Integrated Design of Mechanical Structure and Control Algorithm for Closed-Chain Five-Bar Mechanism

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Abstract: In the traditional mechatronic systems, the controller designs are usually separated from the mechanical structure designs. Consequently, the controller algorithms become more and more complex, i.e., from the PD controller, to the computed-torque controller, and to all sorts of the adaptive controller. These complex control algorithms encounter many difficulties in implementation of the control algorithms to closed-chain mechanisms. The main reason for these difficulties is the so-called computational overhead with the dynamic model of the closed-chain mechanisms. The computational overhead is even experienced in a parallel manipulator system (one of the closed-chain types). One of the solutions to this problem is to seek a smart design of mechanical structures based on the methodology called Design For Control (DFC). DFC specifically proposes the procedure, i.e., design of a mechanical structure, simplification of the dynamic model of the mechanical structure, and design of a simple control algorithm (such as PD controller). In this paper, a 2-dof five-bar closed-chain mechanism (called Delta mechanism for short) is studied. It is shown that the smart design of the mechanical structure of the Delta mechanism can lead to some configuration invariants in the dynamic model. Therefore, the design of PD controllers, incorporating these invariants, can achieve excellent system performance.

Keyword: DFC, five-bar mechanism, control algorithm, tracking performance
1. Introduction

Robot manipulators may be classified into two categories in terms of their architecture. In the first category, all of the links in the kinematics chain are connected sequentially starting from a fixed base. The last link in the chain is connected from one end to a previous link but is free from the other end resulting in an open link chain. We refer to these systems as “opened kinematics chain”. Typically, in these systems each link is connected to the previous link in the link chain by a joint and all of the joints are actuated. In the robotics literature, this kind of open chain systems is usually called “serial robots”. In the second category, the links in the kinematics chain are connected in series as well as in parallel combinations forming one or more closed-link loops. We refer to these mechanisms as “closed-chain systems”. Typically, in these systems not all joints in closed-chain mechanisms are actuated. In the robotics literature, this kind of closed-chain system is usually called “parallel robots”. The simplest characterization of this category is that of the so-called fully parallel robots which consist of a set of serial links each connected to a fixed base from one end, and connected to a common moving endpoint (or platform) on the other end, and resulting in a closed-chain mechanism, for example, steward platform [1].

During the last two decades, the serial robot control has been a long-standing field. Since the serial robots are governed by a well-established formulation of the equations of motion [4,5], which possess many excellent properties, a lot of control results [4-8] have been developed for them. When the structure parameters of the serial robots are known exactly, the PD controller applied to the regulation problem, and the computed-torque controller applied to the trajectory tracking problem [4,5] are widely employed. To deal with the uncertainties in the structure parameters of the serial robots, Craig [6] developed an adaptive controller based on computed-torque control, and Soltine and Li [7] developed another adaptive controller based on passive property. The readers are invited to see the literatures [5,8] for all sorts of controllers applied to serial robots. However, the open-chain mechanisms possess some inherent disadvantages, for example, the position accuracy at the endpoint of the long robot arm is considerably low; a small amount of error at each revolution joint is magnified at the endpoint of arm as its length gets longer; most importantly, the mechanical stiffness of the open-chain construction is inherently poor since each joint is equipped with an actuator. As a result, the accuracy of motion tracking
performance can be deteriorated. The research trend in modern machinery development therefore shifts toward the parallel robots for the position and the trajectory tracking purpose.

The equations of motion governing the parallel robots are the same in form as the equations of motion governing the serial robots, and also possess almost the same properties. It seems that all control methods in the serial robots may apply to the parallel robot control. Nevertheless, because they are local and implicit, the equations of motion of the parallel robots include more structure parameters and are of more complexity than those of the serial robots with the same degree of freedoms (dof). This results in the computation complexity in designing a controller. Several methods reported in the literature were proposed to design the controller for parallel robots [9-11]. As suggested by Lin and Chen [9], a very complex control structure that is composed of several sub-control algorithms, such as a model reference adaptive control, a disturbance compensation loop, with a modified switching controller plus some feedback loops, was proposed to control the four-bar-link mechanism to follow a pre-specification trajectory. Ghorbel [10,11] presented the PD control with simple gravity compensation for a two-dof closed-chain mechanism for the point-to-point control task. Although Ghorbel’s controller is the same in form as the controller used for the open-closed robot, the complexity of its computation is by far more. In general, the intensive computation can result in the difficulty in physical realization of a controller for high-speed performance. There is indeed a trend to apply a parallel computation/processing technique [12] for controlling parallel mechanism systems with multi-dof.

Another method to overcome computation complexity is to design the mechanical structure for a relatively simpler dynamic model. Comparing with serial robots, the dynamical model of the parallel robot includes more structure parameters. Therefore, more opportunities are present for the judicious selection of these parameters. Along this line of thinking, Asada and Toumi [13,14] designed a parallel drive five-bar mechanism. One of the most advantageous properties for this mechanism is that its mass matrix is configuration-invariant, and thus, it is a time-invariant diagonal matrix. This consequently leads to the cancellation of the centrifugal and Coriolis terms. They demonstrated that the design of the controller was simpler, and the performance of the closed-loop system was improved. Following Toumi’s design strategy, Diken [15] improved
the motion tracking performance of an open-loop robot manipulator by applying a mass redistribution scheme. In his study, the structure of a robot arm was first reduced into dynamically equivalent point masses so as to eliminate the gravitational term in the dynamic model. A simple algorithm was then applied to control the system for satisfactory performance of trajectory tracking.

Recently, a more general concept called “Design For Control (DFC)” was proposed [2,16]. The essence of the concept suggests designing the mechanical structure of a programmable machine by fully exploring the physical understanding of overall system with its consideration of facilitation of controller design as well as the execution of control action with the least hardware restriction. An intuitive way to realize this objective is to design an appropriate structure for the mechanical part such that it can result in a “simple” dynamic model and thus a more predictable dynamic response. The four-bar-link mechanism was studied and the gravitational term in the dynamic model was cancelled by using complete shaking force balancing scheme. In essence, this methodology makes the center of general mass of the mechanism configuration-invariant by mass redistribution design. Further results of the four-bar-link mechanism based on DFC were reported in [17].

From the discussion above, it can be seen that the studies in [2,16] took advantage of the property called configuration-invariance of potential energy (CIPE), while the studies in [13,14] took advantage of the property called configuration-invariance of generalized inertial (CIGI). Note that both partial CIPE and partial CIGI are possible (PCIGE and PCIGI for short, respectively). The motivation of this paper is to investigate what effects could be on system performances by integrating CIPE and CIGI. Based on this motivation, a study taking a closed-loop five-bar mechanism with 2-dof was made, and will be presented in this paper. Section 2 describes the dynamic model of a 2-dof closed-chain five-bar mechanism, which gives a foundation for both mechanical structure design and controller design. In Section 3, synthesis of the mass redistribution design is presented to derive configuration-invariant properties. In Section 4, the controller design and simulations are presented. In Section 5, a conclusion is drawn.
2. Dynamic Models for Five-bar Mechanism

As suggested in [3], a closed-chain mechanism can be thought of as consisting of a free system to which constraints are applied. In closed-chain five-bar mechanism (Fig. 1), the free system is two open-chain serial links, each of which contains two links, and the constraints are two independent scleronomic holonomic constraint equations as follows:

\[
\begin{bmatrix}
    \phi_1(q') \\
    \phi_2(q')
\end{bmatrix} = \begin{bmatrix}
    L_1 \cos(q_1) + L_3 \cos(q_1 + q_3) - L_4 \cos(q_2 + q_4) - L_5 \\
    L_4 \sin(q_1) + L_3 \sin(q_1 + q_3) - L_2 \sin(q_2) - L_4 \sin(q_2 + q_4)
\end{bmatrix} = 0 \quad (1)
\]

In order to obtain the dynamic model of the five-bar mechanism, we first describe the dynamic model of the free system. For Link \(i\), the location of the center of mass which is denoted by a darkened circle shown in the figure is described by variables \(r_i\) and \(\delta_i\). Furthermore, \(m_i\) and \(L_i\) denote the mass and the length of the link, respectively, and \(J_i\) is the moment of inertia with respect to the centroid of Link \(i\). Employing the Lagrangian method may derive the dynamic model of the free system as follows:

\[
D'(q')\ddot{q}' + C'(q',\dot{q}')\dot{q}' + g'(q') = B\tau \quad (2)
\]

where \(q' = [q_1 \quad q_2 \quad q_3 \quad q_4]^T\) is the vector of the generalized coordinates of the free system. \(D'(q')\) is the inertia matrix defined as follows

\[
D'(q') = \begin{bmatrix}
    d_{11}' & 0 & d_{13}' & 0 \\
    0 & d_{22}' & 0 & d_{24}' \\
    d_{31}' & 0 & d_{33}' & 0 \\
    0 & d_{42}' & 0 & d_{44}'
\end{bmatrix} \quad (3)
\]

where

\[
d_{11}' = m_1 r_1^2 + m_3 (L_2^2 + r_2^2 + 2L_4 r_3 \cos(q_3 + \delta_3)) + J_1 + J_3, \\
d_{13}' = d_{31}' = m_3 (r_3^2 + L_3 r_3 \cos(q_3 + \delta_3)) + J_3, \\
d_{22}' = m_2 r_2^2 + m_4 (L_2^2 + r_2^2 + 2L_4 r_4 \cos(q_3 + \delta_3)) + J_2 + J_4, \\
d_{24}' = d_{42}' = m_4 (r_4^2 + L_2 r_4 \cos(q_4 + \delta_4)) + J_4, \\
d_{33}' = m_3 r_3^2 + J_3, \\
d_{44}' = m_4 r_4^2 + J_4,
\]
$C'(q', \dot{q}')q'$ is the centrifugal and Coriolis terms, and $C'(q', \dot{q}')$ is defined as follows:

$$
C'(q', \dot{q}') = \begin{bmatrix}
    h_1 \ddot{q}_3 & 0 & h_1 (\dot{q}_1 + \dot{q}_3) & 0 \\
    0 & h_2 \ddot{q}_4 & 0 & h_2 (\dot{q}_2 + \dot{q}_4) \\
    -h_1 \ddot{q}_1 & 0 & 0 & 0 \\
    0 & -h_2 \ddot{q}_2 & 0 & 0 \\
\end{bmatrix}
$$

(4)

where

$h_1 = -m_1 L_1 r_1 \sin(q_3 + \delta_3)$, and $h_2 = -m_2 L_2 r_4 \sin(q_4 + \delta_4)$.

$g'(q')$ is the gravity vector and is defined as follows:

$$
g'(q') = g \begin{bmatrix}
    m_1 r_1 \cos(q_1 + \delta_1) + m_3 (L_1 \cos(q_1) + r_3 \cos(q_1 + q_3 + \delta_3)) \\
    m_2 r_2 \cos(q_2 + \delta_2) + m_4 (L_2 \cos(q_2) + r_4 \cos(q_2 + q_4 + \delta_4)) \\
    m_3 r_3 \cos(q_1 + q_3 + \delta_3) \\
    m_4 r_4 \cos(q_2 + q_4 + \delta_4) \\
\end{bmatrix}
$$

(5)

where $g = 9.81 m/s^2$ is the gravitational acceleration constant.

$B\tau$ is the input torque. Since the actuated joints are Joint 1 and 2, where

$$
B = \begin{bmatrix}
    1 & 0 \\
    0 & 1 \\
    0 & 0 \\
    0 & 0 \\
\end{bmatrix}, \quad \tau = \begin{bmatrix}
    \tau_1 \\
    \tau_2 \\
\end{bmatrix}
$$

(6)

where $\tau_1$ and $\tau_2$ are the input torque applied to Joint 1 and 2, respectively.

Due to the selection of the actuated joints, we may parameterize the 2-dof five-far mechanisms by employing the mapping

$$
q = \begin{bmatrix}
    q_1 \\
    q_2 \\
\end{bmatrix} = \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
\end{bmatrix} q' = \alpha(q').
$$

(7)

According to the derivation in reference [10,11], the dynamic model of the 2-dof five-far mechanisms is given as follows:

$$
\begin{cases}
    D(q')\ddot{q} + C(q', \dot{q}')\dot{q} + g(q') = \tau \\
    \dot{q}' = \rho(q')\dot{q} \\
    q' = \sigma(q)
\end{cases}
$$

(8)

where

$$
D(q') = \rho^T(q')D'(q')\rho(q')
$$

(9)
\[ C(q', \dot{q}') = \rho^T(q') C'(q', \dot{q}') \rho(q') + \rho^T(q') D'(q', \dot{q}') \rho(q', \dot{q}') \]  
\[ g(q') = \rho^T(q') g(q') \]

where

\[ \rho(q') = \Psi_q^{-1}(q') \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \]  

where \( \Psi_q(q') \) is the Jacobian matrix of the following valued-vector function \( \Psi(q') = \begin{bmatrix} \phi(q') \\ \alpha(q') \end{bmatrix} \)

with respect to the vector \( q' \). The mapping \( \rho(q', \dot{q}') \) is the derivative of the mapping \( \rho(q') \) with respect to the time variable \( t \).

Solving the constraint equations can obtain the mapping \( q' = \sigma(q) \) as follows:

\[ q_3 = \tan^{-1}(\sqrt{A^2 + B^2 - C^2/C}) + \tan^{-1}(A/B) - q_2 \]

where \( A = 2L_4 \lambda, B = 2L_4 \mu \), and \( C = L_3^2 - L_4^2 - \lambda^2 - \mu^2 \), and \( \lambda = L_2 \cos(q_2) - L_1 \cos(q_1) + L_3 \),

\[ q_3 = \tan^{-1}((\mu + L_4 \sin(q_2 + q_4))/((\lambda + L_4 \sin(q_2 + q_4))) - q_1 \]  

3. MODIFICATION OF THE FIVE-BAR MECHANISM

As shown in equations (8)-(11), the dynamic model of the five-bar linkage is quite complicated. To design a control algorithm for this system to achieve high performance is not a simple task. Following the DFC concept, this section will present the modification of the mass distribution of the five-bar mechanism, with the aim to find out some invariances of mechanism, which may simplify the dynamic model of the mechanism so as to facilitate the controller design.

3.1. Configuration-Invariance of the Potential Energy (CIPE)

CIPE means that the potential energy of a system does not change with respect to a different configuration. It is sufficient for CIPE that the global center of mass (GCM) of a mechanism
should keep stationary during the operation of the system. The mass distribution is represented by \( m_i, r_i, \delta_i \), as shown in Fig. 1. The GCM of a five-bar mechanism can be expressed by

\[
r_c = \frac{1}{m} \sum_{i=1}^{4} m_i r_{ci}
\]  

(15)

where \( m = \sum_{i=1}^{4} m_i \); \( r_c \) denotes the position vector of the GCM, \( r_{ci} \) \((i=1,2,3,4)\) the position vector of the center of mass of link \( i \). The position vector \( r_{ci} \) are expressed by

\[
r_{ci} = r_i e^{i(q_i+\delta_i)}
\]

\[
r_{c1} = L_4 e^{i\theta_1} + r_3 e^{i(q_3+\delta_3)}
\]

\[
r_{c2} = L_5 + r_2 e^{i(q_2+\delta_2)}
\]

\[
r_{c4} = L_5 + L_2 e^{i\theta_2} + r_4 e^{i(q_4+\delta_4)}
\]

Substitution of the above equations into equation (15) leads to:

\[
m r_e = (m_1 r_1 e^{i\delta_1} + m_3 L_4) e^{i\theta_1} + m_2 r_3 e^{i\delta_3} e^{i(q_3+\delta_3)} + (m_3 r_2 e^{i\delta_2} + m_4 L_2) e^{i\theta_2} + m_4 r_4 e^{i\delta_4} e^{i(q_4+\delta_4)} + m_3 + m_4 L_3 + m_4 L_3
\]  

(16)

The unit vector \( e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3}, \) and \( e^{i\theta_4} \) are constrained by the kinematics loop equation, i. e.,

\[
L_1 e^{i\theta_1} + L_3 e^{i(q_3+\theta_3)} - L_4 e^{i(q_4+\theta_4)} - L_2 e^{i\theta_2} - L_5 = 0
\]

Substitution of the vector \( e^{i(q_3+\theta_3)} \), solved out of the above equation, into equation (16) leads to:

\[
m r_e = (m_1 r_1 e^{i\delta_1} + m_3 L_4 - m_3 L_3 r_3 e^{i\delta_3}) e^{i\theta_1} + (m_2 r_3 e^{i\delta_3} + m_4 L_2 + \frac{L_2}{L_3} m_3 r_3 e^{i\delta_3}) e^{i(q_3+\theta_3)}
\]

\[
+ (m_4 r_4 e^{i\delta_4} + m_3 \frac{L_4}{L_3} r_4 e^{i\delta_4}) e^{i\theta_2} + m_3 \frac{L_3}{L_3} r_3 e^{i\delta_3} + m_3 L_5 + m_4 L_5
\]  

(17)

In the above equation, in order to keep the stationary GCM, the coefficients of the vectors \( e^{i\theta_1}, e^{i(q_3+\theta_3)} \), and \( e^{i\theta_4} \) must vanish, i. e.,

\[
m_1 r_1 e^{i\delta_1} + m_3 L_4 - m_3 \frac{L_1}{L_3} r_3 e^{i\delta_1} = 0
\]  

(18)

\[
m_2 r_3 e^{i\delta_3} + m_4 L_2 + \frac{L_2}{L_3} m_3 r_3 e^{i\delta_3} = 0
\]  

(19)

\[
m_4 r_4 e^{i\delta_4} + m_3 \frac{L_4}{L_3} r_4 e^{i\delta_4} = 0
\]  

(20)
By employing equation (20), cancellation of the term $m_3 r_3 e^{i\delta_3}$ in equation (19) leads to:

$$m_3 r_3 e^{i\delta_3} + m_4 L_2 - \frac{L_2}{L_4} m_4 r_4 e^{i\delta_4} = 0$$  \hspace{1cm} (21)

From Fig.1, the following relationship holds:

$$r_1 e^{i\delta_1} = L_3 + r_3' e^{i\delta_3}$$  \hspace{1cm} (22)

$$r_4 e^{i\delta_4} = L_4 + r_4' e^{i\delta_4}$$  \hspace{1cm} (23)

Substitution of the equation (22) into the first equation in (18) yields:

$$m_1 r_1 e^{i\delta_1} - m_3 \frac{L_3}{L_3} r_3' e^{i\delta_3} = 0$$  \hspace{1cm} (24)

and substitution of the equation (23) into the first equation in (21) yields:

$$m_2 r_2 e^{i\delta_2} - m_4 \frac{L_2}{L_4} r_4' e^{i\delta_4} = 0$$  \hspace{1cm} (25)

Therefore, from the equations (24), (25) and (20), the design conditions of the configuration-invariance of the potential energy for the five-bar mechanism are

$$m_1 r_1 L_3 = m_3 L_3 r_3', \quad \delta_1 = \delta_3'$$  \hspace{1cm} (26)

$$m_2 r_2 L_4 = m_4 L_2 r_4', \quad \delta_2 = \delta_4'$$  \hspace{1cm} (27)

$$m_4 r_4 L_3 = m_3 L_4 r_3, \quad \delta_4 = \delta_3 + \pi$$  \hspace{1cm} (28)

From equations (26), (27), and (28), it can be seen that whenever the mass and the location of the center of the mass of one of the links are given, the mass distribution of the remaining three links can then be determined. Furthermore, equations (26), (27), and (28) can also be applied to determine the size and location of counterweights or negative masses that may need to be added to the mechanism for the configuration-invariance of the potential energy.

It can be verified that when equations (26), (27), and (28) hold, the following relationship exists:

$$g(q') = \rho'(q')g'(q') = 0$$  \hspace{1cm} (29)

Physically, when equations (26), (27), and (28) hold, the GCM of the five-bar mechanism is configuration-invariant. This implies that its potential energy is invariant with its configuration. Therefore the gravity in its dynamic equation is zero. At this point, the dynamic model of the mechanism is given as follows:
\[ D(q')\ddot{q} + C(q', \dot{q'})\dot{q} = \tau \]  

(30)

3.2. Configuration-Invariance of the Generalized Inertia (CIGI)

CIGI means that the generalized inertial matrix \( D(q') \) is a configuration-invariant matrix. The benefit of a dynamic system with CIGI is that a multi-variable system is decoupled into \( n \) single variable (where \( n \) is the number of variable). The work reported in [13,14] showed a parallel drive five-bar mechanism, in which the distance between the two motors is zero and the two pairs of opposite links are parallel. In this case, we have:

\[
L_1 = L_4, \quad L_2 = L_3, \quad L_5 = 0
\]  

(31)

This then leads to the following kinematics motion behavior, i.e.,

\[
q_3 = 2\pi - q_1 + q_2, \quad q_4 = q_1 - q_2
\]  

(32)

From equation (32) it follows:

\[
\rho(q') = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
-1 & 1 \\
1 & -1
\end{bmatrix}, \quad \text{and} \quad \dot{\rho}(q, \dot{q}) = 0
\]  

(33)

By employing equation (33), equation (9) is reduced to

\[
D(q') = \rho^T(q')D(q')\rho(q') = \begin{bmatrix} d_{11} & d_{12} \\
d_{21} & d_{22} \end{bmatrix}
\]  

(34)

where

\[
d_{11} = d_{11}' - 2d_{13}' + d_{33}' + d_{44}' = m_4r_4^2 + m_3L_4^2 + J_1 + m_4r_4^2 + J_4
\]  

(35)

\[
d_{22} = d_{22}' - 2d_{24}' + d_{33}' + d_{44}' = m_2r_2^2 + m_4L_2^2 + J_2 + m_3r_3^2 + J_3
\]  

(36)

\[
d_{12} = d_{21} = d_{31}' - d_{33}' + d_{44}' - d_{44}' = m_3L_4r_3\cos(q_4 + \delta_4) + m_4L_2r_4\cos(q_4 + \delta_4)
\]  

(37)

It is easy to prove that \( d_{12} = 0 \) implies that equation (28) must hold. Furthermore, equation (10) is reduced to

\[
C(q', \dot{q}') = \rho^T(q')C'(q', \dot{q}')\rho(q') + \rho^T(q')D'(q', \dot{q}')\dot{\rho}(q', \dot{q}') = \begin{bmatrix} 0 & c_{12} \\
c_{21} & 0 \end{bmatrix}
\]  

(38)
where
\[
\begin{align*}
    c_{12} &= (m_3 L_3 r_3 - m_4 L_4 r_4) \cos(q_2 + \delta_2) \\
    c_{21} &= -(m_4 L_4 r_4 - m_4 L_2 r_4) \cos(q_1 + \delta_1)
\end{align*}
\]

(39) \quad (40)

It is easy to show that when equation (28) holds:
\[
c_{12} = c_{21} \equiv 0
\]

(41)

Thus the dynamic equation at this point is reduced to:
\[
D\ddot{q} + g(q') = \tau
\]

(42)

where the generalized inertial matrix \( D = \text{diag}[d_{11}, d_{22}] \) is a constant matrix, and \( d_{11} \) and \( d_{22} \) are defined by equations (35) and (37), respectively.

It may be seen that the conditions for CIPE and CIGI have a overlapping relationship, i.e., equation (28). In order that the system possesses both CIPE and CIGI, equations (26)-(28) and equation (31) must hold simultaneously. In this case, the dynamic equation is further reduced to:
\[
D\ddot{q} = \tau
\]

(43)

3.3. Partial Configuration-Invariance of the Generalized Inertia (PCIGI)

The five-bar mechanism with a parallel structure has, however, a limited application scope. It should be interesting to consider partial CIGI for a five-bar mechanism with non-parallel structure. In particular, the impact of a partial CIGI on the design is studied.

Through some empirical studies, we have found a special situation that can lead to a simple dynamic model with a partial CIGI. Let the mass centers of the link 3 and 4 be adjusted to the ends of Link1 and Link 2, respectively, i.e.
\[
r_3 = r_4 = 0
\]

(44)

For the system to be of CIPE, equation (28) should obviously hold, and equation (26) and (27) are reduced to, respectively:
\[
m_4 r_4 = m_4 L_4, \quad \delta_4 = \pi
\]

(45)
\[ m_2 r_2 = m_4 L_2, \quad \delta_2 = \pi \]  

In this situation, the generalized inertia of the free system \( D'(q') \) is reduced to a constant matrix, and the Coriolis/centripetal matrix of the free system \( C'(q',\dot{q}') \) is a zero matrix. On the other hand, when equation (45) holds, the terms related to \( r_3 \) and \( r_4 \) in the generalized inertia \( D(q') \) are zero, and therefore, the system is partial CIGI (PCIGI). It is further noted that the terms related to \( r_3 \) and \( r_4 \) in \( D(q') \) involve trigonometric functions, which cause the generalized inertial disturbance and may deteriorate the performance of the mechanical system. Therefore the PCIGI in this case is expected to improve the performance of the system.

4. Controller Design and Simulation Studies

Theoretically, the control algorithms applied to the serial robots, for instance, the computer-torque control (CTC) method [6], and adaptive control methods based on CTC [6,7], may be used to the trajectory tracking problems for the parallel robots. The flaw of these algorithms is that their performance depends heavily on the dynamic model of mechanisms. Comparing with the serial robots, the dynamic models of the parallel robots are more complicated, so these algorithms, as anticipated, have encountered some difficulties in implementation.

In the traditional mechatronic systems, the main task was placed on the controller design after the mechanical structure design, which may be not concurrent, is finished. This caused that the control algorithms, from the PD control to the CTC to the adaptive control, became more and more complex. Consequently, the implementing of the controller become more and more difficult. On contrary, in this section, our idea is that employing the simple controller achieves the excellent performance for the complex mechanism when the last section has expended more attention on the mechanical design to reduce the dynamic models of a complex mechanism. The parallel drive five-bar mechanism with CIPE, whose dynamic equation has been reduced to the linear time-invariant system described in equation (43), is a special kind of the five-bar mechanism.
To investigate the effectiveness of our presented method, simulation studies were carried out for the five-bar mechanism of different cases with different controllers. The parameters of the five-bar mechanism under different cases are recorded in Table I. Case 1 describes the original mechanical without any mass redistribution, Case 2 describes the mechanism with only CIPE; and Case 3 describes the mechanism with CIPE and PCIGI. In order to modify easily the existing mechanism, we employed the added mass method from Case 1, to Case 2, and to case 3.

In this simulation study, the desired trajectories for two actuated links are expressed as follows:

\[
q_{1d} = 0.5\pi t, \quad q_{2d} = 2\pi\left(6\frac{t^5}{t_f^5} - 15\frac{t^4}{t_f^4} + 10\frac{t^3}{t_f^3}\right), \quad \mathbf{q}_d = \begin{bmatrix} q_{1d} \\ q_{2d} \end{bmatrix}
\]

where the scale \( t_f = 4(s) \) is the time span during which the simulation holds.

### 4.1 Analysis of Simulation 1

In this subsection, we consider tracking performance of the five-bar mechanism of three cases with the following PD controller:

\[
\mathbf{\tau} = K_p e + K_d \mathbf{\dot{e}}
\]

where the vector \( e = \mathbf{q}_d - \mathbf{q} \) is the trajectory tracking error, and \( K_p \) and \( K_d \) are the gain matrices and are positive definite. In the simulation, the gain matrices were selected to be \( K_p = 5\mathbf{I} \) and \( K_d = 10\mathbf{I} \), where \( \mathbf{I} \) expresses the second order identity matrix. The motions in simulation start from \( t_0 = 0 \) and end at \( t_f = 4(s) \), and the initial values are chosen as \( e = 0 \) and \( \mathbf{\dot{e}} = 0 \).

Comparing the simulation results, it is observed that, after the mechanical structure is carefully designed by applying the DFC method, the motion tracking performance of the mechanism is improved significantly step by step from Case 1, to Case 2, and to Case 3. Fig. 2 (a) and (c) show that the angular displacement tracking errors of two input links are largely reduced in Case 2, and are further reduced in Case 3. Fig. 2 (c) and (d) show that the angular velocity tracking errors of two input links are largely improved in Case 2, and the better results are obtained in Case 3.
From the control torque profiles shown in Fig. 2 (e) and (f), it is indicated that less control energy is consumed in Case 2 and Case 3.

In a word, the simulation results shown in Fig. 2 reveal that, by applying the DFC method, not only can the motion tracking performance be improved, but also the control energy is largely cut.

4.2 Analysis of Simulation 2

In this subsection, we compare the original five-bar mechanism with the CTC controller with the five-bar mechanism with CIPE and PCIGI with the PD controller. The CTC controller is chosen as follows:

\[ \tau = D(q')(\dot{q}_d + K_p e + K_d \dot{e}) + C(q', \dot{q}')\ddot{q} + g(q') \]  

(49)

The PD controller is the same as equation (48). The gain matrices of two controllers are chosen to be \( K_p = 10I \) and \( K_d = 50I \).

The simulation results are shown in Fig. 3. Fig. 3 (a) and (c) depict profiles of the angular displacement errors of two input links under the two different cases, which indicate that although the tracking errors are less in Case 1 with the CTC controller than in Case 3 with the PD controller, the difference between them is only by 0.00025 (rad). Fig. 3 (b) and (d) depict profiles of the angular velocity errors of two input links under the two different cases, which indicate again that although the tracking errors are in less Case 1 with the CTC controller than in Case 3 with the PD controller, the difference between them is only by 0.001 (rad/s). It is interesting to note that the maximum errors of the second input link in two cases are almost the same. In a word, Fig. 3 (a)-(d) have shown that the difference of tracking performances of Case 1 with the CTC controller and Case 3 with the PD controller is not significant. However, Fig. 3 (e) and (f), which depict profiles of the torques of the two input links, show that Case 1 with the CTC controller consumes ten times more control energy than Case 3 with the PD controller.

5. Conclusion
It is shown from the preceding discussion that the careful design of the mechanical structure with the properties such as CIPE, CIGI and PCIGI can significantly facilitate the controller design. And this can further lead to the improvement of system performance in terms of trajectory tracking errors and torques in the two servomotors. When the PD controller is applied to the case without CIPE, CIGI and PCIGI (Case 1), the case with CIPE (Case 2), and the case with both CIPE and PCIGI (Case 3), the system performance are significantly improved from Case 1, to Case 2, and to Case 3. The CTC controller applied to Case 1 does not improve the trajectory tracking performance with respect to Case 3; however, produces about 10 times higher torque in Case 1 than in Case 3.

Acknowledgement

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References


Table I

Mechanical parameters of three cases:

Case 1: original mechanical without any mass redistribution;
Case 2: CIPE;
Case 3: CIPE and PCIGI

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Fig. 1 Five-Bar Mechanism
Fig. 2. Tracking performance of five-bar mechanism with PD controller. (a) Profiles of the angular displacement error of the first input link. (b) Profiles of the angular velocity error of the first input link. (c) Profiles of the angular displacement error of the second input link. (d) Profiles of the angular velocity error of the second input link. (e) Profiles of the torque of the first input link. (f) Profiles of the torque of the second input link. (Solid line: for Case 1; dashed line: for Case 2; dotted line: for Case 3.)
Fig. 3. Tracking performances of Case 1 with the CTC controller and Case 3 with the PD controller. (a) Profiles of the angular displacement error of the first input link. (b) Profiles of the angular velocity error of the first input link. (d) Profiles of the angular displacement error of the second input link. (e) Profiles of the torque of the first input link. (f) Profiles of the torque of the second input link. (Solid line: for Case 1 with the CTC controller; dotted line: for Case 3 with the PD controller.)