6. Deductive Arguments:
Categorical Logic

Review: Deductive Relationships

- You’ll recall: Statement A *deductively entails* statement B if and only if it is impossible for B to be false *given that* A is true.

- Logical Impossibility: A state of affairs that cannot exist.

  E.g., If $x > 1$, $x$ cannot also be $< 1$

  If Dion is a bachelor, then Dion cannot also be married (since bachelor just means ‘unmarried male’).
Recall also that what we are ultimately after in this course is to be able to evaluate arguments. (e.g., in terms of ARG conditions)

Deductive entailment is important to evaluation in that, if an argument is deductively valid, then it must satisfy R.

Moreover, if its premises also satisfy A (i.e., if its premises are all true), then it satisfies G and is said to be sound.

1. All pacifists are opponents of war
2. No opponents of war are arms traders
   Therefore,
3. No pacifists are arms traders

Deductively valid, by virtue of its categorical form (i.e., formally valid)

We can see more clearly what is meant by formal validity by substituting new values for subject and predicate in each premise ...
1. All alephs are bleeps
2. No bleeps are cheeps
Therefore,
3. No alephs are cheeps

We can see intuitively that the argument form is valid, even if we have no idea what an aleph or bleep or cheep is. Similarly ...

1. All A are B
2. No B are C
Therefore,
3. No A are C
Categorical Logic

- The logic of “class membership” or membership in categories.

Logic which uses “all,” “no,” “some,” “not” and so forth as its basic logical terms.

- Categorical form: A **subject** category is connected to a **predicate** category ...

“All consistent vegetarians are opponents of using animals for leather.”

A categorical statement, in which “consistent vegetarian” is the **subject** and “opponents of using animals for leather” is the **predicate**.
Four Categorical Forms

**Universal Affirmation** (A – Affirmo)
All S are P
(All members of the category S are included within category P. – “All bachelors are males”)

**Universal Negation** (E – nEgo)
No S are P
(All members of category S are excluded from category P. – “No bachelors are married persons.”)

**Particular Affirmation** (I - affIrmo)
Some S are P
(Some members of category S are included in category P – i.e., there is at least one S which is P)

**Particular Negation** (O – negO)
Some S are not P
(Some members of category S are excluded from category P – i.e., there is at least one S which is not P)
The Square of Opposition

A: All S are P  E: No S are P

I: Some S are P  O: Some S are not P

Contradictories

- In the square of opposition, each form of proposition is the *contradictory* of the one diagonally opposed to it.

  When one statement is the contradictory of another, the two must always opposite **truth values**.

  I.e., if it is *true* that “No C are O” then it must *false* that “Some C are O”; if it is false that “All P are Q” then it must be true that “Some P are Q”
Contrariety

- A and E statements are *contraries*.

Contrary statements cannot both be true, but they *can* both be false.

E.g., “All dogs are black” and “No dogs are black” cannot both be true at the same time, but they could both be false. (And, in this case, both statements are in fact false.)
Subcontraries

- I and O statements are said to be *subcontraries*.

Subcontrary statements cannot both be false, (on the ‘existential import’ assumption in categorical logic) but they *can* both be true.

E.g., “some dogs are black” and “some dogs are not black” cannot both be false, but both can be true. (In this case, both are true.)
Distribution

- A term (i.e., a subject or predicate) is said to be distributed if the statement that contains that term says something (makes some assertion about) about every item falling under that term.

- A and E statements are fully universal; their subject term is always distributed ...

For example:

“All bachelors are males” (A)

Says something about all bachelors (namely, that they are male), but it does not say anything about all males.

The subject term of an A statement is distributed, but its predicate term is not distributed.
“No married men are bachelors” (E)

Says something about all married men (namely, that none of them are bachelors) and it also says something about all bachelors (namely, that none of them are married men).

Both the subject and the predicate terms of an E statement are distributed.

“Some men are bachelors” (I)

Some men ≠ all men, so the statement does not say anything about all members of its subject category. It also does not anything about all bachelors (it says only the category men and the category bachelor overlap)

So, in an I statement neither the subject nor the predicate term is distributed.
“Some dogs are not black” (O)

Some dogs ≠ all dogs, so the statement does not say anything about all dogs. But this statement does say something about the entire category of black things (namely, that there is at least one thing – a non-black dog – that is excluded from that category).

So, in an O statement, the subject term is not distributed, but the predicate term is distributed.

To Recap …

**A:** Subject is D; Predicate is ~D

**E:** Subject is D; Predicate is D (both)

**I:** Subject is ~D; Predicate is ~D (neither)

**O:** Subject is ~D; Predicate is D
Translating Natural Language Statements into Categorical Form

- In the section on Natural Language and Categorical Form (pp. 210-5), Govier offers some advice about translating natural language statements into categorical form.

Translation is a useful thing to do, because, once an argument is put into categorical form, we can make use of rules of inference that make evaluating arguments much easier. ...

... but, alas, not many statements in “real world” natural language arguments are found in perfect categorical form.

However, by paying attention to the principles of translation, we can sometimes put subject/predicate type statements into categorical form ...
Universal Affirmative (A)

*Any S is P*
*Every single S is P*
*The Ss are all Ps*
*Whatever S you look at, it is bound to be a P*
*Each S is a P*
*If it’s an S, it’s a P ... etc.*

All of these can rightly be translated into an A statement of the form “All S are P”

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Quantity of the Statement

- Often, it is important to figure out if the statement is intended to be universal or particular:
  
  “Nurses are retiring early due to stress and ill health” – All of them or only some?
  
  “Men feel uncomfortable discussing their emotions.” – All of them or only some?
Stereotyping

- While some arguers may indeed intend to make universal claim with such statements, we should be cautious.

The danger of attributing a universal claim when only a particular claim is intended can be called stereotyping.

For obvious reasons, stereotyping is not only logically inaccurate, but ethically and politically objectionable as well.

“Only”

- The word “only” does typically make an implicitly universal claim:

“Only members who have paid their dues in full will be allowed to attend the next meeting of the Aerosmith Fan Club.”

Only D are A = All A are D

Notice that we cannot conclude from this that All D are A (some members might stay home, e.g.)
Universal Negative (E)

Pigs can’t fly
No pig can fly
Pigs are not able to fly
There has never been a pig that was able to fly.
If it’s a pig, then it can’t fly

All of these are universal negative statements that can be translated into categorical form as:

No pigs are creatures that can fly (No P are F)

Some slightly tricky cases

"Not all lawyers are rich people”

"All racial minorities are not black people”

The first looks like an E statement, the second looks like an A statement; but both are correctly translated as O statements:

"Some lawyers are not rich people”

"Some racial minorities are not black people”
Particular Affirmative (I)

To repeat: In categorical logic, “some” is understood to mean “at least one” – no distinction is made between one, several, many or most.

- So, a particular affirmative (I) statement picks out “greater than none, less than all”

- Particular affirmations too can create some tricky situations:

  "Some lawyers do not take bribes"

  It seems natural to infer from this that some lawyers then *do* take bribes, but that is not necessarily what the arguer has asserted.

  A) “Some L are not B” and B) “Some L are B” cannot both be false, but the truth of A) does not guarantee the truth of B) (subcontrariety).
Particular Negative (O)

- The expression “not all” occurring before the subject of a categorical statement is usually a denial of a universal affirmation.

"Not all famous plays are written by Shakespeare"

= there is at least one famous play not written by Shakespeare.

Accordingly, “Some P are not S”

Venn Diagrams

- Venn diagrams are named after the English mathematician John Venn (1834-1923).

- Venn diagrams can be used to show the meaning of categorical statements, to describe rules of immediate inference and to work with categorical syllogisms
Areas of the Venn Diagram

An A Statement: “All S are P”

No S are outside of P; i.e., the domain of things that are S and not P is empty, blacked out.
An E Statement: “No S are P”

No S are inside of P; i.e., the overlap between S and P is empty, the categories have no members in common.

An I Statement: “Some S are P”

There is at least one thing that is both S and P; that thing is represented by the X.
An O Statement: “Some S are not P”

There is at least one thing that is S but is not P; that thing is represented by the X.

Rules of Immediate Inference

- If you have two categorical statements and the first deductively entails the second, you can *immediately* (i.e., without any intermediate steps), infer the second from the first.

- Immediate inference, in the present context, refers to operations involving A, E, I, and O statements that yield valid inferences.
Conversion

- The converse of a statement in categorical form is obtained by transposing the subject and predicate terms. So, ...

<table>
<thead>
<tr>
<th>Statement</th>
<th>Converse</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: All S are P</td>
<td>All P are S</td>
</tr>
<tr>
<td>E: No S are P</td>
<td>No P are S</td>
</tr>
<tr>
<td>I: Some S are P</td>
<td>Some P are S</td>
</tr>
<tr>
<td>O: Some S are not P</td>
<td>Some P are not S</td>
</tr>
</tbody>
</table>

Logical Equivalence

- E and I statements are *logically equivalent* to their converse; i.e., they logically entail each other and they are either both true or both false. It is impossible for one to be false given that the other is true.

E.g.,

If “no women are men,” then “no men are women."

If “some lawyers are not crooked,” then “some not crooked things are lawyers”
‘E’ Conversion Venn Diagram

No S are P
No P are S

‘I’ Conversion Venn Diagram

Some S are P
Some P are S
- So, to repeat, E and I statements are both logically equivalent to their converses; because they logically equivalent, they must have the same truth value.

- This does not hold for the conversion or A and O statements, however, as can easily be seen by inspecting the relevant Venn diagrams ...
So, the repeat A statements and O statements are NOT logically equivalent to their converses.

As Govier notes, this is actually a fairly common error in reasoning. E.g.:

Just because “All Muslims are religious believers” we certainly cannot infer that “All religious believers are Muslims”
Contraposition

To contrapose a statement in categorical form, one first converts and then adds a “non” to each category. So, ...

<table>
<thead>
<tr>
<th>Statement</th>
<th>Contraposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: All S are P</td>
<td>All non-P are non-S</td>
</tr>
<tr>
<td>E: No S are P</td>
<td>No non-P are non-S</td>
</tr>
<tr>
<td>I: Some S are P</td>
<td>Some non-P are non-S</td>
</tr>
<tr>
<td>O: Some S are not P</td>
<td>Some non-P are not non-S</td>
</tr>
</tbody>
</table>

Contraposition & Inference

- A and O statements are logically equivalent to their contrapositives.
- E and I statements are not logically equivalent to their contrapositives.
Contraposition and A Statements

- Given the truth of an A statement, its contrapositive must also be true:

  All ants are insects (All S are P)
  All non-insects are non-ants (All non-P are non-S)

  If the truth of A statement did not guarantee the truth of its contrapositive, there would be a counterexample to the A statement, a non-P that was S: a non-insect that was an ant.

Contraposition and O Statements

“Some insects are not wasps”
“Some non-wasps are not non-insects”

Just as with integers in mathematics, the double negation (“not non-”) cancels out.

To say that “some non-wasps are insects” is clearly to say something logically equivalent to “some insects are not wasps”
Contraposition and E Statements

E: No S are P

Contrapositive: No non-P are non-S

The first statement that categories S and P do not intersect at all (their intersection is empty)

The second says that whatever is outside of the P category does not intersect with whatever is outside of the S category ...

E: No cats are dogs
Contrapositive: No non-dogs are non-cats

The first happens to be true (in our world there is nothing that is both a cat and a dog); but the second is obviously false, since all sorts of things are both non-dogs and non-cats.
Contraposition and I Statements

I: Some $S$ are $P$
Contrapositive: Some non-$P$ are non-$S$

The first asserts that there is at least one thing that is in both $S$ and $P$; the contrapositive asserts that there is at least one thing outside of both categories.

Some numbers are prime numbers (true a priori)
Some non-prime numbers are non-numbers (false a priori)

Obversion

The obverse of a categorical statement is formed by first adding a "non-" to the predicate category, then changing the quality (affirmative or negative) of the statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Obversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: All $S$ are $P$</td>
<td>No $S$ are non-$P$</td>
</tr>
<tr>
<td>E: No $S$ are $P$</td>
<td>All $S$ are non-$P$</td>
</tr>
<tr>
<td>I: Some $S$ are $P$</td>
<td>Some $S$ are not non-$P$</td>
</tr>
<tr>
<td>O: Some $S$ are not $P$</td>
<td>Some $S$ non-$P$</td>
</tr>
</tbody>
</table>
Obversion and Inference

- The obverse of any categorical statement yields a logically equivalent statement.

*All guitarists are musicians*

*No guitarists are non-musicians*

Contradictories

- As you may have noticed, when there are differences among rules of immediate inference, A and O statements ‘go together’ and E and I statements go together

- To see why this is the case, think back to the Square of Opposition ...
The Square of Opposition

A: All S are P    E: No S are P

I: Some S are P    O: Some S are not P

Contradictory

Contradictory

- A and O are contradictories
- E and I are contradictories

Which means, you’ll the recall, that the truth of one guarantees the falsehood of the other and vice versa.

If it is true that “All cows are ungulates,” it must be false that “Some cows are not ungulates”
Counterexample

- This provides with another way of looking at the idea of counterexample:

  We can show *definitively* that the statement “All philosophy Ph.D.s are unemployed” is false if we can point to even a single example of a person with a philosophy Ph.D. who has a job.

False Dichotomies

1) “All refugees are non-happy persons”

It may tempting to translate this as

2) “All refugees are unhappy persons”

But these are non logically equivalent. 1) says only some refugees are outside of the category of happy persons; 2) says that refugees belong within the class of unhappy persons
• “Non-happy” is the complementary predicate of “happy”

As basic truth of logic, everything in the universe is either happy or non-happy (T a priori)

But it is not the case that everything in the universe must be either happy or unhappy.

(Some things, e.g., simply be mildly bored; for other things, the notions might not be applicable at all, e.g., rocks, trees, oil slicks ...)

Contrary Predicates

• “happy” and “unhappy” are instead said to be contrary predicates.

Something cannot be both happy and unhappy, but it need not be either
Fallacy of False Dichotomy

- Mistaking contraries for logically exhaustive complementaries

- Also called “polarized thinking” or “binary thinking”

Consider: “You are either with us or you are against us.”
Categorical Logic:
Some Philosophical Background

- First developed by Aristotle in the 3rd century BCE.

- (False) Assumption: That all statements can be put into subject/predicate form.

Consider:

"Either Martin will win or Harper will win”

"If Harper wins, our country is doomed."

These clearly have truth values, but they are not subject/predicate statements.

Individuals

- Also, categorical logic cannot easily deal with individuals:

  "George W. Bush has two daughters."

- We could get around this in CL, by creating a class that contains only one individual:

  "All persons identical with George W. Bush are persons that have two daughters."

...
In propositional logic, however, this sort of move can be avoided by symbolizing logical individuals directly.

(B . D)

Existential Import

- Classical (or Aristotelian) CL assumes that we only make assertions about things that are real (i.e., that exist).

For Aristotle, the idea of speaking about non-existent things was irrational.

So, if we assert in traditional categorical logic:

“All humans are mortal” (A)

This carries with it the assumption that human beings do in fact exist.
Hypothetical (Boolean) Interpretation

- The existential assumption may seem harmless enough in everyday contexts, but consider:

"All black holes are invisible"

A scientist who makes this assertion may not know for certain that black holes exist. Instead, she claims only that if black holes exist, they will be invisible. This latter claim has a truth value (i.e., it can be true or false) whether or not black holes exist.

- So, in modern (non-Aristotelian) logic, universal statements are understood as hypothetical statements:

"All humans are mortal" ->

"If anything is a human being, then that thing is mortal"

On the hypothetical interpretation, the existence of the subject term is not assumed.
Similarly,

"(All) unicorns have a single horn"

On a hypothetical interpretation, this statement has a truth value even though no unicorns exist (we think).

On the existential interpretation it, weirdly, commits us to the claim that they do exist.

Particulars

- In both modern and Aristotelian logic, particular statements are understood to assert existence.

- *BUT*: On a hypothetical interpretation of, say, an A statement, we cannot validly infer an I statement from it.

  "All mermaids have no legs" (A)

  "Some mermaids have no legs" (I)
That is, on the modern view, we cannot validly infer actual existence from hypothetical existence.

This may seem strange in some practical contexts. Consider Govier’s example ...

(a) “All lawyers are rich people”

(b) “Some lawyers are rich people”

It seems natural to think that if all lawyers are rich, then some of them are.
How to Deal With This Difference

- In this course, our main task is to gain skill in interpreting and evaluating “real world” natural language arguments.
  
  So, in that context, we need not be ‘purists’ about either the modern or the Aristotelian views.

- In interpreting (and translating arguments), Govier advises us to pay attention to the context of the argument and, if necessary, to ‘read in’ the existential assumption if that best makes sense of the argument.
The Categorical Syllogism

- An argument with two premises and a conclusion, all of which are statements in categorical form.

In a valid CS, each of the categories is (must be) mentioned in two different statements. E.g.,

All men are mortal
Socrates is a man
Therefore,
Socrates is mortal

Govier’s example

1. All consistent vegetarians are opposed to using animals for leather.
2. No opponents of using animals for leather are fur trappers
Therefore,
3. No consistent vegetarians are fur trappers.
Formalized ...

1. All C are O
2. No O are T
Therefore
3. No C are T

T = major term (predicate in conclusion)
C = minor term (subject in conclusion)
O = middle term (appears in both premises but not in the conclusion)

Categorical Syllogisms and Venn Diagrams

- Venn diagrams can be used both i) to represent categorical syllogisms and ii) to determine their deductive validity.

It’s almost magical:

Once we have successfully represented the premises in the diagram, we can simply look to see if the conclusion is also represented ...
area 1: S, not P, not M
area 2: S, not P, M
area 3: S, P, not M
area 4: S, P, M
area 5: not S, P, not M
area 6: not S, P, M
area 7: not S, not P, M
area 8: not S, not P, not M
Diagramming Categorical Syllogisms

1. Identify and represent the major (S), minor (P) and middle (M) terms with circles corresponding to each category.

2. Represent universal premises

3. Represent particular premises
   (This last step can be tricky)

Some examples ...

1. All C are O
2. No O are T
   Therefore,
3. No C are T
1. All whales are mammals
2. All mammals are warm-blooded creatures
   Therefore,
3. All whales are warm-blooded creatures

1. All W are M
2. All M are C
   Therefore,
3. All W are C

1. All philosophers are liars
2. Some Greeks are philosophers
   Therefore,
3. Some Greeks are liars

1. All P are L
2. Some G are P
3. Therefore,
4. Some G are L
1. Some persons who pursue extreme sports are snowboarders
2. Some snowboarders are persons who enjoy taking risks

Therefore,
3. Some persons who pursue extreme sports are persons who enjoy taking risks

1. Some E are S
2. Some S are R
3. Therefore,
4. Some E are R

Rules of the Categorical Syllogism

- Besides using Venn diagrams, we can also assess the deductive validity of categorical syllogisms by applying the five rules of the syllogism.

If none of these rules are violated (i.e., broken), then we can conclude that the syllogism is a valid deductive argument.

First, a little review ...
Distribution of Terms

- You’ll recall that we said a term of a categorical statement is distributed if that statement asserts something about all members of the category to which that term refers.

A: Subject is D; Predicate is ~D
E: Subject is D; Predicate is D (both)
I: Subject is ~D; Predicate is ~D (neither)
O: Subject is ~D; Predicate is D

Recall:

“All whales are warm-blooded creatures” (A)

Says something about all whales (namely, that they are warm-blooded), but it does not say anything about all warm-blooded creatures.

The subject term of an A statement is distributed, but its predicate term is not distributed.
“No reptiles men are warm-blooded” (E)

Says something about all reptiles (namely, that none of them are warm-blooded) and it also says something about all warm-blooded creatures (namely, that none of them are reptiles).

Both the subject and the predicate terms of an E statement are distributed.

“Some dogs are friendly” (I)

Some dogs ≠ all dogs, so the statement does not say anything about all members of its subject category. It also does not anything about all friendly things (it says only the category dogs and the category friendly things overlap)

So, in an I statement neither the subject nor the predicate term is distributed.
“Some dogs are not friendly” (O)

Some dogs ≠ all dogs, so the statement does not say anything about all dogs. But this statement does say something about the entire category of friendly things (namely, that there is at least one thing – a non-friendly dog – that is excluded from that category).

So, in an O statement, the subject term is not distributed, but the predicate term is distributed.

The Middle Term and the Fallacy of the Undistributed Middle

- The middle term, we just saw, is the term which occurs in both premises, but not in the conclusion, of a categorical syllogism.

- For a categorical syllogism to be valid, its middle term must be distributed in at least one of its premises.

Consider ...
1. No Bush supporters are pacifists
2. All pacifists are Democrats
   Therefore,
3. No Bush supporters are Democrats

“Pacifists” is the middle term; it is distributed in both premise 1 (E) and premise 2 (A).
*Valid.*

1. All logic students are persons who political debate
2. Some Republicans are persons who enjoy political debate
   Therefore,
3. Some Republicans are logic students

“persons who enjoy political debate” is the middle term; it is not distributed in either premise 1 (A) or in premise 2 (I). *Invalid*
Rules of the Categorical Syllogism

For a syllogism to be valid,

1. Its middle term must be distributed in at least one premise.

2. No term can be distributed in the conclusion unless it is also distributed in at least one premise.

3. It must have at least one affirmative premise.

4. If it has a negative conclusion, it must also have a negative premise. Conversely, if it has one negative premise, it must also have a negative conclusion.

5. If it has two universal premises, it cannot have a particular conclusion. *

The fifth rule (If it has two universal premises, it cannot have a particular conclusion), you’ll note, follows from the hypothetical interpretation of universal categorical statements.

Apart from that qualification, all of the other rules apply under either interpretation.
Govier’s (Possibly Too Cute) Example:

1. All invalid syllogisms contain an error
2. This syllogism contains an error
Therefore,
3. This syllogism is invalid

Notice that in this self-referential syllogism all of the premises and the conclusion are true (A is satisfied), but because it is invalid R and, therefore, G are not satisfied.

Enthymemes

- An incompletely expressed categorical syllogism, one in which a premise or the conclusion is unstated.

  The bigger the burger, the better the burger
  The burgers are bigger at Burger King
Enthymemes give rise to the problem (which we have encountered several times before) of when it is appropriate to supply a missing conclusion or premise when translating, formalizing or evaluating natural language arguments.

1. All persons who love children are persons who would make a good kindergarten teacher.
2. Sue loves children.
   Therefore,
3. Sue would make a good kindergarten teacher.

Enthymemes & Sorites

Enthymemes are often found strung in a pattern called a sorites (< Greek “heap”)

A typical pattern:

1. All A are B
2. All B are C
3. All C are D
4. All D are E
   Therefore,
5. All A are F
Since a categorical syllogism, by definition, has two premises, a sorites has between three and \( n \) (i.e., there is no upward limit).

To see if a sorites is valid, we can break it up into a succession of enthymemes ...

1. All A are B
2. All B are C
   So,
   2x. All A are C (valid)

2x. All A are C
3. All C are D
   So,
   3x. All A are D (valid)

... and so on through to “All A are F.”