Long-run economic growth

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Outline

1. Introduction
2. Accumulation of capital
3. Population growth
4. Technical progress
5. Problems of the model
6. Economic policy

International Differences in the Standard of Living: 1999

Country	Income per Person (in U.S. dollars)	Country	Income per Person (in U.S. dollars)
United States	$31,910	China	3,550
Japan	25,170	Indonesia	2,660
Germany	23,510	India	2,230
Mexico	8,070	Pakistan	1,860
Russia	6,990	Bangladesh	1,530
Brazil	6,840	Nigeria	770

Source: World Bank.

Introduction

$$Y = BK^{\alpha}L^{1-\alpha}$$

Output can grow b/c

1. More capital
2. More labour
3. Better technology

Introduction

Standards of living (GDP per capita) can grow b/c

1. More capital (per person)
2. Better technology

Introduction

Kaldor's (1961) stylized facts:

1. Factor distribution of income shows no trend
2. GDP per capita exhibits steady and sustained growth
3. Ratio capital/output shows no trend
4. Real rate of return to capital shows no trend
5. Wages exhibit sustained growth

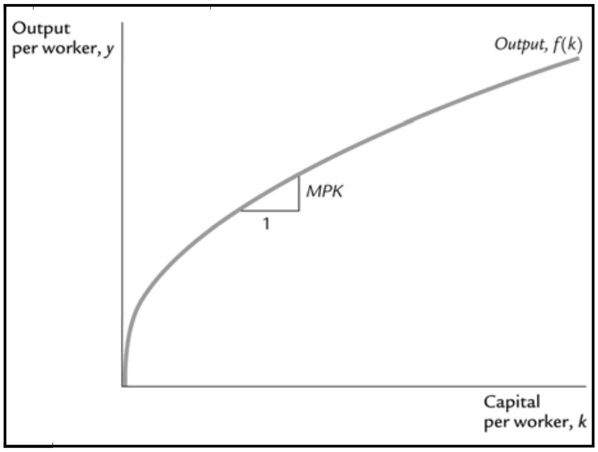
Accumulation of capital

Assumptions/simplifications:

- Do not differentiate private and public
- Labour force = population

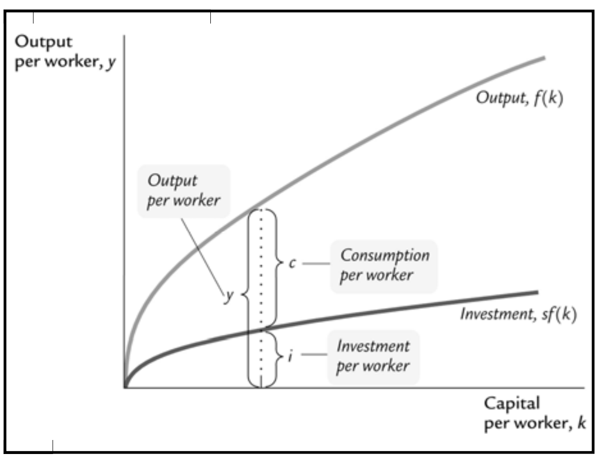
Accumulation of capital

- Capital letters indicate aggregate (total) values.
- Lower case letters indicate values in per capita terms.
- C-D production function $Y = K^{\alpha}L^{1-\alpha}$.
- In per capita terms
  $y = Y/L = K^{\alpha}L^{1-\alpha}/L = K^{\alpha}/L^{\alpha} = (K/L)^{\alpha} = k^{\alpha}$
- Shape due to diminishing marginal returns to capital.



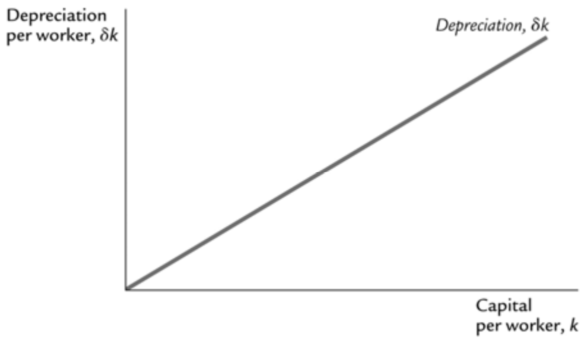
Accumulation of capital

- Assumption: saving = constant proportion of income σ (exogenous)
- $I = S = \sigma \bullet Y$
- In per capita terms $i = s = \sigma \bullet y$



Accumulation of capital

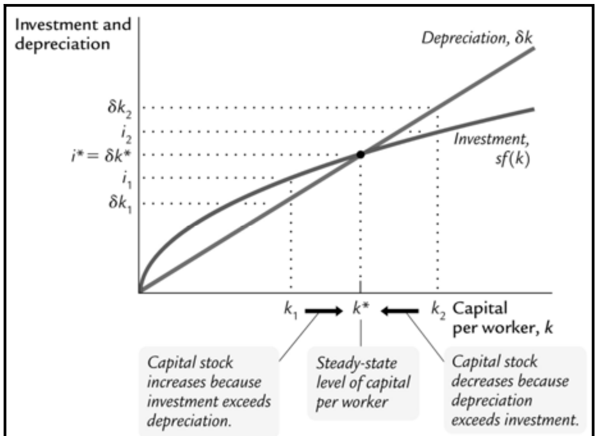
- d = depreciation ratio
- capital next period $K' = K - d \bullet K + I$
  $\Delta K = K' - K = I - d \bullet K$
- In per capita terms: $k' = k - d \bullet k + i$
  $\Delta k = k' - k = i - d \bullet k$



Accumulation of capital

Steady state (SS) = situation in which all variables are constant.

- If net investment = zero, capital will be constant → output, consumption and saving will be constant.
- Net investment = zero when $\sigma \cdot y^* = d \cdot k^*$ (SS condition)
- Asterisks denote SS values.



Accumulation of capital

Calculating the SS.

$\sigma \cdot k^{\alpha} = d \cdot k$

$\sigma / d = k^{1-\alpha}$

$k = (\sigma/d)^{1/(1-\alpha)}$

$y = (\sigma/d)^{\alpha/(1-\alpha)}$

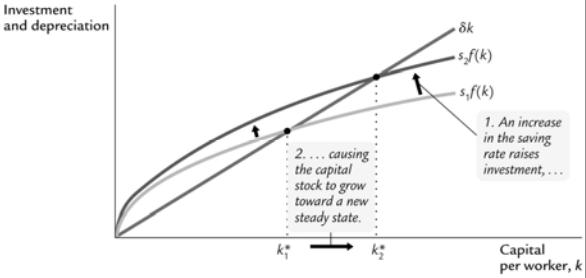
Accumulation of capital

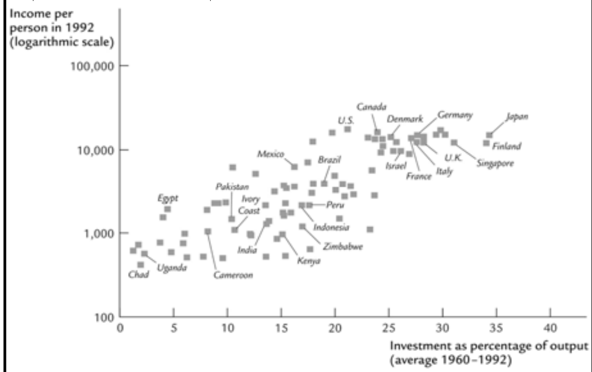
- There is a tendency to go to the SS.
  - if $k < k^*$, economic growth
  - if $k = k^*$, no growth
  - if $k > k^*$, negative growth

Accumulation of capital

An increase in the saving rate.

- Economic growth for a while
- Eventually the economy goes back to no economic growth.
- In the new SS: more k and more y → the higher investment rate, the higher output per capita.





Accumulation of capital

Moral.

1. Accumulation of capital does not explain sustained growth but ...
2. differences in investment rates explain differences in standards of living

Accumulation of capital

Numerical example.

- Calculate the SS if σ = 30%, d = 10% and α = ½

  $k^* = (0.3/01)^2 = 9$

  $y^* = 9^{1/2} = 3$

  $s^* = 0.3 \bullet 3 = 0.9 = 0.1 \bullet 9$

  $c^* = 0.7 \bullet 3 = 2.1$

Accumulation of capital

- Transition: What happens if the starting k = 4?

	k	y	g	s	c	dk	Δk
1	4	$4^{0.5}=2$		0.3•2= 0.6	0.7•2= 1.4	0.1•4= 0.4	0.60.4= 0.2
2	4+0.2= 4.2	2.049	(2.05-2)/2= 2.5%	0.615	1.435	0.42	0.195
3	4.395						

Approaching the Steady State: A Numerical Example

Assumptions: $y = \sqrt{k}$; $s = 0.3$; $\delta = 0.1$; initial $k = 4.0$

Year	k	y	c	i	δk	Δk
1	4.000	2.000	1.400	0.600	0.400	0.200
2	4.200	2.049	1.435	0.615	0.420	0.195
3	4.395	2.096	1.467	0.629	0.440	0.189
4	4.584	2.141	1.499	0.642	0.458	0.184
5	4.768	2.184	1.529	0.655	0.477	0.178
. . .						
10	5.602	2.367	1.657	0.710	0.560	0.150
. . .						
25	7.321	2.706	1.894	0.812	0.732	0.080
. . .						
100	8.962	2.994	2.096	0.898	0.896	0.002
. . .						
∞	9.000	3.000	2.100	0.900	0.900	0.000

Accumulation of capital

- Calculate the SS if σ = 40%

  $k^* = (0.4/01)^2 = 16$

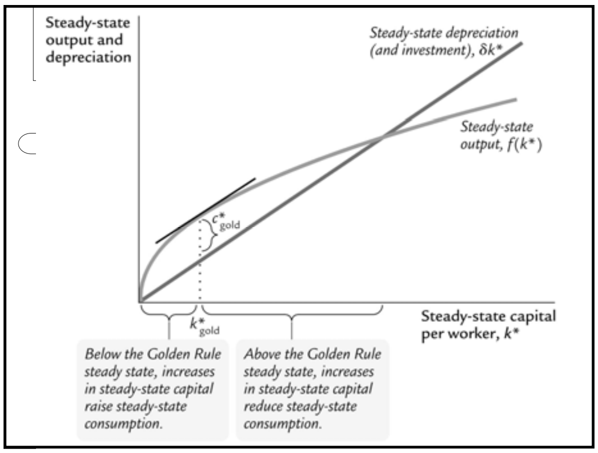
  $y^* = 16^{1/2} = 4$

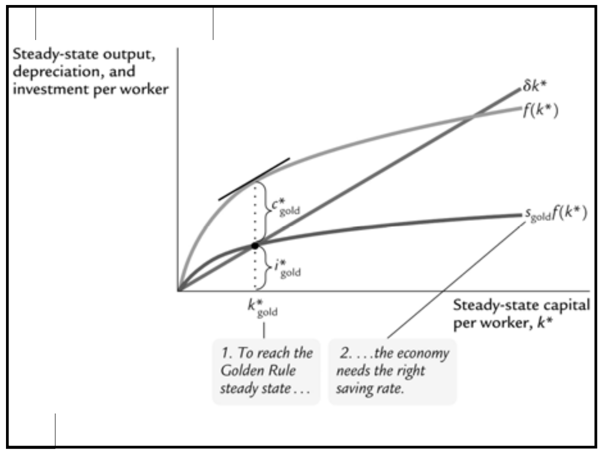
  $s^* = 0.4 \cdot 4 = 1.6 = 0.1 \cdot 16$

  $c^* = 0.6 \cdot 4 = 2.4$

Accumulation of capital

- **Golden Rule (GR):** SS at which consumption is maximized.
- Graphically, consumption is maximized when $MPK^{**} = d$
- Double asterisk denotes GR values.





Accumulation of capital

- Analytically (you do not need to know the proof):

  $c^* = y^* - d\,k^* = y(k^*) - d\,k^* = c(k^*)$

  Maximize c over k^*

  $y'(k^*) - d = 0 \rightarrow \quad MPK^* = d$ (condition)

1. MPK anywhere: $MPK = \alpha k^{\alpha-1}$
2. MPK at any SS: $MPK^* = \alpha\, ((\sigma/d)^{1/(1-\alpha)})^{\alpha-1}$

   $= \alpha\, (\sigma/d)^{-1} = \alpha\, (d/\sigma)$

   Substitute in condition $\alpha\, (d/\sigma) = d \quad \sigma^{**} = \alpha$

- Golden rule: investment rate should equal α (capital intensity).

Accumulation of capital

Numerical example

- Calculate the SS for the following σ: 0. 1, 0.2, 0.3, 0.4, 0.5 and 0.6.

Finding the Golden Rule Steady State: A Numerical Example

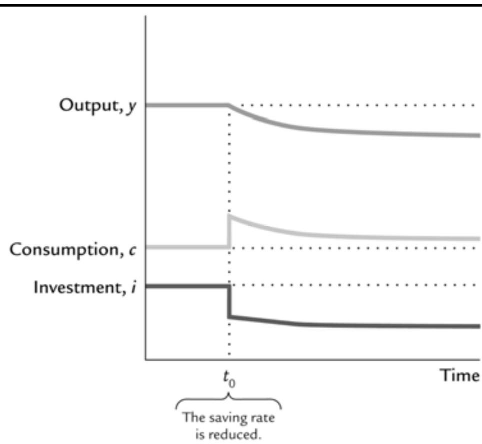
Assumptions: $y = \sqrt{k}$; $\delta = 0.1$

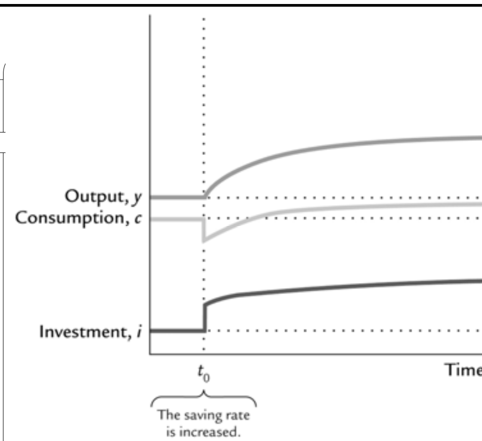
s	k^*	y^*	δk^*	c^*	MPK	$MPK - \delta$
0.0	0.0	0.0	0.0	0.0	∞	∞
0.1	1.0	1.0	0.1	0.9	0.500	0.400
0.2	4.0	2.0	0.4	1.6	0.250	0.150
0.3	9.0	3.0	0.9	2.1	0.167	0.067
0.4	16.0	4.0	1.6	2.4	0.125	0.025
0.5	**25.0**	**5.0**	**2.5**	**2.5**	**0.100**	**0.000**
0.6	36.0	6.0	3.6	2.4	0.083	−0.017
0.7	49.0	7.0	4.9	2.1	0.071	−0.029
0.8	64.0	8.0	6.4	1.6	0.062	−0.038
0.9	81.0	9.0	8.1	0.9	0.056	−0.044
1.0	100.0	10.0	10.0	0.0	0.050	−0.050

Accumulation of capital

Problem with the Golden Rule

- Saving too much = all generations better off
- Saving too little = current generation needs to sacrifice for future generations.
- Try problem 1 in chapter 8 (p. 268).





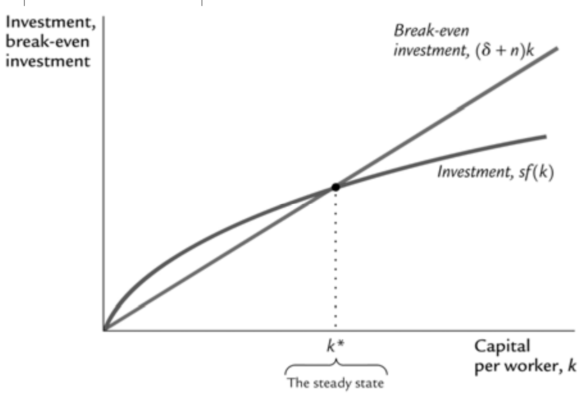
Population growth

Properties of growth rates.

1. If a variable is the product of two variables, its growth rate is the sum of their growth rates.
2. Conversely, if a variable is the quotient of two variables, its growth rate its the subtraction of their growth rates

Population growth

- Assumption: Population (labour force) growth rate = n (exogenous)
- (d + n) = adjusted depreciation to maintain k



Population growth

- **SS:** $i = (d+n) \bullet k$

  $\delta \bullet y = (d+n) \bullet k$

  Substituting

  $\sigma \bullet k^{\alpha} = (d+n) \bullet k$

  $\sigma/(d+n) = k^{1-\alpha}$

  $k = (\sigma/(d+n))^{1/(1-\alpha)}$

  $y = (\sigma/(d+n))^{\alpha/(1-\alpha)}$

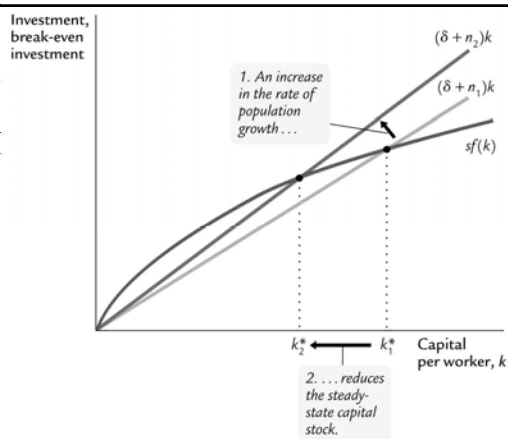
Population growth

- Per capita variables are constant.
- Total (aggregate) variables are growing.
  - $Y = y \bullet L \rightarrow \Delta Y/Y = \Delta y/y + \Delta L/L = 0 + n = n$
  - $K = k \bullet L \rightarrow \Delta K/K = \Delta k/k + \Delta L/L = n$

Population growth

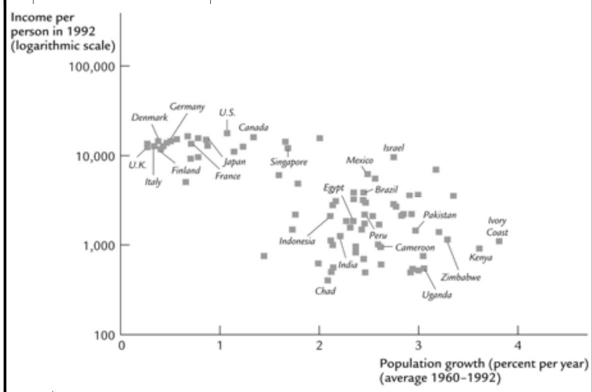
Comparative statics:

- Population growth increases → Savings not enough to maintain k → economy moves to a lower SS
- During transition: negative growth in income per capita (not necessarily in total output).
- Model predicts an inverse correlation between population growth rate and standards of living; supported by data.



Population growth

- Golden rule not affected.
- Economy tends to a SS in per capita terms
- **Summary**: Population growth explains sustained growth in GDP (total output); it does **not** explain sustained growth in GDP per capita.



A useful property

- $y = k^{\alpha}$
- $MPK = \alpha \bullet k^{\alpha-1} = \alpha \bullet k^{\alpha}/k = \alpha \bullet y/k$
- We have already seen this:

  $$r K = \alpha Y$$

Technological progress

- "Labour augmenting" = increases efficiency of labour.
- (Reason why technological progress increases wages)
- Efficiency of labour also increased b/c education (accumulated as capital)
- Exogenous: unexplained. Rate fairly constant = g (≈ 2%)

Technological progress

- **Balanced Growth Path (BGP):** per capita variables all grow at the rate g.
  - $\Delta y/y = g$
  - $\Delta k/k = g$
- Total (aggregate) variables grow at the rate g+n
  - $Y = y \bullet L \rightarrow \Delta Y/Y = \Delta y/y + \Delta L/L = g + n$
  - $K = k \bullet L \rightarrow \Delta K/K = \Delta k/k + \Delta L/L = g + n$

Technological progress

- **Question**. Suppose that y = 10 today and it is growing at an average annual rate of 1.5%. What will y be in 100 years?

Technological progress

- Growth rate determined by rate of technological progress.
- Higher investment rate means higher standards of living in the SS,
- Same with lower rate of population growth.
- Tendency to go to the SS.

Technological progress

- Golden rule does not change: consumption at the SS is maximized when investment rate equals α (between 30% and 40%)

The Kaldor facts

1. Use C-D to replicate 1
2. At the BGP y grows at rate g.
3. If K & Y grow at the same rate, K/Y constant.
4. Since $r = \alpha \bullet Y/K$, r constant
5. Since $\omega L = (1-\alpha)Y$, $\omega = (1-\alpha)y$, ω grows at the rate g.

Problems of the model

- Very useful model: around since the 1950s.
- Main shortcoming: taking technological progress as exogenous.

Economic policy (recommendations)

1. Decrease population growth. More relevant to LDCs
2. Increase investment rate
   1. Increase taxes. •
   2. Decrease public consumption.
   3. Increase public investment:
3. Increase technological progress.
   1. Build human capital
   2. Encourage R&D