4 Homework #2

1. **Risk premium.** (This is a rewording of exercise 1 in page 586 of Ray's textbook). In each of cases (a)–(d), assume that you always have the option to keep extra funds in the bank at a 10% rate of interest with no fear of losing any of these funds. For each case, calculate the minimum rate of interest and, therefore, the risk premium, at which you would lend $1,000 on the informal market.

   (a) With probability 1/2 the loan will be repaid with interest, and with probability 1/2 the loan will not be repaid at all.

   The interest rate i needs to be such that

   $$\frac{1}{2}1000(1+i) = 1000 \times 1.1.$$

   Thus, $i = 1.2 = 120\%$ which implies a risk premium of 110%.

   (b) With probability 1/2 the loan will be repaid with interest, and with probability 1/2 only the principal will be repaid.

   The interest rate needs to be such that

   $$\frac{1}{2}1000(1+i) + \frac{1}{2}1000 = 1000 \times 1.1.$$

   Therefore, $i = 0.2 = 20\%$ or a risk premium of 10%.

   (c) With probability 1/3 the loan will be repaid with interest, with probability 1/3 only the principal will be repaid, and with probability 1/3 the loan will not be repaid at all.

   The interest rate needs to be such that

   $$\frac{1}{3}1000(1+i) + \frac{1}{3}1000 = 1000 \times 1.1.$$

   So $i = 1.3 = 130\%$ or a 120% risk-premium.

   (d) With probability 1/2 the loan will be repaid with interest, and with probability 1/2 the loan will not be repaid but there is a 1/2 probability of recovering assets from the borrower worth $500.

   The interest rate needs to be such that

   $$\frac{1}{2}1000(1+i) + \frac{1}{2}\frac{1}{2}500 = 1000 \times 1.1.$$

   In this case, $i = 0.95 = 95\%$ or an 85% risk-premium.

Table 1

Loan size ($)	**Loans defaulted (%)**
50-99	5
100-149	10
150-199	20
200-249	25
250-300	30
>300	50

2. **Credit rationing**. (This is a rewording of exercise 2 in page 586 of Ray's textbook.) Table 1 gives default risks for various loan sizes in an informal credit market. Suppose that the rate of interest in the informal sector is 18% per year and that in the formal sector (with no default) is 10% per year.

   (a) Calculate the maximum loan size that will be offered in the informal-sector credit market. (Show your work)

   You need to calculate a probability of default π such that

   $$1.18 \times P \times (1 - \pi) = 1.1P,$$

   where P denotes the principal. Thus, $\pi = 6.8\%$: if the probability of default is lower than that, the risk is worth taking. Only loans in the first category have a lower probability of default; therefore, the maximum loan size is $99.

   (b) For what minimum rate of interest will loans in the $250-300 category be offered?

   The interest rate needs to be such that

   $$0.7P(1 + i) = P \times 1.1;$$

   i.e., $i = 0.571 = 57.1\%$.

3. In the town of Ondartza the harvest can take two values: $3,000 if all works well and $1,000 if there is some damage to the crop. With greater care and better application of inputs, each farmer can produce the better output with probability 0.7, but if he underapplies inputs the probability falls to 0.4. Suppose that their utility function is given by $u(x) = x$. Finally, let the additional cost of diligence assume the value 500 in utility units.

   (a) Check that in isolation each farmer will indeed put in the higher level of effort.

   The expected utility if the farmer puts the higher level of effort equals

   $$0.7 \times 3000 + 0.3 \times 1000 - 500 = 1900$$

and this is greater than the expected utility if the farmer puts the lower level of effort,

$$0.4 \times 3000 + 0.6 \times 1000 = 1800$$

(b) Explain the incentive problem that perfect insurance faces in this case. What is the solution to this problem?

Under perfect insurance each farmer receives the average yield,

$$0.7 \times 3000 + 0.3 \times 1000 = 2400,$$

whether they put the effort or not. If they put the lower level of effort, their utility is 2400 while it is 1900 if they put more effort. But if nobody makes the extra effort, the average yield is only 1800 so the insurance scheme is not sustainable.

The solution to this problem is to offer incomplete insurance so if a farmer's harvest is worth \$1000, the farmer will consume less than 2400, and if a farmer's harvest is worth \$3000 the farmer will consume more than 2400. In this way they have an incentive to put more effort.

(c) Solve for the second-best insurance scheme for the town of Ondartza.

It needs to fulfill two conditions:

i. feasibility:

$$0.7 \times X + 0.3 \times Y = 2400,$$

where X refers to consumption when the harvest value is high and Y when it is low

ii. incentive compatibility:

$$(0.7 - 0.4)(X - Y) = 500.$$

Solving for these two equations simultaneously yields

$$\begin{aligned} X &= 2900, \text{ and} \\ Y &= 1233. \end{aligned}$$

(The second best is far off from the first best)