Why a fixed workweek? *

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Why a fixed workweek?
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Abstract

The main goal of this article is to explain why the fixed workweek appeared. To this purpose we differentiate between "jobs" and "hours per job". We consider an economy where hours and number of workers are substitutes in production but in which hiring a worker entails a fixed cost plus a variable cost per hour worked. This fixed cost implies that, for firms, it is not equivalent to hire a worker for eight hours or eight workers for an hour. Therefore, firms would like workers to work as many hours as physically possible. In an unregulated economy, workers work more hours that they would like to at the ongoing wage rate. This situation characterizes the economy of today's industrialized countries in the 19th centuries.
1. Introduction

The main purpose of this article is to explain why the fixed workweek appeared and it is a feature of almost all industrialized countries. We consider an economy where hours and number of workers are substitutes in production but in which hiring a worker entails a fixed cost plus a variable cost per hour worked. This fixed cost implies that the total cost of hiring a worker for eight hours is not the same as that of hiring eight workers for an hour. Therefore, firms would like workers to work as many hours as physically possible. In an unregulated economy, workers work more hours that they would like to at the ongoing wage rate. This is so because the hourly wage determines not only the number of hours demanded per body but also the number of bodies demanded, given the workweek. Thus, the hourly wage that guarantees employment to all workers is not necessarily the same that makes number of hours demanded per person equal to number of hours supplied per person.

This situation characterizes the economy of today’s industrialized countries in the 19th centuries. Because the workers could not fight this situation, with which they were very dissatisfied, in an individual fashion, they organized themselves against the long working hours to institute a fixed workweek. In the words of Marx: "The establishment of a norm for the working day presents itself as a struggle over the limits of that day, a struggle between collective capital, i.e. the class of capitalists, and collective labour, i.e., the working class" (p. 344) It was this struggle which, allegedly, gave birth to the labour movement. Its first important battle was against the long workweek and its first victory was to institute a fixed workweek.

In this paper we compare the outputs of an unregulated economy and an economy in which a fixed workweek has been instituted but wages and other labour costs are such that unemployment does not exist. We also study the effects on output of varying the length of the workweek. Further we introduce some other labour market restrictions ("unions" in this article) that generate unemployment to study the
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Effects not only on output but also on unemployment of varying the length of the workweek.

That is, we analyze three economies:

1. A no regulated economy where cost per hour worked and the length of the workweek is competitively determined,

2. An economy where the workweek is fixed but the wage is competitively determined, and

3. An economy in which the workweek is fixed and unions fix the offer supply to maintain wages at a certain level.

Recent studies that show that subjective well-being or happiness does not increase with income levels in developed countries (see, for instance, Easterlin 2001 or the best-seller from Layard 2005) suggest that people would be better off if they worked less and had more leisure time. The usual explanation for this phenomenon is what is called the treadmill effect (see, for example, Binswanger 2006). The positional treadmill effect means that relative consumption matters and, if this is the case, consumption imposes a negative externality on neighbors. Fiscal policy to correct for this negative externality may not be politically feasible. In such an environment a regulated fixed workweek provides an alternative policy tool to induce a market outcome closer to the efficient allocation, as Álvarez Cuadrado (2006) explains.

We propose an alternative explanation for both the fact that the distribution of time between work and leisure does not correspond to workers’ wishes and, thus, for the rationale for a fixed workweek. Our explanation is on line with Alesina et al’s (2005) explanation of the difference in working hours between US and Europe which argues that this difference is due to the push of unions advocating "work less, work all".

There are some papers related to ours. For instance, Osuna and Ríos-Rull (2003) study the implications of taxing overtime work in
order to reduce the workweek. They use the standard framework found in Horstein and Prescott (1993). In their setup, technology uses capital, hours (the workweek) and bodies. There are constant returns to scale in capital and bodies but increasing returns to scale in the three factors. Thus, if firms are competitive, equilibrium does not exist. This is why they assume that firms demand workers for a given workweek. That is, firms offer a wage schedule. The length of the workweek is set by workers that act in a coordinated fashion: there is a trade union that offers a lottery—the probability of working for any possible workweek. In absence of such a union, equilibrium may not exist.

Osuna and Ríos-Rull (2003) find that taxing overtime work increases employment but reduces output and productivity and volatility of output in the business cycle frequency. The quantitative results depend on the degree of substitutability between hours and bodies. In their framework there is no perfect substitution between hours and bodies in the technology, as in ours, and working entails commuting costs for workers. These commuting costs depend on the size of the labor force and play a similar role to the personnel cost we assume here. Nevertheless, in our case personnel cost is born by entrepreneurs whereas in their framework workers bear the cost of commuting. Moreover, commuting time creates an externality, which makes the competitive equilibrium inefficient.

The critical difference between their setup and ours is the technology. In their framework there are increasing returns to scale in the workweek, whereas in ours there are not. Thus, equilibrium exists in our framework even in the absence of a trade union. The similarity between both frameworks is the friction in place: working or not is an indivisible commodity.

Ueberfeldt (2006) uses a setup very similar to that of Osuna and Ríos-Rull to account for the secular decline in the workweek. He finds that increases in labor productivity are largely responsible for such a decline.

Marimón and Zilibotti (2000) use a labor search framework where
creating vacancies entails a fixed cost for the firm. Thus, the cost of creating jobs plays a similar role to our personnel costs. Nevertheless, in their framework a firm can hire only one worker. This implies that all the trade-offs between hours and bodies are ignored at the plant level and play some role only at the aggregate level. In this search theoretical model the length of the workweek determines the amount of output produced by the plant to be split between the firm and the worker. Thus, shortening the workweek amounts to imposing an upper bound on the amount of output produced by the plant. Consequently, the wage has to decrease and more plants (more jobs) will be created if the wage is low enough. These results are straight implications of the assumptions made. In this setup, the lower the bargaining power of the worker, the longer the workweek. The problem is that Marimón and Zilibotti need a non-Walrasian mechanism to obtain this result, whereas we do not. There exists another key difference: in their framework, utility while not working is well defined whereas in our framework is not. That is, in Marimón and Zilibotti marginal utility when consumption is zero is finite, whereas in our setup is not. This assumption allows them to obtain the same result as they would have they assumed the existence of an unemployment subsidy.

Ortega (2003) also uses a labor search framework. Firms hire only one worker. They do so before knowing the demand for the final good produced. The firm and the worker bargain over the wage and, after the demand is revealed, the workweek is decided. Opening a vacancy entails a fixed cost. The problem with the last two papers is that they do not have a notion of a "too long workweek" as we do. Marimón and Zilibotti can argue that the length of the workweek depends on the workers bargaining power but the relationship is not monotone. Moreover, in their search framework there is always unemployment, which was negligible before the end of the 19th century, as we will see below.

The rest of the paper is organized as follows: the next section, section 2, looks at the interactions between labour relations and fixed workweek during the nineteenth century in the United States. Sec-
tion 3 sets the environment of the three models. The models follow a stylized chronological sequence (see next section): the first model in section 4 corresponds to the laissez faire economy of the 19th century; the second model in section 5 corresponds to a world in which the labour movement has already attained its vindication of a fixed workweek, however wages are fixed by supply and demand; in the last model in section 6 the labour movement not only has attained its vindication of a fixed workweek but also indirectly controls the wage level with the result of possible unemployment. We study the effects of shortening the fixed workweek in the last two cases. The last section, section 7, summarizes the results and delineates future lines of research.

2. Labour relations and fixed work week

2.1. Length of the workweek

In pre-industrial times, non-enslaved people in today’s industrialized countries generally worked fewer hours per year than they do today, though on a less regular cycle since agriculture demands a variable amount of effort over the course of the year. The industrial revolution made it easier to find work year-round, since labor was not tied to the season, and artificial lighting made work possible for the greater part of the day. Thus, technological advances during early capitalism made it possible to extract upwards of seventy hours per week of working time from a person. As Schor, a sociologist, states it: "Once capital is invested, its owner has strong financial incentives to see that it is used as intensively as possible." (1991, p. 59) Thus, before collective bargaining and worker protection laws, "eighteenth- and nineteenth-century Europe and America (...) witnessed what were probably the longest and more arduous work schedules in the history of humankind." (Schor, 1991, p. 6) Records indicate that work schedules as arduous as sixteen hours per day were demanded of wage earners, including children (Brown, 1982, p. 66, Marx p. 356).
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The decline of working time since mid-nineteenth century is not a consequence of the market system; on the contrary, it happened because workers join forces against the market forces since, in an individual fashion, the market determines a long work schedule.

The eventual recovery of leisure, the decline in working hours, came about because the labour movement fought a lengthy battle for shorter hours. The workweek, in most of the industrialized world, dropped steadily, to about forty hours after World War II. The table below indicates the approximate annual working hours across the U.K and the U.S over eight centuries.

<table>
<thead>
<tr>
<th>Year</th>
<th>Type of worker</th>
<th>Country</th>
<th>Annual hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>13th century</td>
<td>Adult male peasant</td>
<td>U.K.</td>
<td>1620</td>
</tr>
<tr>
<td>14th century</td>
<td>Casual labourer</td>
<td>U.K.</td>
<td>1440</td>
</tr>
<tr>
<td>Middle ages</td>
<td>English worker</td>
<td>U.K.</td>
<td>2309</td>
</tr>
<tr>
<td>1400-1600</td>
<td>farmer-miner (male)</td>
<td>U.K.</td>
<td>1980</td>
</tr>
<tr>
<td>1840</td>
<td>Average worker</td>
<td>U.K.</td>
<td>3105-3588</td>
</tr>
<tr>
<td>1850</td>
<td>Average worker</td>
<td>U.S.</td>
<td>3150-3650</td>
</tr>
<tr>
<td>1987</td>
<td>Average worker</td>
<td>U.S.</td>
<td>1949</td>
</tr>
<tr>
<td>1988</td>
<td>Manufacturing workers</td>
<td>U.K.</td>
<td>1856</td>
</tr>
</tbody>
</table>


In Britain, and according to Brown (1982), the agitation first began in 1831.

In 1847, John Fielden finally secured the passage of a bill restricting the hours of young persons and females in textile factories to ten a day. Subsequent measures blocked loopholes and extended protection to the same vulnerable groups in other industries. The Factory Act (extension) of 1967 led to the application of existing legislation to all factories employing more than fifty workers and also to specified places of work such as iron and steel mills, blast furnaces, and paper and tobacco works. In the same year
The Workshops Regulation Act was passed, covering all establishment with workforce of under fifty. (p. 160)

He continues (p. 165): "The legislation of the early 1879, like the earlier repeal of the Combination Acts, created a climate conducive to union growth. The spread of unionization, especially among previously unorganized groups of workers, was encouraged mainly by the boom conditions which characterized the first half of the decade. ... The immediate spur seems to have been the long and successful strike conducted in 1871 by north-easter engineers to secure a nine-hour day. It triggered off an astonishing response." Thus, he ties the struggle for a shorter day to the spread of unionization and, later on (p. 173), he remarks on the emphasis the trade unions placed on state intervention in labour matters, on the importance for the trade unions of legal regulations of the work schedule.

In the United States, in the early 1830s the average workweek was 69 hours. At the end of the century was about 59 hours per week. The cause was a decline in daily hours: from 11 hours and 20 minutes in 1832 to just over ten hours in 1880 (Margo 2000).

2.2. Unions and government regulation in the Nineteenth century in the United States

According to Margo (2000, p. 231),

In explaining the decline in weekly hours, historians have traditionally emphasized the twin roles of organized labour and the state. According to this view, employers steadfastly resisted a decline in the weekly hours and they could only be convinced by strike or government edict. The union push for shorter hours essentially began in the late 1820s and early 1830s as workers in Philadelphia, Boston and New York clamored for a 10 hour day ... leading to the passage of the first "maximum hours" laws in New Hampshire (1847) and Pennsylvania (1849).
President van Buren established a ten hour day for manual labour in 1840. After the American Civil War the vindication of the organized labor changed form the ten hour to the eight hour day. "Strikes for shorter hours became more common, reaching a peak in the mid-1880s." (Margo, 2000) In fact, as Alesina and Glaeser (2004, p. 124) remind us, "the international workers' holiday of May 1 commemorates the Chicago Haymarket riot of 1886."

Margo continues (p. 235): "the first recognizable attempt at a labour movement in the United States occurred in the 1820s." It took the form of union organizing and great frequency of strikes. It embraced causes such as shorter hours, higher wages, and better working conditions. "Despite their limited penetration into the labor force, antebellum labor organizations were far from total failures. Many strikes did raise wages, forestall wage cuts, reduce hours, and improve working conditions." (p. 236)

Unemployment starts being a problem in the States in the late 19th century. Since there was no unemployment insurance, workers would rely on savings, odd jobs, the earnings of other family members, relatives, friends, churches, and unions. According to Margo, "in occupations or locations in which unemployment was predictable, wages were higher: unemployment risk commanded a wage premium" (p. 242). However, the causality might have been the other way around. What seems relevant is that there was a correlation between unemployment and higher wages.

A stylized chronological sequence of the labour movement in the States is as follows: labour gets organized pushing for shorter hours at the beginning of the 19th century. Although formal unionization was never large, unions were successful at organizing labour. By the end of the century unemployment is a fact, probably brought up by higher wages. The unemployed would survive by using their social network, including unions. "The earliest known (trade union) plan was established in 1831... practically all trade unions gave assistance to their unemployed members and many unemployment benefit relief plans sprang up" (Blaustein 1993, p. 108). There were also joint
agreement plans between unions and management and some company plans started in 1917, almost at the same time that the first attempts to enact state unemployment insurance. "As the depression of the 1930s deepened, an increasing number of progressive reformers and representatives sensitive to workers interests were elected to governorships and state legislatures." (Blaustein 1993, p. 107). These voluntary schemes plus the labour movement push through the above mentioned representatives lead to the passage of the Social Security Act in 1935.

3. The environment

There are $N$ workers and $m$ entrepreneurs. We take both numbers as exogenously given. Each entrepreneur has available a technology that uses only labor. According to this technology, output equals

$$a(n) \cdot (nh)^\theta, \theta \in (0, 1),$$

where $h$ is hours per worker and $n$ is the number of workers. The factor $a(n)$ is a productivity factor that results from firm expenditures at the workplace, which we are going to call personnel costs. The productivity factor also captures the impact of capital, assumed away.²

Personnel costs are a function of the number of workers, $t(n)$. Since our model is just a one-period model, they encompass the annual cost of training³ plus the cost of recruiting and hiring and other fixed costs that need to be incurred every production period. These costs nowadays include pensions, health and life insurance, paid vacations and taxes, although this was not the case in the nineteenth century.

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¹Assuming away capital and taking the number of entrepreneurs as given are two assumptions that go hand in hand if we think of an entrepreneur as being somebody with enough savings (rather than somebody with special skills).

²Behind this assumption we have in mind a question of timing: the investment decision is taken prior to the hiring decision; thus, in a one-period model, labour is the only variable factor of production.

³That would be the total cost prorated among the years in which the benefits of training accrue.
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For simplicity of exposition we assume that $a(n) = an^\alpha$, (where $\alpha \in (0,1)$, $\alpha + \theta < 1$, $\alpha + 2\theta \leq 1$) and that personnel costs are linear, $t(n) = tn$.

Workers have preferences on consumption and leisure. We assume that their preferences are represented by the utility function

$$u(c, l) = \frac{c^\rho}{\rho} + \gamma \frac{l^\rho}{\rho},$$

where $\rho \in (0, 1)$. Workers have a maximum amount of available time equal to $\bar{h}$. Therefore, $0 \leq l \leq \bar{h}$. Entrepreneurs maximize profits.

4. The laissez-faire economy

The basic model would characterize the situation during the 19th century in which there were no unions, no unemployment insurance and no legislation over the workweek.

Entrepreneurs maximize on $h$ and $n$ the following objective function:

$$an^{\alpha+\theta}h^{\theta} - whn - tn,$$

where $w$ refers to compensation (wages plus benefits) henceforth called wages. The solution to this problem is given by

$$n^d = a^{\frac{1}{1-\alpha-\theta}} \left(\frac{\alpha}{t}\right)^{\frac{1-\theta}{\alpha}} \left(\frac{\theta}{w}\right)^{\frac{\theta}{1-\alpha-\theta}},$$

and

$$h^d = \frac{\theta t}{\alpha w}. (1)$$

On their turn, the workers

$$\max_{c,h} \frac{c^\rho}{\rho} + \gamma \frac{(\bar{h} - h)^\rho}{\rho}$$

s.t. $c \leq wh$. (2)
As a result, the individual labor supply is
\[ h^* = \frac{\gamma^{\frac{1}{\rho-1}} n}{w^{\frac{1}{\rho-1}} + \gamma^{\frac{1}{\rho-1}}}. \]

Notice that, since there is no unemployment insurance, workers always want to work and labor supply is an increasing function of the wage, \[ h^* \to \bar{h}, \] given the assumption \( \rho \in (0, 1). \)

Before characterizing the equilibrium, it will be useful to compare our setup with other approaches to study the effects of changes in the length of the workweek (see, for instance Fitzgerald 1998 or Osuna and Ríos-Rull 2003). Hornstein and Prescott (1993) set the framework and assume that a firm (actually, a plant in their theory) produces final output with the technology
\[ y = h^{\xi-\phi} k^{1-\phi} (n h)^{\phi}, \quad \phi \in (0, 1). \]

In this setup, \( n \) is the number of workers and \( k \) is the stock of capital. That is, \( k^{1-\phi} n^\phi \) is the production per hour and \( h \) is the number of hours the plant is operated. If we take the stock of capital as given, and call it \( a = k^{1-\phi} \) the technology becomes \( y = a h^{\xi-\phi} n^\phi \), which is equivalent to ours. In Hornstein and Prescott (1993) and Fitzgerald (1998), \( \xi = 1 \). This is the case called “team production” in the literature—production per hour worked does not change with the length of the workweek. Osuna and Ríos-Rull (2003) calibrate their economy so that \( \xi < 1 \), rationalized as “fatigue”, and assume \( \xi + \phi > 1 \). In either case, there are increasing returns to scale in hours and bodies. Our case is one in which \( \xi + \phi < 1 \); that is, there is “severe fatigue”. Although both frameworks may seem equivalent, they are conceptually different. In the Hornstein and Prescott’s framework, productivity at the plant depends on the workweek (in our notation, \( h^\alpha (nh)^\theta \), \( \theta \in (0, 1) \)) whereas in our framework it depends on the number of workers. Thus, in the Horstein and Prescott’s framework, \( \xi \geq \phi \), whereas we assume \( \xi < \phi \). The issue is, then, quantitative: since there are no reliable estimates for \( \xi \), we do not have a way of discriminating between both theories of production.
4.1. Equilibrium

Proposition 1 In equilibrium, the wage is such that \( n_d m = N \).

Proof. Let us denote as \( \bar{w} \) the wage at which \( n_d m = N \). The market wage cannot be below \( \bar{w} \) because in that case entrepreneurs would be rationed and they would have incentives to offer a higher wage. Let us think of the case \( w > \bar{w} \). In this case, the demand would be lower than the number of workers willing to work (remember that they always want to work). Thus, unemployed workers would have incentives to supply hours at a lower wage. Hence, in equilibrium \( w = \bar{w} \).  

Since in equilibrium \( n_d = N/m \), the equilibrium wage rate is a negative function of this ratio: the higher the number of workers in relation to the number of firms, the lower the equilibrium wage rate and vice-versa.

Proposition 2 Let us denote as \( h^d \) the number of hours demanded at the wage \( \bar{w} \) and \( h^* \) the hours that maximize worker’s utility at that wage. If \( h^d \) \( \leq \) \( h^* \), then hours worked per worker \( h = h^d \). If \( h^d > h^* \), then \( h = \min\{h^d, \bar{h}\} \).

Proof. The first part of the proposition is trivial: given the wage \( \bar{w} \), hours worked in equilibrium cannot be higher than the hours demanded at that wage. In the second case, \( h^d > h^* \), if hours worked in equilibrium were equal to \( h^* \) competition among entrepreneurs would drive the wage above \( \bar{w} \). In that case, not all workers would be employed, which would drive the wage down again.

If the wage \( \bar{w} \) is such that hours demanded are fewer than hours offered, hours traded would equal hours demanded, as usual. However, if the wage \( \bar{w} \) is such that hours demanded are greater than what workers would like to work, in equilibrium workers work more hours than desired, contrary to the usual solution.\(^4\)

\(^4\)Notice that we obtain the result of workers working more hours than wanted without imposing any kind of monopolistic practices in the labor market.
We are concerned here with the second situation. As explained above, the larger $N$ is in relation to $m$, the lower the wage rate and the most likely is this second situation to occur. For historical reasons, we can think of the 19th century as characterized by such a large ratio—a large labor force. The equilibrium would explain the struggles of the 19th century labor force for shorter workweeks (see Marimón and Zilibotti 2000) as arising from a situation in which workers feel compelled to work too many hours, given the wage rate.

4.2. Characterization of the equilibrium

As explained above, we define the equilibrium as a full employment situation and, hence, workers per firm are $N/m$. Equating equation (1) to $N/m$ allows us to calculate the equilibrium wage rate,

$$w^* = a^{1/\theta} \left( \frac{\alpha}{t} \right)^{\frac{1-\theta}{\vartheta}} \theta \left( \frac{m}{N} \right)^{\frac{1-\alpha-\theta}{\vartheta}}.$$

Substituting $w^*$ in (2) we obtain hours per worker,

$$h^d = \left( \frac{1}{a} \right)^{1/\theta} \left( \frac{\alpha}{t} \right)^{\frac{1}{\vartheta}} \left( \frac{N}{m} \right)^{\frac{1-\alpha-\theta}{\vartheta}}.$$

Thus, total output equals

$$Y = man^{\alpha+\theta}h^\theta = N\left( \frac{t}{\alpha} \right).$$

Each firm generates a profit,

$$(1 - \alpha - \theta) \frac{t \ N}{\alpha \ m}$$

(positive since $1 > \alpha + \theta$) and, thus, total profits in the economy $\Pi = (1 - \alpha - \theta)Nt/\alpha = (1 - \alpha - \theta)Y$. On the other hand, consumption (or wages) per worker, $C = \theta t/\alpha$. Thus the total wage bill equals $\Omega = N\theta t/\alpha = (\alpha + \theta)Y - tN$ (or the remainder minus the personnel costs). Hence, total profits, the wage bill and the personnel costs exhaust the economy’s output.

$^{5}$Remember that profits in this context are returns to capital in actuality.
5. The regulated economy

This second model is meant to reproduce the point in history when workers have already achieved their vindications of a shorter work-week. We will assume that the situation is such that \( h^{ds} > h^{ss} \) and will take the length of the workweek, \( \tilde{h} < h^{ds} \) as given (being decided by the government or the product of regulation) as a start without worrying, at this point, about how the government decides on the value of \( \tilde{h} \).

In this case, the workers’ problem is trivial since their supply is inelastic: they always want to work at the market wage because there is no unemployment insurance. Thus, the market wage is the one at which there is no unemployment.

The entrepreneurs’ problem equals

\[
\max_{n} \quad an^{\alpha+\theta} \tilde{h}^\theta - w\tilde{h}n - tn.
\]

The solution to this problem is

\[
n^d = \tilde{h}^{\frac{\theta}{1-\alpha}} \left( \frac{a(\alpha + \theta)}{w\tilde{h} + t} \right)^{\frac{1}{1-\alpha}}; \tag{3}
\]

i.e., the demand of workers is increasing on the number of hours, \( \tilde{h} \), if \( t > ((1-\theta) / \theta) w\tilde{h} \) and decreasing otherwise. In other words, if personnel costs are important, an increase in the length of the workweek increases the demand of workers because a longer workweek allows entrepreneurs to better recoup the personnel spending on each individual worker. On the other hand, if personnel costs are not large, a longer workweek means that entrepreneurs substitute people by hours per worker to obtain a total number of hours; therefore, a longer workweek decreases the demand for workers.

By introducing the difference between “jobs” and hours per “job” (rather than assuming that all hours of work are made equal) in our model, we find some justification for the popular belief than a lengthened workweek decreases the demand for “jobs”.


Notice that, since in equilibrium, \( n^d = N/m \), in the first case (important personnel costs) a shorter workweek would decrease the wage rate while in the second case (relatively unimportant personnel costs) it would increase the wage rate.

5.1. Comparison of both economies

Since hours per worker are fewer by assumption and everybody is employed, output,
\[
Y = m^{1-\alpha-\theta}aN^{\alpha+\theta}\hat{h}^\theta,
\]
is obviously smaller.

We obtain \( w^* \) by equating the individual firm’s demand for workers, (3), to \( N/m \) so every worker is employed:
\[
w^* = \hat{h}^{\theta-1}a(\alpha + \theta)\left(\frac{m}{N}\right)^{1-\alpha-\theta} - \frac{t}{\hat{h}}.
\]
As explained above, the wage rate increases with \( \hat{h} \) if the personnel costs are important enough (if \( t > ((1 - \theta)/\theta)w\hat{h}) \) and it decreases otherwise. Since \( \hat{h} < h^{dx} \), if the personnel costs are important enough, the wage rate is smaller in this case than in the unregulated economy. Otherwise, the wage rate is larger in the regulated economy.

In this case the profits that each firm generates,
\[
\left(\frac{N}{m}\right)^{\alpha+\theta}a\hat{h}^\theta(1 - \alpha - \theta)
\]
(positive since \( 1 > \alpha + \theta \)); and the total profits in the economy equal
\[
\Pi = N^{\alpha+\theta}m^{1-(\alpha+\theta)}a\hat{h}^\theta(1 - \alpha - \theta) = (1 - \alpha - \theta)Y.
\]
Since they represent the same proportion of output as in the previous case but total output is smaller in this case, total profits are smaller in this case. Since the number of firms is the same, profits per firm are smaller in this case.
Consumption (wages) per worker equals
\[
\hat{h}^a (\alpha + \theta) \left( \frac{m}{N} \right)^{1-\alpha-\theta} - t.
\]
Thus, the total wage bill equals
\[
\Omega = \hat{h}^a (\alpha + \theta) m^{1-\alpha-\theta} N^{\alpha+\theta} - tN = (\alpha + \theta) Y - tN.
\]
As above, the wage bill represents the same proportion of a smaller output and the personnel costs are the same. Therefore, the wage bill is smaller and, since the number of workers is the same, wages (consumption) per worker are smaller, independent of the wage rate being smaller or larger than in the previous case. However, since the previous leisure/consumption allocation was not optimal on the workers’ eyes, that does not mean that workers’ are worse off in this case.

To see under which conditions workers are better off with a shorter workweek, we differentiate their utility with respect to hours worked. For them to be better off with a shorter workweek the following condition:
\[
\frac{dU(w(h)h, \bar{h} - h)}{dh} = U_1 \cdot \left( \frac{dw}{dh} h + w(h) \right) - U_2 < 0,
\]
is needed. In other words,
\[
\frac{dw}{dh} h + w(h) < MRS.
\]
The marginal rate of substitution is greater than \(w\) when workers are working more hours than desired at the going wage rate. On the other hand, if the economy is regulated,
\[
\frac{dw}{dh} h + w(h) = \frac{1}{h} [\theta t - (1 - \theta) hw] + w.
\]
Hence: if \(\theta t < ((1 - \theta) / \theta) hw\),
\[
\frac{dw}{dh} h + w(h) < w < MRS
\]
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and workers’ utility increases with a shorter workweek. If \( t > ((1 - \theta)/\theta)\hat{h}w > 0 \),

\[
\frac{dw}{dh}h + w(h) > w
\]

and in this case it is not possible to say with certainty what will happen to the utility if the workweek is shortened.

Workers’ utility increases with a shortened workweek in the case in which a shortened workweek implies an increase in the wage rate; i.e., as long as the personnel costs are not too large.

6. The economy with insurance contracts

To say something meaningful about the effect on employment we need a variation of the model in which unemployment exists. We assume in this section the existence of a labour movement; i.e., that workers act in a coordinated fashion, as a “union”. As we saw in section 2, workers do not need to be formally unionized to act in a coordinated fashion. As Blanchard and Summers (1986) argue, the union model can be interpreted as describing the behavior of workers when they act as a group, even if they are not formally unionized.

The usual union models propose unions demanding wages above equilibrium, knowing that this will generate unemployment and, therefore, establishing an unemployment insurance fund.\(^6\) In this model we change the control variable of the union with similar results: the union restricts the offer of labor to keep wages higher and institutes an employment insurance fund to this effect: they open an insurance market where \( \pi \) denotes the probability of working. The problem faced by entrepreneurs does not change.\(^7\)

\(^6\)As shown in section 2, unemployment insurance was not originally provided by the state but by unions and other workers’ or workers’ oriented groups. Later on, the labour movement induces the state to introduce an UI scheme.

\(^7\)The appendix shows the benchmark case in which there exist insurance contracts but the length of the workweek is not regulated.
Why a fixed workweek?

Since the wage rate is determined by the equation \( n(w, \hat{h}) = N/m \), by effectively restricting the amount of workers offering their services, the union can push wages up. It decides which proportion of the labor force will work (by the law of large numbers, this is the individual probability of being employed) to maximize workers’ expected utility given the labor market structure. To this purpose, it establishes an insurance employment fund. Workers are then randomly assigned to jobs. If employed, the worker consumes \( c_1 \) and, if unemployed, \( c_0 \). Employed workers compensate unemployed workers and feasibility demands that \( \pi c_1 + (1 - \pi) c_0 \leq w \hat{h} \pi \).

Thus, the union’s problem is

\[
\max_{c_1, c_0, \pi} \pi \left( \frac{c_1}{\rho} + \gamma \left( \frac{\overline{h} - \hat{h}}{\rho} \right)^\rho \right) + (1 - \pi) \left( \frac{c_0}{\rho} + \gamma \left( \frac{\overline{h}}{\rho} \right)^\rho \right)
\]

s.t. \( \pi c_1 + (1 - \pi) c_0 \leq w \hat{h} \pi \).

Notice that in the individual optimal allocation \( c_1 = c_0 \). Therefore, we can write the union’s problem as

\[
\max_{c, \pi} \frac{c}{\rho} + \chi - \phi \pi
\]

s.t. \( c \leq w \pi \hat{h} \),

where \( \chi = \gamma \overline{h}^\rho / \rho \), and

\[
\phi = \gamma \left( \frac{\overline{h}^\rho}{\rho} - \left( \frac{\overline{h} - \hat{h}}{\rho} \right)^\rho \right) > 0, \quad \frac{d\phi}{dh} > 0.
\]

The solution to this problem is the probability of working, \( \pi(w, \hat{h}) \).

**Proposition 3** The equilibrium wage \( w^* \) equates

\[
\pi^*(w^*, \hat{h}) \cdot N = m \cdot n^d(w^*).
\]

Since the firms’ problem is exactly the same, the demand of workers is given by function 3.
The supply of workers (more precisely, the proportion of the labor force that will work) is obtained by solving (4) and it equals

\[ \pi = \phi^{\frac{1}{\rho+1}} \frac{w^{\frac{\rho}{\rho+1}}}{\hat{h}}. \]  

(6)

Notice that the labor supply is an increasing function of the wage and it decreases with the length of the workweek.

Substituting (3) and (6) into the equilibrium condition (5) and rearranging terms we obtain

\[ F(w, \hat{h}) = \Gamma \left( \frac{\hat{h}^{1-\alpha}}{wh + t} \right)^{\frac{1}{1-\alpha-\theta}} - w^{\frac{\rho}{\rho+1}} = 0 \]

where

\[ \Gamma = \frac{m}{N} \phi^{\frac{1}{\rho+1}} \left( a(\alpha + \theta) \right)^{\frac{1}{1-\alpha-\theta}} > 0. \]

Thus, the equilibrium wage should satisfy this condition.

According to the implicit function theorem,

\[ \frac{dw}{dh} = -\frac{\delta F/\delta \hat{h}}{\delta F/\delta w}, \]

and, since

\[ \frac{\delta F}{\delta \hat{h}} = \frac{\Gamma}{1-\alpha-\theta} \left( \frac{\hat{h}^{\theta}}{(wh + t)^{2-(\alpha+\theta)}} \right)^{\frac{1}{1-\alpha-\theta}} \left[ (1-\alpha) t - \alpha \hat{h}w \right] \]

and

\[ \frac{\delta F}{\delta w} = -\frac{\Gamma}{1-\alpha-\theta} \left( \frac{h^{2(1-\alpha)-\theta}}{(wh + t)^{2-(\alpha+\theta)}} \right)^{\frac{1}{1-\alpha-\theta}} - \frac{\rho}{1-\rho} w^{\frac{\rho}{\rho+1} - 1} < 0, \]
whenever $t > (\alpha / (1 - \alpha))\hat{hw}$ is positive, a decrease in the workweek length decreases the wage and, if the opposite occurs, a decrease in the workweek length increases the wage. The condition is similar as the condition in the case in which there is no unemployment insurance: if personnel costs are not excessively large, decreasing the workweek increases wages.

To see the effect on employment, from equation (6)

$$\frac{d\pi}{dh} = \frac{\delta\pi}{\delta w} \frac{dw}{dh} + \frac{\delta\pi}{\delta h} = \frac{\phi^{\frac{1}{1-\rho}} w^{1-\rho}}{h} \left[ \frac{\rho}{1 - \rho} \frac{dw}{dh} - \frac{w}{h} \right].$$

Then, if personnel costs are not excessively large, $dw/dh < 0$, and therefore $d\pi/dh < 0$: shortening the workweek actually increases the employment rate $\pi$. If $dw/dh > 0$, the effect is not clear.

Comparing with the previous situation, since the workweek length $\hat{h}$ is the same (or assuming it is the same) the output should be smaller since now not everybody is employed.

7. Conclusion

As we saw, the introduction of the fixed workweek meant a decrease of the total output and a decrease of profits. However, workers were better off. This distributional effects are similar to the ones in Marimón and Zilibotti (2000). With fixed costs not being very large, which we think is the case, the demand for workers increases with a shortening of the fixed workweek. Not only the demand for workers increases, even the compensation (wage plus benefits) increases with a shortened workweek. Thus, total utility of workers increases with a shortened workweek. This distinction between jobs and hours per job, contrary to the usual assumption than an hour of labour is equal to other hour of labour at the same skill, effort, etc. level, legitimizes the arguments of the labour movement in industrialized countries.
Given the existence of a fixed workweek, the existence of unemployment, propitiated by the existence of unemployment insurance, obviously decreases total output but the effect on profits is not clear. Once we accept the existence of unemployment, not only demand for workers and the wage increase with a shortening of the fixed work, the unemployment decreases, consistent with an increase in the demand for workers (always assuming not very large personnel costs). In the unemployment existence case, it is not clear what is the effect of a shortening of the fixed workweek on total output and profits.

Defining, as we did, an entrepreneur as somebody with enough saving, the number of entrepreneurs is given in a one-period model; in a dynamic model, however, it is a state variable. Since, as explained in section 4, the likelihood of workers working more hours than they desire in an unregulated equilibrium depends on the wage rate, which in turn depends on the amount of capital and the level of technology, the need for regulation may weaken with economic prosperity. A dynamic model, our next step, should produce rich across-time and across-country comparisons to this respect as well as allow us to differentiate between two types of costs: the first, an initial cost of training people—a large cost (one and a half years of wages on average, according to some business literature) whose benefits extend over more than one period of time; the second, an on-going personnel cost per worker—a cost not so large but that needs to be incurred on every period of time the worker is employed.

We will also like to investigate the mechanism by which the labour movement convinces the state to introduce a fixed workweek.
A A benchmark economy

We can think of this economy as an economy in which there is no regulation about the length of the workweek but there exists employment insurance. The problem faced by entrepreneurs does not change.

The workers open an insurance market where \( \pi \) denotes the probability of working. Since the workers know that the entrepreneurs will fix the number of hours, they take this number as given (i.e., this is the number of hours that workers provide in the case they work). They adjust the probability of working to respond both to the wage rate and the workweek, both taken as given.

Thus, the union’s problem is

\[
\max_{c_1, c_0, \pi} \pi \left( \frac{c^\rho}{\rho} + \gamma \frac{(\bar{h} - h)^\rho}{\rho} \right) + (1 - \pi) \left( \frac{c^\rho}{\rho} + \gamma \frac{\bar{h}^\rho}{\rho} \right)
\]

s.t. \( \pi c_1 + (1 - \pi) c_0 \leq w h \pi \).

Notice that, as in the text, \( c_1 = c_0 \). Therefore, the union’s problem equals

\[
\max_{c, \pi} \frac{c^\rho}{\rho} + \chi - \phi \pi
\]

s.t. \( c \leq w \pi h \),

where

\[
\phi = \gamma \left( \frac{\bar{h}^\rho}{\rho} - \frac{(\bar{h} - h)^\rho}{\rho} \right).
\]

The probability of working, \( \pi(w, h) \), constitutes the solution to this problem.

**Proposition 4** Let us denote as \( w^* \) the wage in equilibrium. Thus, in equilibrium, hours worked, \( h^* = h^d(w^*) \), are equal to hours demanded at the wage \( w^* \).

**Proof.** Suppose that \( h^* \), is higher than the number of hours demanded at the wage \( w^* \). For entrepreneurs to employ more hours, the
wage would have to fall; thus, $w^*$ would not be the equilibrium wage. On the other hand, suppose that $h^*$ is lower than hours demanded. Because of competition among entrepreneurs, the wage would have to rise. ■
References


