Control performance improvement of a parallel robot via the design for control approach

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Abstract

Parallel structure robots have been receiving growing attentions from both academia and industries in recent years. This is due to their advantages over serial structure robots, such as high stiffness, high motion accuracy and high load-structure ratio. Control of a parallel robot, however, renders a difficult problem to control engineers. To obtain the same degrees of freedom (DOF), a parallel structure is more complex than a serial one, and so is its dynamic model in general. To effectively control a complex mechanical structure for precise and fast performance, an advanced controller embedded with the system’s dynamic model is usually desired. In cases of controlling parallel robots, however, the intensive computation due to the complexity of the dynamic model can result in difficulties in the physical implementations of the controllers for high-speed performance. To avoid heavy computation, simplified dynamic models can be obtained by applying simplification techniques, nevertheless, performance accuracy will suffer due to modeling errors. This paper suggests applying a general mechatronics design approach, i.e., the design for control (DFC) approach, to handle this problem. The underlying idea of the DFC approach is that, no matter how complex a system is, as long as its mechanical structure can be judiciously designed such that it can result in a simple dynamic model, a simple control algorithm may be good enough for a satisfactory control performance. As such, complicated controller design can be avoided, on-line computation load can be reduced and better control performance can be achieved. Through out the discussion in the paper, the design and control of a two DOF parallel robot is studied as an illustration example. Resulted control performances of several different mechanical designs
1. Introduction

According to their structures, robots can be classified into two categories, namely, serial and parallel. The serial type robots consist of links sequentially connected forming an open chain. Normally the first link of the open chain starts from a fixed base and the end effector is connected to its previous link by one end and with the other end open. This kind of robots is characterized by its large workspace and its high flexibility. However, the open-chain mechanisms have some inherent disadvantages, for example, the position accuracy at the endpoint of a robot with many links is considerably low, as a small amount of error at each joint is magnified and accumulated by its subsequent links; further, the mechanical stiffness of the open-chain construction is inherently poor since each link has to carry its subsequent links and their actuators. As a result, for tasks that require high positioning/accuracy, serial robots are not a preferable choice.

In contrast with the serial robots, the end effector of a parallel robot is connected to the fixed base via multiple kinematic chains. Thus all the actuators can be located on or closer to the base. This leads to high stiffness and high load-carrying capacity. Since the inertia of the moving parts is drastically reduced, this also leads to better dynamic properties. Moreover, joint errors will not be accumulated by link lengths. The main drawback of this kind of robots is its limited workspaces. However, by integrating a parallel robot (for micro motion) with a serial robot (for macro motion), a large workspace is achievable. Therefore, for precise position/force tasks, parallel robots are favored.

Although the first parallel robot, i.e., the six DOF Steward platform, was developed in 1965 [1], most current industrial applications are still dominated by serial robots. The dynamic equations describing the motion of serial robots are well established [2,3]. Many effective control algorithms are developed based on these knowledge. For example, under the situation when the dynamic model of a robot is accurately known, the PD-plus-gravity controller is good for point-to-point control [4], and the computed-torque controller (CTC) is well suited for trajectory tracking control [4]. If there are unstructured modeling errors, Craig’s adaptive CTC derived based on the linear property of the parameters and Slotine and Li’s adaptive CTC developed based on the passive property of the dynamics are able to capture the parameter variations [5,6]. When there exist both structured and unstructured modeling errors, neural network CTC provides a better solution for robot control [7].

On the other hand, the dynamic models of the parallel robots are far more complex than those of the serial ones. The major difficulty lies in finding a solution.
that not only can sufficiently describe the real system, but also can possibly be calculated in real time for implementation into control algorithms. To speed up the computation, a parallel computational algorithm which makes use of the particular geometry of the parallel structures has been proposed [8]. Another method suggests to remove the mass and inertia of the links in the dynamic model as they are negligible as compared with the dynamics of the platform and the actuators. This method leads to simplified dynamic models but introducing modeling errors in turn [9,10].

Although with the above mentioned parallel computation algorithm and simplification method, the resultant dynamics of the parallel robots can still be highly nonlinear and highly coupled. It thus demands a highly advanced control algorithm for a satisfactory dynamic performance. However, it may be rather difficult to derive such a control algorithm. Further, the control performance is hard to predict due to modeling uncertainties and unmodeled dynamics. In this paper, another effective engineering approach, namely, design for control (DFC) approach, will be adopted to the control of the parallel robots. This DFC approach is a general design approach which can be applied to any mechatronic system design. More explanation of this approach can be found in reference [11]. Different from most of the existing works for robot control, where a control design is usually considered at the completion of the design and construction of a mechanical structure, DFC suggests that the facilitation of a control design as well as the execution of a control action with the least hardware restriction must be taken into consideration at the stage of the mechanical structure design. An intuitive way to implement this idea is to design an appropriate mechanical structure so that it can result in a simple dynamic model with simple dynamic response characteristics. To design and implement a controller for such a system is thus a less difficult task. Successful examples applied DFC approach can be found in the authors’ previous works [12,13], where a four-bar mechanism controlled by a servo-motor is studied. Those works have shown that, by applying a mass-distribution scheme to redesign the mechanical structure of the four-bar mechanism, the gravitational term in the dynamic model can be canceled. This design not only simplifies the dynamic model, but also balances the shaking force transferred to the ground. As a result, the vibration effect during motion is significantly decreased, and satisfactory control performance can be achieved by simply using a PD controller. On the contrary, if the conventional “design-then-control” sequence is adopted to control a similar four-bar mechanism, a much more complicated control structure must be used. As the example reported in [14], the controller used is a combination of several sub-control algorithms, namely, a model reference adaptive control, a disturbance compensation loop, a modified switching controller and some feedback loops. For this particular four-bar mechanism system, one may argue that implementing an advanced control structure may be easier than physically rebuilding the mechanical structure. However, for any mechatronic system, if the mechanical structure is not properly designed, it is not guaranteed that a possible controller is always available. Furthermore, if a proper mechanical structure is designed based on the idea of DFC, the effort needed to search for a complicated controller can be spared.
The design examples presented in this paper extend the application of the mass-distribution scheme from the four-bar linkage to a parallel robot. In the first design case, the mechanical structure is designed following the conventional design procedure. As the general cases, the resultant dynamic model consists of the inertia term, the Coriolis plus centrifugal term and the gravitational term. In the subsequent three design cases, the mass-distribution of the mechanical structure is carefully synthesized following the idea suggested by the DFC approach. The resultant dynamic models are simplified to different extents respectively. For Case 2, the gravitational term is eliminated, for Case 3, both the gravitational term and the Coriolis plus centrifugal term are cancelled, the inertia matrix is also linearised, for Case 4, simplification of the inertia matrix is considered. Given these simplified dynamics, the controller design will be significantly facilitated accordingly.

In the remainder of this paper, Section 2 will describe the parallel robot studied in this work and also present its dynamic model. In Section 3, three mass-distribution schemes together with their effects on the simplification of the dynamic modeling will be discussed in details. Section 4 will present the control performances obtained from the different mechanical structures, followed by Section 5 which concludes the work in this paper and indicates directions for future studies.

2. Dynamic models of the parallel robot

In a broad sense, the concept of a parallel robot can be defined as “A mechanism or mechanical device is considered to be parallel when the relative motion between two bodies (usually the output link or gripper, and some reference or grounded link) is controlled by a combination of constraints and actuators acting in parallel [15].” Following this definition and also considering the simplicity for illustration, a two DOF parallel robot, sometimes also referred to as a five-bar closed-loop mechanism, is chosen as a design example throughout the discussion in this paper. The structure of the 2 DOF parallel robot is shown in Fig. 1. Links 1 and 2 are driven by two servo motors independently.

To effectively derive a dynamic model for this robot, its structure can be considered as a constrained system consisting of a free system and constraints as suggested by [16]. The free system is constructed by two serial open-chains, namely, Chain $L_1L_3$ and Chain $L_2L_4$. To describe Link $i$ in the free system, $m_i$ and $L_i$ are used to denote the mass and the length of the link respectively; $J_i$ is to denote the moment of inertia with respect to the centroid of the link; and finally, two variables $r_i$ and $\delta_i$ are used to denote the location of the mass center, indicated by a darkened circle in Fig. 1. Employing the Lagrangian method, the following dynamic model of the free system can be derived [16]:

$$D(q')\ddot{q} + C'(q', \dot{q'})\dot{q} + g'(q') = B\tau$$  \hspace{1cm} (1)

where $q' = [q_1 \ q_2 \ q_3 \ q_4]^T$ is the vector of the generalized coordinates of the free system; $D(q')$ is the inertia matrix defined as follows:
\[ D'(q') = \begin{bmatrix} d_{11}' & 0 & d_{13}' & 0 \\ 0 & d_{22}' & 0 & d_{24}' \\ d_{31}' & 0 & d_{33}' & 0 \\ 0 & d_{42}' & 0 & d_{44}' \end{bmatrix} \]  

with

\[
\begin{align*}
    d_{11}' &= m_1 r_1^2 + m_3 (L_1^2 + r_2^2 + 2L_1r_3 \cos(q_3 + \delta_3)) + J_1 + J_3 \\
    d_{13}' &= d_{31}' = m_3 (r_3^2 + L_1r_3 \cos(q_3 + \delta_3)) + J_3 \\
    d_{22}' &= m_2 r_2^2 + m_4 (L_2^2 + r_4^2 + 2L_2r_4 \cos(q_4 + \delta_4)) + J_2 + J_4 \\
    d_{24}' &= d_{42}' = m_4 (r_4^2 + L_2r_4 \cos(q_4 + \delta_4)) + J_4 \\
    d_{33}' &= m_3 r_3^2 + J_3 \\
    d_{44}' &= m_4 r_4^2 + J_4
\end{align*}
\]

and

\[ C'(q', \dot{q'}) \dot{q'} \] is the centrifugal and Coriolis term, and \( C'(q', \dot{q'}) \) is defined as:

\[
C'(q', \dot{q'}) = \begin{bmatrix} h_1 \dot{q}_3 & 0 & h_1 (\dot{q}_1 + \dot{q}_3) & 0 \\ 0 & h_2 \dot{q}_4 & 0 & h_2 (\dot{q}_2 + \dot{q}_4) \\ -h_1 \dot{q}_1 & 0 & 0 & 0 \\ 0 & -h_2 \dot{q}_2 & 0 & 0 \end{bmatrix}
\]
with
\[ h_1 = -m_3 L_1 r_3 \sin(q_3 + \delta_3) \quad \text{and} \quad h_2 = -m_4 L_2 r_4 \sin(q_4 + \delta_4) \]

\( g'(q') \) is the gravitational term defined as:
\[ g'(q') = \begin{bmatrix}
  m_1 r_1 \cos(q_1 + \delta_1) + m_3 (L_1 \cos(q_1) + r_3 \cos(q_1 + q_3 + \delta_3)) \\
  m_2 r_2 \cos(q_2 + \delta_2) + m_4 (L_2 \cos(q_2) + r_4 \cos(q_2 + q_4 + \delta_4)) \\
  m_3 r_3 \cos(q_1 + q_3 + \delta_3) \\
  m_4 r_4 \cos(q_2 + q_4 + \delta_4)
\end{bmatrix} \]

with \( g \) as the gravitational acceleration constant; and \( B \tau \) is the input torque, with
\[ B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}^T \]

and
\[ \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \]

where \( \tau_1 \) and \( \tau_2 \) are the input torques applied to Joints 1 and 2 respectively. This free system is further subject to two independent holonomic constraints described as follows:
\[ \phi(q') = \begin{bmatrix} \phi_1(q') \\ \phi_2(q') \end{bmatrix} = \begin{bmatrix} L_1 \cos(q_1) + L_3 \cos(q_1 + q_3) - L_2 \cos(q_2) - L_4 \cos(q_2 + q_4) - L_5 \\ L_1 \sin(q_1) + L_3 \sin(q_1 + q_3) - L_2 \sin(q_2) - L_4 \sin(q_2 + q_4) \end{bmatrix} = 0 \]

Note when deriving Eqs. (1) and (7), assumption \( q_5 = 0 \) is imposed.

Since there are only two active joints, the coordinate vector \( q' \) can be parameterized as follows:
\[ q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} q' = \mathbf{a} q' \]

by introducing a constant matrix \( \mathbf{a} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \). Substituting Eqs. (7) and (8) into Eq. (1), a reduced dynamic model of the parallel robot is obtained as follows (for more details, please refer to [16]):
\[ \begin{cases}
  D(q') \ddot{q} + C(q', \dot{q}') \dot{q} + g(q') = \tau \\
  \dot{q}' = \rho(q') \dot{q} \\
  \dot{q}' = \sigma(q)
\end{cases} \]
where
\[
\begin{align*}
D(q') &= \rho^T(q')D'(q')\rho(q') \\
C(q', q') &= \rho^T(q')C'(q', q')\rho(q') + \rho^T(q')D'(q')\rho(q', q') \\
g(q') &= \rho^T(q')g'(q')
\end{align*}
\]
with
\[
\rho(q') = \Psi^{-1}(q') \begin{bmatrix} 0 & 0 & 1 & 0^T \\
0 & 0 & 1 \end{bmatrix}
\]

further with definitions
\[
\Psi(q') \triangleq \begin{bmatrix} \varphi(q') \\
z(q') \end{bmatrix} \quad \text{and} \quad \Psi_q(q') \triangleq \frac{\partial \Psi}{q},
\]
the details of the latter is given as:
\[
\Psi_q(q') = \begin{bmatrix} \Psi_q(1, 1) & \Psi_q(1, 2) & -L_3 \sin(q_1 + q_3) & L_4 \sin(q_2 + q_4) \\
\Psi_q(2, 1) & \Psi_q(2, 2) & L_3 \cos(q_1 + q_3) & -L_4 \cos(q_2 + q_4) \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\end{bmatrix}
\]

where
\[
\begin{align*}
\Psi_q(1, 1) &= -L_1 \sin(q_1) - L_3 \sin(q_1 + q_3) \\
\Psi_q(1, 2) &= L_2 \sin(q_2) + L_4 \sin(q_2 + q_4) \\
\Psi_q(2, 1) &= L_1 \cos(q_1) + L_3 \cos(q_1 + q_3) \\
\Psi_q(2, 2) &= -L_2 \cos(q_2) - L_4 \cos(q_2 + q_4)
\end{align*}
\]

and
\[
\Psi_q(q') \triangleq \frac{\partial \Psi}{q}
\]

Also note that in Eq. (10), \(\dot{\rho}(q', q')\) is the derivative of \(\rho(q')\) with respect to time; further note that it is not possible to derive an analytic expression for \(q' = \sigma(q)\) in Eq. (9), it must be computed using numerical methods. For the robot structure discussed in this paper, the following result is obtained:
\[
q' = \begin{bmatrix} q_1 \\
q_2 \\
q_3 = \tan^{-1}((\mu + L_4 \sin(q_2 + q_4))/(\lambda + L_4 \sin(q_2 + q_4))) - q_1 \\
q_4 = \pm \tan^{-1}(\sqrt{A^2 + B^2 - C^2/C}) + \tan^{-1}(B/A) - q_2
\end{bmatrix}
\]

where \(\lambda = 2L_2 \mu, \quad B = 2L_4 \mu, \quad C = L_4^2 - L_2^2 - \lambda^2 - \mu^2, \quad \text{with} \quad \lambda = L_2 \cos(q_2) - L_1 \cos(q_1) + L_5, \quad \text{and} \quad \mu = L_2 \sin(q_2) - L_1 \sin(q_1).

The above description shows that, after the parameterisation of the joint coordinates, although the general dynamic model of the parallel robot given in Eq. (1) can be simplified as the reduced one given in Eq. (9), the latter is still quite complicated. To design a control algorithm based on this model is thus not a simple task.
3. Redesign of the parallel robot

To facilitate the control design, redesign of the parallel robot will be considered in this section. Re-exam Eq. (9), it can be seen that the nonlinear and the coupling effects in the model come from three factors: the inertia matrix, the Coriolis and centrifugal matrix and the gravitational vector. The underlying principle adopted in the redesign is to linearise and decouple these three terms to the most extent by using mass-distribution schemes. Three redesign cases will be discussed as follows.

3.1. Case 1—invariant potential energy

It is well known that the Lagrangian formulation of the robot dynamics is derived from the Lagrangian, which is defined as the difference between the kinetic and potential energy of the robot. If the robot structure is designed in such a way that the potential energy can be maintained as a constant during motion, the dynamic model can thus be simplified. This is the motivation behind the first mass-distribution scheme discussed in this case. For the robot structure studied in this paper, gravity is the sole contributor to the potential energy. If the global mass center of the parallel robot is kept stationary during motion, the potential energy is thus invariant.

The global mass center can be described by the following equation:

\[ \mathbf{r}_c = \frac{1}{m} \sum_{i=1}^{4} m_i \mathbf{r}_{ci} \]  

where \( m = \sum_{i=1}^{4} m_i \), \( \mathbf{r}_c \) denotes the position vector of the global mass center, \( \mathbf{r}_{ci} \) \((i = 1, 2, 3 \text{ and } 4)\) is the position vector of the mass center of Link \( i \). Based on the illustration shown in Fig. 1, the position vector \( \mathbf{r}_{ci} \) can be expressed as:

\[ \mathbf{r}_{c1} = r_1 e^{i(q_1 + d_1)} \]
\[ \mathbf{r}_{c3} = L_1 e^{i q_1} + r_3 e^{i(q_3 + q_1 + d_3)} \]
\[ \mathbf{r}_{c2} = L_5 + r_2 e^{i(q_2 + d_2)} \]
\[ \mathbf{r}_{c4} = L_5 + L_2 e^{i q_2} + r_4 e^{i(q_4 + q_2 + d_4)} \]  

Substituting Eq. (14) into Eq. (13) leads to:

\[ m \mathbf{r}_c = (m_1 r_1 e^{i \delta_1} + m_3 L_3) e^{i q_1} + m_3 r_3 e^{i \delta_3} e^{i(q_3 + q_1 + d_3)} + (m_2 r_2 e^{i \delta_2} + m_4 L_2) e^{i q_2} \]
\[ + m_2 r_4 e^{i \delta_4} e^{i(q_4 + q_2 + d_4)} + m_2 L_5 + m_4 L_5 \]  

(15)

Based on the constraints given in Eq. (7), the unit vectors \( e^{i q_1}, e^{i q_2}, e^{i q_3}, \text{ and } e^{i q_4} \) must satisfy the following kinematics loop equation:

\[ L_1 e^{i q_1} + L_3 e^{i(q_3 + q_1)} - L_4 e^{i(q_4 + q_2)} - L_2 e^{i q_2} - L_5 = 0 \]  

(16)
Solving $e^{i(q_3 + q_4)}$ from Eq. (16) and substituting the result into Eq. (15) leads to:

$$mr_c = \left( m_1 r_1 e^{i\delta_1} + m_3 L_1 - m_3 \frac{L_1}{L_3} r_3 e^{i\delta_3} \right) e^{i q_1}$$

$$+ \left( m_2 r_2 e^{i\delta_2} + m_4 L_2 + \frac{L_2}{L_3} m_3 r_3 e^{i\delta_3} \right) e^{i q_2} \left( m_4 r_4 e^{i\delta_4} + m_3 \frac{L_4}{L_3} r_3 e^{i\delta_3} \right) e^{i(q_2 + q_4)}$$

$$+ m_3 \frac{L_5}{L_3} r_3 e^{i\delta_3} + m_2 L_5 + m_4 L_5$$  \hspace{1cm} (17)

From the above equation, it can be seen that in order to maintain the global mass center stationary during motion, the coefficients of the vectors $e^{i\delta_1}$, $e^{i\delta_2}$, and $e^{i(q_2 + q_4)}$ must be zero, i.e.,

$$m_1 r_1 e^{i\delta_1} + m_3 L_1 - m_3 \frac{L_1}{L_3} r_3 e^{i\delta_3} = 0$$  \hspace{1cm} (18)

$$m_2 r_2 e^{i\delta_2} + m_4 L_2 + \frac{L_2}{L_3} m_3 r_3 e^{i\delta_3} = 0$$  \hspace{1cm} (19)

$$m_4 r_4 e^{i\delta_4} + m_3 \frac{L_4}{L_3} r_3 e^{i\delta_3} = 0$$  \hspace{1cm} (20)

Substituting Eq. (20) into Eq. (19) leads to:

$$m_2 r_2 e^{i\delta_2} + m_4 L_2 - \frac{L_2}{L_4} m_4 r_4 e^{i\delta_4} = 0$$  \hspace{1cm} (21)

Further, from Fig. 1, the following relationships hold

$$r_3 e^{i\delta_3} = L_3 + r'_3 e^{i\delta'_3}$$  \hspace{1cm} (22)

$$r_4 e^{i\delta_4} = L_4 + r'_4 e^{i\delta'_4}$$  \hspace{1cm} (23)

Substituting Eq. (22) into Eq. (18) yields:

$$m_1 r_1 e^{i\delta_1} - m_3 \frac{L_1}{L_3} r'_3 e^{i\delta'_3} = 0$$  \hspace{1cm} (24)

and substituting Eq. (23) into Eq. (21) yields:

$$m_2 r_2 e^{i\delta_2} - m_4 \frac{L_2}{L_4} r'_4 e^{i\delta'_4} = 0$$  \hspace{1cm} (25)

Further rearranging Eqs. (20), (24) and (25), the following results can be derived:

$$m_1 r_1 L_3 = m_3 L_1 r'_3, \quad \delta_1 = \delta'_3$$  \hspace{1cm} (26)

$$m_2 r_2 L_4 = m_4 L_2 r'_4, \quad \delta_2 = \delta'_4$$  \hspace{1cm} (27)

$$m_4 r_4 L_3 = m_3 L_4 r'_3, \quad \delta_4 = \delta_3 + \pi$$  \hspace{1cm} (28)

Eqs. (26)–(28) are the mass-distribution conditions. In order to make sure that the potential energy is invariant during motion, these conditions must be satisfied when designing the robot links. It is noted that, which ever link mass and the location of its
mass center are given, the rest three link masses and their mass centers are determined.

When the conditions given in Eqs. (26)–(28) are held, it can be derived that
\[ g(q') = \rho^T(q')g'(q') = 0 \] (29)
This result is not difficult to explain, as the gravitational term in the dynamic model is the derivative of the potential energy. Since the potential energy is invariant during motion, \( g(q') \) is therefore vanished. Eq. (29) thus leads to the following simpler dynamic model:
\[ D(q')\ddot{q} + C(q', \dot{q}')\dot{q} = \tau \] (30)

3.2. Case 2—Invariant generalized inertia

In this design case, apart from making \( g(q') \) zero, the possibility is explored to further linearise and decouple the inertia matrix and the Coriolis plus centrifugal matrix in the dynamic model. First, consider imposing the following constraints to the structure in Fig. 1,
\[ L_1 = L_4, \ L_2 = L_3, \ L_5 = 0 \] (31)
These conditions result in the following kinematics motion behavior:
\[ \begin{cases} q_3 = -(q_1 - q_2) \\
q_4 = q_1 - q_2
\end{cases} \] (32)
Substituting Eq. (32) into Eq. (11), it leads to the following result:
\[ \rho(q') = \begin{bmatrix} 1 & 0 \\
0 & 1 \\
-1 & 1 \\
1 & -1 \end{bmatrix} \text{ and } \dot{\rho}(q', \dot{q}') = 0 \] (33)
By employing Eq. (33), the inertia matrix will be reduced to
\[ D(q') = \rho^T(q')D'(q')\rho(q') = \begin{bmatrix} d_{11} & d_{12} \\
d_{21} & d_{22} \end{bmatrix} \] (34)
where
\[ d_{11} = d'_{11} - 2d'_{13} + d'_{33} + d'_{44} = m_1r_1^2 + m_3L_1^2 + J_1 + m_4r_4^2 + J_4 \] (35)
\[ d_{12} = d'_{22} - 2d'_{24} + d'_{33} + d'_{44} = m_2r_2^2 + m_4L_2^2 + J_2 + m_3r_3^2 + J_3 \] (36)
\[ d_{22} = d'_{21} + d'_3 - d'_{33} + d'_{44} = m_3L_1r_3 \cos(q_3 + \delta_3) + m_4L_2r_4 \cos(q_4 + \delta_4) \] (37)
Similarly, the Coriolis plus centrifugal matrix will be reduced to
\[ C(q', \dot{q}') = \rho^T(q')C'(q', \dot{q}')\rho(q') + \rho^T(q')D'(q')\dot{\rho}(q', \dot{q}') = \begin{bmatrix} 0 & c_{12} \\
c_{21} & 0 \end{bmatrix} \] (38)
where

\[ c_{12} = (m_3 L_1 r_3 - m_4 L_2 r_4) \cos(q_2 + \delta_2) \]  
(39)

\[ c_{21} = -(m_3 L_1 r_3 - m_4 L_2 r_4) \cos(q_1 + \delta_1) \]  
(40)

When the mass-balance condition in Eq. (28) holds, Eqs. (37), (39) and (40) become:

\[ c_{12} = c_{21} = d_{12} \equiv 0 \]  
(41)

The outcome in Eq. (41) indicates that the inertia matrix is invariant and the Coriolis plus centrifugal term vanishes during motion. Thus the dynamic equation of the parallel robot is reduced to

\[ D \ddot{q} + g(q) = \tau \]  
(42)

where the generalized inertial matrix \( D = \text{diag}[d_{11}, d_{22}] \) is a constant matrix.

It is interesting to note that, for this particular robot structure where the two pairs of opposite links are parallel, result (41) is true as long as the condition in Eq. (28) holds. This means that the “invariant potential energy” design implies the “invariant generalized inertia” design, which means \( g(q') = 0 \) in this case. As such, the final dynamic model of the system is greatly simplified:

\[ D \ddot{q} = \tau \]  
(43)

This is in fact a typical second-order linear system. Designing a controller for this linear system is relatively a much simpler task. The dynamic response of the control performance can be predicated if the modeling is error-free.

3.3. Case 3—partially invariant generalized inertia

Although the design in Case 2 results in an ideal linear and simple dynamic model, the structure is degenerated into a one DOF system. Therefore, to obtain a fully invariant generalized inertia is not a practical design. The constraint that requires the opposite two links be parallel must be removed. However, it is still possible to obtain a partially invariant generalized inertia. In this case, the design objective is to, without varying the topology of the parallel robot, linearise and decouple the matrices \( D(q') \) and \( C(q', \dot{q}') \) to the most extent.

Let the mass centers of Links 3 and 4 be placed at the ends of the input Links 1 and 2 respectively, i.e.,

\[ r_3 = r_4 = 0 \]  
(44)

If the condition in Eq. (44) is held, Eq. (28) is always true, further Eqs. (26) and (27) are reduced as:

\[ m_1 r_1 = m_3 L_1, \quad \delta_1 = \pi \]  
(45)

\[ m_2 r_2 = m_4 L_2, \quad \delta_2 = \pi \]  
(46)

Substituting the above two results into Eqs. (2) and (3), the dynamic terms of the free system, it is observed that \( D'(q') \) is reduced to a constant matrix and \( C'(q', \dot{q}') \) becomes a zero matrix. However, when the free system is subject to the
holonomic constraints, $D(q')$ can not be fully reduced to a constant matrix, neither $C(q',\dot{q}')$ a zero matrix. Denote the resultant dynamic model of the full system as follows:

$$
\begin{align*}
\bar{D}(q')\ddot{q} + \bar{C}(q',\dot{q}')\dot{q} &= \tau \\
\dot{q}' &= \rho(q')\dot{q} \\
q' &= \sigma(q)
\end{align*}
$$

(47)

with the hat ‘–’ indicating that the matrices are partially linearised and decoupled. For the inertia matrix $D(q')$, all the elements associated with $r_3$ and $r_4$ are vanished. Therefore $\bar{D}(q')$ is partially invariant. For the Coriolis plus centrifugal matrix $C(q',\dot{q}')$, refer to Eq. (10), the first term becomes zero, and the second term is partially invariant. Hence, $\bar{C}(q',\dot{q}')$ is also simplified. It is noted that, in the above dynamic model, $\dot{q}'$ and $q'$ are the same as those in Eq. (9). This is because these two vectors are related to the topology of the structure. Relocating mass centers has no effect on them. Further note that the gravitational term is disappeared as the conditions in Eqs. (26)–(28) are still held.

4. Controller design

In the previous section, three schemes were proposed for the redesign of the parallel robot. All these designs lead to simpler dynamic models as summarized in Table 1. Case 1 refers to the original design without applying the DFC approach. Case 2 refers to the design where the potential energy is made invariant. Case 3 is the degenerated situation where the system is ended up with one DOF. Case 4 is the design where partially invariant generalized inertia is resulted; and finally. For the generality of discussion, Case 3 will not be considered in the rest of this section. The detailed design parameters of the parallel robot under different design situations are recorded in Table 2. Note that, for all the redesign cases, it is assumed that the material-addition method was employed for the mass redistribution.

As mentioned earlier, the basic idea of the DFC approach is to spare control design effort and improve real-time performance by providing a simple dynamic model through judicious mechanical design. Hence, in this section, a simple proportional-plus-derive (PD) control will be implemented to control the robots with simpler dynamic models. For comparison purpose, a more complicated computed-

<table>
<thead>
<tr>
<th>Case</th>
<th>Dynamic models</th>
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<tbody>
<tr>
<td>Case 1</td>
<td>$D(q')\ddot{q} + C(q',\dot{q}')\dot{q} + g(q') = \tau$</td>
</tr>
<tr>
<td>Case 2</td>
<td>$D(q')\ddot{q} + C(q',\dot{q}')\dot{q} = \tau$</td>
</tr>
<tr>
<td>Case 3</td>
<td>$D\ddot{q} = \tau$</td>
</tr>
<tr>
<td>Case 4</td>
<td>$\bar{D}(q')\ddot{q} + \bar{C}(q',\dot{q}')\dot{q} = \tau$</td>
</tr>
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</table>
torque control (CTC) will be used to control the original parallel robot. Trajectory tracking control will be studied with the desired trajectories for the two actuated links respectively given as follows:

\[ q_{1d} = 0.5\pi t, \quad q_{2d} = 2\pi \left( 6\frac{t^5}{t_f^5} - 15\frac{t^4}{t_f^4} + 10\frac{t^3}{t_f^3} \right) \]  

(48)

where \( t_f = 4s \) is the time span of tracking.

4.1. PD control results

In this sub-section, the following PD controller

\[ \tau = K_p e + K_d \dot{e} \]  

(49)

is implemented for the trajectory tracking control. In the control algorithm shown in Eq. (49), vector \( e = q_d - q \) is the trajectory tracking error, and \( K_p \) and \( K_d \) are the positive definite gain matrices. In this simulation study, the gain matrices were selected to be \( K_p = 5I \) and \( K_d = 10I \) respectively, with \( I \) as a second-order identity matrix.
The simulation results are shown in Fig. 2. It is observed that the tracking performance of the robot has been improved from case to case, coinciding with the depth of the simplification of the dynamic model. Fig. 2(a) and (c) show that the angular displacement tracking errors of the two input links are greatly reduced from Case 1 to
Case 2, and further reduced in Case 4. Similar phenomenon is also observed for the angular velocity tracking errors as depicted in Fig. 2(b) and (d). From the control torque profiles shown in Fig. 2(e) and (f), it is indicated that less control energy is consumed in Case 2, and the energy consumption is further reduced in Case 4.

Fig. 3. Tracking performances with CTC control: (a) profiles of the angular displacement errors of the first input link, (b) profiles of the angular velocity errors of the first input link, (c) profiles of the angular displacement errors of the second input link, (d) profiles of the angular velocity errors of the second input link, (e) profiles of the torques of the first input link and (f) profiles of the torques of the second input link (dotted line: for Situation 1; solid line: for Situation 2).
In a word, these simulation results reveal that, through redesign, not only can the motion tracking performance be improved, but also the control energy is significantly reduced. These advantages become more obvious when more efforts on the simplification of the dynamic model are spent.

4.2. CTC control results

The results in the previous simulation study show that a simple PD controller is able to generate good tracking performances for Cases 2 and 4 where the system’s dynamic models are simplified. However, for Case 1, the original robot design, the PD algorithm is failed to produce a satisfactory control. In this sub-section, a more complex CTC algorithm is employed to control the structure in Case 1. The motivation behind this study is to compare the performances between two situations arisen from this particular mechatronic system: Situation (1) a complex controller + a complex dynamic model; Situation (2) a simple controller + a simple dynamic model.

The CTC algorithm is chosen as follows:

$$\tau = D(q')(\dot{q}_d + K_p e + K_d \dot{e}) + C(q', \dot{q}')\dot{q} + g(q')$$

with $K_p$ and $K_d$ as the feedback PD loop. In this simulation, the PD gain matrices are chosen to be $K_p = 10I$ and $K_d = 50I$, respectively.

The simulation results are shown in Fig. 3, where the dotted lines are for Situation (1), i.e., PD algorithm + Case 4; and the solid lines are for Situation (2), i.e., CTC algorithm + Case 1. Fig. 3(a) depicts profiles of the angular displacement errors of the first input link under these two situations. Although it seems that the tracking error for Situation (2) is much smaller than that for Situation (1), by taking into consideration of the scale of the figure, the actual difference between two results is
only 0.00025 rad, which is negligible. Similar phenomenon also occurs to the angular velocity errors of the first input link, as shown in Fig. 3(b), where the difference between two errors is 0.001 rad/s, which is also negligible. From Fig. 3(c) and (d), it is interesting to note that the maximum trajectory and velocity errors of the second input link in both situations are almost the same. From these observations, it reveals that the difference of the tracking performance between two design situations is not significant. However, for Situation (2), to produce the similar performance as Situation (1), it has to generate much higher control torques, as shown in Fig. 3(e) and (f).

It can thus be concluded that, for this mechatronic system, the tracking performance of the original parallel robot controlled by the complex CTC algorithm does not surpass that of the redesigned robot structure controlled by the simple PD algorithm. However, the former design requires much more control energy.

5. Conclusions

Control of serial robots for high performance and high speed tasks has always been a challenge to control engineers. Due to the demands from the robotic industry, the robot structures have evolved from serial to parallel, and the robot application tasks have been extended from the macro world to the micro world. Control engineers thus are facing a more difficult problem.

Following the traditional control engineering approach, it is possible to develop advanced algorithms for parallel robot control. This approach, however, may encounter difficulties such as heavy computational load and modeling errors, to name it a few. The satisfactory control performance may not be obtainable. In this paper, we have adopted a new control concept which suggests a feasible solution from a different point of view. Instead of handling the control problem by only looking at the controller design, this idea suggests to tackle this problem by reversing one step backward, that is, at the stage when the design of the mechanical structure is considered, the controller design, which normally occurs after the construction of the mechanical structure, should also be taken into consideration. Following this suggestion, this paper employs the DFC approach, which is generally a feasible approach for all sort of mechanic system design, to design and control a two DOF parallel robot. The basic argument of DFC can be summarised as follows. A simple control algorithm can control a structure with a simple dynamic model quite well in general. Therefore, no matter how sophisticate a desired motion task is, the mechanical structure should be designed such that it results in a linearised and decoupled dynamic model. Then, to design a controller for such a system will not be a difficult issue. Based on this argument, several design versions of a 2 DOF parallel robot are studied in this work. The simulation results have demonstrated that, a simple PD algorithm can control the robot with the simplest dynamic model very well.

The derivation of the dynamic models of the robot studied in this paper is based on the Lagrangian formulation. When other dynamic modeling methods are applied, such as the body-oriented method [17], for example, the dynamic models will be
different. It will be very interesting to investigate how the redesign of the robot will be influenced by different modeling methods.

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References