hold onto the basic motivation of
his theory, since the second way in which Kitcher distances himself from
mathematical truth—treating mathematics as fiction—can be used to avoid
realism. In ([1988a], p. 313) we find Kitcher saying that:

One way to articulate the content of the science is to conceive of mathematics
as a collection of stories about the performances of an ideal subject to whom we
attribute powers in the hope of illuminating the abilities we have to structure
our environment.

Further, Kitcher also says that in ‘both ideal gas theory and in mathematics,
we tell stories—stories designed to highlight the salient features of messy
reality’ ([1988a], p. 324, n. 33). Ideal gases and ideal agents are fictions,
and the theories that describe them are stories. As such, we need not treat
their contents as true descriptive accounts of the world.

Of course this move does not reinstate truth for mathematical theories.
If mathematical theories are stories about an ideal agent, then they can
only straightforwardly be true if that ideal agent exists. Merely calling a
stretch of discourse a story does not avoid standard semantic requirements
for truth. Indeed the situation may now be worse. We can not magically
get truth from such a move unless we are willing to accept fictional objects,
and fictional objects may generate more ontological angst for the meta-
physically timid than mathematical objects. Calling mathematics ‘fiction’
could then to commit us to an ontology as troubling or more troubling than
the platonist’s. To remedy this, what we need is an acceptable antirealist
account of the semantics of stories. Happily, there is just such an account

3 Balaguer ([1998], pp. 107–108) points out that Kitcher’s account is best understood
as an anti-realist account of mathematics and that it can be given a fictionalist reading.

4 Kitcher also highlights the fictionalism in the previous note, saying, ‘I have been
taken to substitute one kind of abstract object (ideal agents) for another (sets). But, as
I took some pains to emphasize, there are no more any ideal agents than there are such
things as ideal gases’ ([1988a], p. 324, n. 33).
to hand: The make-believe theory of fiction developed by Walton [1990] and Currie [1990].

The problems of mathematical truth as they arise for an empiricist like Kitcher are in fact parallel to the problems of truth for fictional discourse. In both cases we have discourse that we are pre-theoretically inclined to accept and engage in, but which also appears to generate ontological commitments that are philosophically worrying. Most people are pre-theoretically inclined to agree that Sherlock Holmes is a detective. Furthermore, even those who post-theoretically deny the truth of ‘Holmes is a detective’ are bound to admit that, as John Woods [1974] has pointed out, it is a bet-sensitive statement. A bet for it would win, while a bet against it would not. And this is the same problem that Kitcher’s account of mathematics encounters: certain statements about numbers compel our assent (‘Every number has a successor’), even if we are post-theoretically inclined to deny their truth. We therefore need some account of the difference between merely false statements and those that are false but bet-worthy. We need, in other words, a way of understanding the notion of true-in-mathematics (or true-in-the-story) that does not entail true-in-the-world. And this is where the make-believe theory of fiction can work for us.

The make-believe theory of fiction accounts for the bet-sensitivity or appropriateness of certain propositions in terms of the games of make-believe that are associated with works of fiction, without adding fictional objects to our ontology. Games of make-believe are ‘one species of imaginative activity; specifically, they are exercises of the imagination involving props’ (Walton [1990], p. 12). Straightforward examples of games of make-believe are those played by kids which typically make use of props like sticks, furniture, bicycles, and Barbie dolls. In Walton’s terms the role these props play in games of make-believe is to make certain propositions fictional. Because the Barbie doll is on the arm of the couch, it is fictional in the game that she has reached the summit of the mountain. Facts about the Barbie make certain things true in the game. More specifically, a proposition is fictional in a game of make-believe if the game includes a prescription to imagine it. And so fictionalism is the property that accounts for bet-worthiness. It is appropriate to utter ‘Holmes is a detective’ while engaging imaginatively with the Sherlock Holmes stories because that proposition is fictional in those stories. Likewise, it is not appropriate to utter ‘Holmes is not a detective’.

On this account we are to understand works of fiction as having the function of props in games of make-believe. For example, certain books are props in the games of make-believe associated with the Sherlock Holmes stories. The role these books play is to make certain propositions fictional: those which the games prescribe us to imagine, propositions like: ‘Holmes is a detective’. This proposition is (on the current theory) false, but fictional
(in the Holmes stories), while a proposition like ‘Sherlock Holmes lives in Saskatoon’ is neither true nor fictional (in the Holmes stories). Notice that fictionality does not imply falsity, however. There can be true propositions that games of make-believe prescribe us to imagine. ‘The English Channel runs between England and France’ is a true proposition. It is also fictional in the Sherlock Holmes stories. This is because this proposition is part of the content of the fiction that we are supposed to imagine. Games of make-believe associated with the Holmes stories prescribe our imagining that the English Channel is in the same place as it actually is. In principle any game of make-believe could be played with any prop. This means that for any work there is a possible game which makes any arbitrary proposition fictional, or fails to make it fictional. And there will be such (possible) games (possibly) played with the Holmes stories. This might appear to present a problem for determining what the content of a fiction is. It may seem to follow from this that all propositions are fictional in every work. For any work of fiction, however, there will be a circumscribed set of authorized games of make-believe and these determine the content of these works—the propositions that are fictional in every game authorized for a work are the propositions that are fictional in the work.

This account of fiction can provide us with a sketch of mathematics as fiction. Begin with the idea that mathematical theories are stories. As parts of a story, the propositions of any mathematical theory are fictional. The authorized games of make-believe associated with mathematical works prescribe imagining the propositions of those works. These prescriptions make the relevant propositions fictional. Even if we take the view that some of them are true and some of them are false, we can say that all the propositions of any mathematical theory are fictional, since fictionality is independent of truth value on this account of fiction.

Imagining a story may be a familiar thing, but the idea of imagining a mathematical proposition probably is not, so a brief discussion of mathematics, make-believe, and imagination is appropriate at this point. I do not have a general theory of imagination to offer, and even if I did space constraints would not allow me to outline it here. This makes it difficult to spell out exactly what is happening when a mathematician participates in a game of make-believe that prescribes the imagining of certain mathematical propositions. Nevertheless, there are a few things that can be said on this topic.

The word ‘imagination’ might suggest to some a visual model. Taking this suggestion too seriously is, however, a mistake. Imagination is not purely, or even primarily, visual. For instance, when you imagine a camel, your imagining may indeed come in the form of an image—a brownish, large, four-legged, humped creature. But imagine, for a moment, the rich ungulate smell of a camel, or the velvety texture of its muzzle. These,
though not visual, are certainly legitimate instances of imagination.

Another general feature of imagining is the extent and kind of involvement of the imagining subject herself in the content of her imagining. Many imaginings are of objects external to the imaginer, but an imaginer herself can also be the object of her imagining. I may, for instance, not just imagine a camel but imagine a camel chasing me. Furthermore, imaginings like this about oneself can be from the inside or the outside. My imagining that a camel is chasing me could be from the perspective of a spectator observing the chase or from my own perspective in the chase. But there is a sense in which all imagining is of oneself and from the inside perspective. Walton suggests that ‘all imagining involves a kind of self-imagining . . . the minimal self-imagining . . . of being aware of whatever else it is that one imagines’ ([1990], p. 29). When I imagine a camel, I am also at least imagining my awareness of the camel. To that extent imagining a camel (or anything else) is imagining something about myself.

And so it is with stories also. When I imagine a story I am also at least imagining that I am aware of what happens in it. In fact, this imagining probably also involves an even stronger reflexive engagement than this. To the extent that the story is successful, my imagining of it will involve imagining of myself that I am experiencing or doing various things—maybe riding along on horseback in pursuit of the hounds. This involvement is part of the game of make-believe in which the story is a prop; it generates fictional truths about me that are part of the game even though they are not part of the work. So, as a narrative unfolds, its audience may merely be caught up in it as if they were invisible spectators of the events that occur. Their involvement may, however, include a more robust reflexive self-imagining than this.

The imagining involved in a mathematical game of make-believe is, I suspect, rather more like this self-imagining than it is like merely imagining external things. It is more like imagining running away from a camel than merely imagining a camel. In fact, I would take this even further. Recall the overall claim stemming from Kitcher’s theory: Mathematical theories are stories about an ideal agent. Playing mathematical games of make-believe, I suggest, often involves imagining that you are the main character

\footnote{Christopher Peacocke proposes that imagining something always involves at least imagining, ‘from the inside, being in some conscious state’ ([1985], p. 21).}

\footnote{This might seem to cause a problem for the make-believe account specifying the content of a fictional work. If there are propositions about me that are fictional in a game of make-believe using a work of fiction as a prop, then does not that make those propositions part of the fictional work? No it does not. The propositions that are fictional in a work are the propositions that are fictional in all authorized games of make-believe in which it is a prop. The propositions about me that are fictional in the games of make-believe I play with a work will not be fictional in the games that others play, so they are not part of the content of the work.}

in the story. In other words, you imagine that you are the ideal agent; you imagine you are doing the things that the theory says that the ideal agent does. This suggestion is partly echoed by something Kitcher says, though he does not put it in terms of make-believe or imagining. He describes the result of idealizing the operations we perform in the following way:

"We obtain a perspicuous way of reflecting on our actual operations—that is, on our structuring of the world of experience—by making up a story, a story in which we effectively treat ourselves as freed from certain well-known limitations." ([1988b], p. 530. My emphasis.)

So there is a sense in which we are ourselves the subjects of mathematical stories—ourselves as we would be if we were ‘freed from certain well-known limitations’.

5. Varieties of Fictionalism

One advantage of the fictionalism I am proposing is that it is consistent with mathematics’ being indispensable to science. This gives it an advantage over any fictionalist account along the lines of Field’s *Science Without Numbers* which depends on the success of the project of nominalizing physical theory. The prospects of establishing the dispensability of mathematics to science in this way are a matter of great disagreement. It seems to me to be an advantage of ideal-agent fictionalism, therefore, that it is consistent with the indispensability of mathematics to science. It is not, however, the only kind of fictionalism with this advantage. Indeed, it may not even seem to be the best kind of account with this advantage. One could adopt a fictionalism that said that mathematics is a set of stories about mathematical objects rather than operations. Such a view shares with platonism the advantage of taking mathematical language at face-value and shares with the ideal-agent formulation the advantages of fictionalism. One could argue that this makes mathematical-object fictionalism preferable to the account I propose. There are, however, reasons to prefer the ideal-agent formulation.

The basic argument against ideal-agent fictionalism here is as follows. Mathematical-object fictionalism is a better view than ideal-agent fictionalism because it does not reconstrue mathematical language in some awkward way, but takes what mathematicians say to be exactly what they seem to say. Articulated this way, the central disagreement between ideal-agent and mathematical-object fictionalism is in how best to characterize what mathematics is *really about*. The ideal-agent fictionalist says that mathematics is really about the ideal agent (though she does not exist) and the operations that she can perform (though most of these do not exist). The mathematical-object fictionalist on the other hand says that mathematics

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7 This variety of fictionalism is proposed by Balaguer [1998].
is really about mathematical objects (though these do not exist). It is not
clear to me that we can or have to decide definitively between these alter-
natives, though I certainly favour the ideal-agent formulation. Fictionalism
tells us that mathematics is a set of stories and we could decide that there
is room for as many stories as can be imagined (and more). But philosophy
of mathematics should presumably give us an account of just exactly what
mathematicians are doing and saying. And here, I believe, the ideal-agent
formulation has some advantages.

One of these advantages is that the ideal-agent formulation takes mathe-
ematics to be about operations rather than abstract mathematical objects.
This allows the account to save some important mathematical truth. If
mathematics is really about abstract mathematical objects then it is en-
tirely false, since there are none of these at all. If mathematics is a set
of stories about operations, however, then the portion of it that is about
operations that actually exist really is true. Though this gets us only a
small amount of truth in comparison with the infinities of ideal operations
that do not exist it is important truth. Many of the operations the ideal
agent performs are operations that do indeed exist and are performed by
actual agents, but no finite subset of abstract mathematical objects exists.
Operations are the kinds of things that really exist; numbers are not.

I should not be understood to be arguing that mathematicians should
adopt mathematical language that refers to operations rather than the lan-
guage they have been using all along. My account is not aimed at the
reform of mathematical practice. And in any case there are good reasons
apparent in the practice and purposes of engaging with (non-mathematical)
fictions that speak against such reforms in general. Engagement requires
imaginining the content of the game of make-believe, and this makes it im-
portant in general for the participants to refrain from drawing attention to
the fact that much of what they are saying is not true. If understanding and
progress requires engagement and immersion in a fictional world, then any
practice that increases the chances of engagement and immersion is clearly
justified and any practice that impedes them is not. It is for mathemati-
cians, and not philosophers, to make judgments about what mathematical
language is best suited to achieving mathematical purposes.

But it is the job of not just mathematics but the philosophy of mathe-
ematics to give us a picture of mathematical activity. As indicated above, the
ideal-agent formulation has an advantage here as well. Mathematics is not
only a tool of natural science, it is also a theory of what mathematicians
do. The mathematical-objects version of fictionalism gives us the wrong
picture of this central aspect of mathematics. It gives us a simpler seman-
tics for mathematical discourse, but at the price, it seems to me, of ignoring
some of the complexity of the situation. I have not provided anything like
a full account of mathematics as an activity, but ideal-agent fictionalism
looks more promising to me in this regard than mathematical-object fictionalism. Mathematical-object fictionalism tells us that mathematics is about nonexistent abstract mathematical objects. There is a reflexivity in mathematical language, however, that is ignored by this view. Object fictionalism implies that when mathematicians engage with mathematics they are imagining things about abstract nonexistent mathematical objects. But this does not seem right. Rather, mathematicians are imagining something about themselves. They are, as I argued above, themselves the subjects of mathematical stories.

These are not knock-down arguments for the ideal-agent formulation of fictionalism. Someone might still prefer the semantic simplicity of the mathematical-object view. But their view will nevertheless need the make-believe account of the nature of fiction. The two separable aspects of my account—mathematics as about the ideal agent, and the make-believe account of the nature of fiction—can be endorsed independently. I believe there are good arguments for both aspects, but the stronger supports the make-believe account of fiction. It should be adopted by any fictionalist because it allows us to explain more fully the mathematical discourse of both mathematicians and non-mathematicians. Simply calling mathematics false and leaving it at that is unsatisfactory. We need a fuller account of what we mean by true in the story, and Waltonian semantics gives us that. Moreover, it is the best approach to fictional discourse available to anyone whose aim is to use fiction to avoid realism. I do not have space here to argue this point at length. It should be sufficient, however, to point out that the alternatives to a make-believe theory of fictional are all in one way or another committed to ontologies that would undermine any anti-realist purpose in invoking fiction. Fictionalists should say more about mathematical discourse than merely that it is false. It is not enough to invoke fiction and leave it at that. A look at the literature on fictional discourse shows that in many ways the debate between realists and anti-realists in the philosophy of mathematics is replicated in discussions about fiction itself. The stakes are even higher in the case of theories of fiction, since even many sanguine platonists about mathematical objects and other abstracta cringe at realism regarding fictional objects. But there is a lot of important truth in fiction, just as there is in mathematics, and theories of fiction need to account for this in some way. If the aim is an anti-realist account, then it is important if invoking fiction to say what fictionality consists in, and say it in a way that does not commit you to the existence of fictional objects. This is precisely what the make-believe account does.

6. Conclusion

I have offered here only a sketch of a full fictionalist account of mathematics. Much work is necessary to fill it out. The use it makes of the notions
of imagination and prop-mediated make-believe, for example, obviously raise questions about the phenomenology of mathematical practice. If the account is broadly correct, there should be strong parallels between this dimension of mathematical practice and the kind of engagement present in our interactions with fictions and works of art. Mathematicians and philosophers of mathematics may have different intuitions about this question, but whether or not the account will be borne out in this regard is a topic for further research.
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References


Abstract. Kitcher urges us to think of mathematics as an idealized science of human operations, rather than a theory describing abstract mathematical objects. I argue that Kitcher’s invocation of idealization cannot save mathematical truth and avoid platonism. Nevertheless, what is left of Kitcher’s view is worth holding onto. I propose that Kitcher’s account should be fictionalized, making use of Walton’s and Currie’s make-believe theory of fiction, and argue that the resulting ideal-agent fictionalism has advantages over mathematical-object fictionalism.