On the origin and significance of subadiabatic temperature gradients in the mantle

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Received 9 November 2006; revised 11 July 2007; accepted 13 August 2007; published 27 October 2007.

[1] It is well established that the temperature gradients in the interiors of internally heated mantle convection models are subadiabatic. The subadiabatic gradients have been explained as arising because of a balance between vertical advection and internal heating; however, a detailed analysis of the energy balance in the subadiabatic regions has not been undertaken. In this paper, we examine in detail the energy balance in a suite of simple, two-dimensional convection calculations with mixed internal and basal heating, depth-dependent viscosity, and continents. We find that there are three causes of subadiabatic gradients. One is the above mentioned balance, which becomes significant when the ratio of internal heating to total surface heat flow is large. The second mechanism involves the growth of the “overshoot” of the geotherm near the lower boundary where the dominant balance is between vertical and horizontal advection. The latter mechanism is significant even in relatively weakly internally heated calculations. For time-dependent calculations, we find that local secular cooling can be a dominant term in the energy equation and can lead to subadiabaticity. However, it does not show its signature on the shape of the time-averaged geotherm. We also compare the basal heat flow with parameterized calculations based on the temperature drop at the core-mantle boundary, calculated both with and without taking the subadiabatic gradient into account, and we find a significantly improved fit with its inclusion.


1. Introduction

[2] It is often assumed that the temperature profile in the Earth’s mantle outside of thermal boundary layers is close to adiabatic due to the dominance of advective heat transfer [e.g., Schubert et al., 2001]. However, numerical simulations of convection scaled to the Earth’s mantle have shown that in the presence of internal heating, which models the effects of radioactive decay, the temperature increases with depth more slowly than would be predicted assuming adiabaticity [e.g., McKenzie et al., 1974; Sotin and Labrosse, 1999; Matyska and Yuen, 2000; Bunge et al., 2001]. Sleep [2003] and Bunge [2005] have estimated that the temperature increase from the base of the surface thermal boundary layer to the top of the basal thermal boundary layer is less, by roughly 400 K, than would be predicted if the mantle temperature profile were purely adiabatic. However, Zhong [2006] argues for only 180 K. This has significant consequences for estimates of the composition of the mantle [Mattern et al., 2005] as well as for mantle transport properties [Monnereau and Yuen, 2002]. The presence of a subadiabatic thermal gradient in the mantle would also lead to a greater temperature drop across the core-mantle boundary (CMB), that would increase estimates of the heat flow in this region [Bunge, 2005]. An increase in the estimated heat flow at the CMB would, in turn, be of significance in determining the energy budgets for the mantle and core and would imply a young inner core [e.g., Butler et al., 2005; Davies, 2007].

[3] Jeanloz and Morris [1987] presented a physical argument for the existence of a subadiabatic gradient in the interior of purely internally heated convecting systems. They assumed that subadiabatic gradients arise due to the balance between internal heating and vertical advection, and because of the asymmetry between upwellings and downwellings in internally heated flows. This argument, and similar ones [Sleep, 2003; Bunge, 2005] have been used to explain the existence of the subadiabatic gradient observed in internally heated convection models. It is also well known that an “overshoot” is often seen inside thermal boundary layers in the horizontally averaged internal temperature in convection models [e.g., McKenzie et al., 1974; Blankenbach et al., 1989], which occurs because vertical flows are forced to turn horizontally at the top and bottom boundaries. The inside of these overshoots also correspond to regions of subadiabatic gradients. We will also show that the basal overshoot becomes more pronounced as internal heating is increased, making a significant contribution to the total subadiabatic gradient.
[4] Another mechanism causing regions of subadiabaticity is only present when convection is time-dependent. Every transient plume has superadiabatic and subadiabatic regions [Matyska and Yuen, 2002]. Although this mechanism is significant, its effects on a time-averaged geotherm are very similar to those of the other two mechanisms. In section 2 we present in greater detail how these mechanisms can result in subadiabatic temperature gradients, and in section 5 we examine in detail the energy balance in regions of subadiabaticity in order to determine the relative importance of each mechanism.

[5] Depth-dependent viscosity [e.g., Gurnis and Davies, 1986; Cserepes, 1993; Bunge et al., 1996] and the thermal effects of continents [e.g., Guillou and Jaupart, 1995; Lenardic et al., 2005] are significant factors affecting convective heat transport in the mantle. In sections 6 and 7, we analyze the effects of depth-dependent viscosity and different surface boundary conditions, representing the effects of continents, on mantle subadiabaticity.

[6] Sotin and Labrosse [1999] presented a series of calculations with mixed basal and internal heating and were able to achieve excellent agreement with a parameterized model for the average temperature and surface heat flow. The CMB heat flow could not be calculated in their parameterized model based on the interior temperature, however, and they argued that the subadiabatic gradient in the lower mantle would need to be taken into account in order to achieve such an agreement. In section 9 we demonstrate that an improved fit is achieved when the subadiabatic gradient is considered.

2. Mechanisms Resulting in Subadiabatic Gradients

[7] In order to achieve the greatest possible simplicity, we consider an incompressible Boussinesq system. The temperature, \( T \), in the energy equation can then be considered as an approximation for the potential temperature in a compressible system [Jarvis and McKenzie, 1980], where we have neglected the terms due to thermal conduction along the adiabat as well as viscous dissipation, which are generally small. Using this approximation, when the temperature is constant with height, we will consider the system to be adiabatic. When temperature increases or decreases with height, the system is then said to be subadiabatic or superadiabatic, respectively. The nondimensional equation governing the temperature, \( T \), is then

\[
\frac{\partial T}{\partial t} - u \frac{\partial T}{\partial x} - w \frac{\partial T}{\partial z} + \nabla^2 T + H = 0, (1)
\]

where \( t, u, w \) and \( H \) are time, horizontal and vertical velocity and the rate of change of temperature due to internal heating, respectively. The terms in the equation (from left to right) represent the addition of heat to an infinitesimal volume by local secular cooling, horizontal advection, vertical advection, diffusion and internal heating, respectively. The equation has been nondimensionalized using scales for distance, \( d \), time, \( d^2/\kappa \), and temperature change, \( \Delta T \), where \( d \), \( \kappa \) and \( \Delta T \) are the depth, thermal diffusivity and the total temperature drop across the mantle, respectively. The resulting nondimensional internal heating parameter, \( H \), is then \( (\chi \alpha \delta \rho)/(\Delta T \kappa) \) where \( \chi \), \( \rho \) and \( \kappa \) are the internal heating rate per unit mass, density and thermal conductivity of mantle material, respectively. In all of our calculations, we have run the models to a steady state, or statistically steady state, so that the volume-averaged local secular cooling term is small. This term could also be combined with \( H \) to give an effective internal heating rate. However, we will show that in time-dependent calculations, the local secular cooling term is often locally a dominant term in the energy balance even when its volume-average is minimal.

[8] Jeanloz and Morris [1987] presented a physical argument for the existence of a subadiabatic temperature gradient in purely internally heated convective systems. They assumed that the horizontal contribution to advection and diffusion are negligible everywhere except within the boundary layers. This leaves a balance between vertical advection and internal heating,

\[
\frac{\partial T}{\partial z} = H. (2)
\]

Since \( H \) is always positive in regions of upwelling \((w > 0)\), the temperature gradient will be subadiabatic \((\partial T/\partial z > 0)\), while in regions of downwelling the temperature gradient will be superadiabatic. Physically, heat is added by internal heating to both hot rising and cold sinking parcels of fluid, leading to subadiabatic and superadiabatic temperature variations. Note that the deviation from a state where \( \partial T/\partial z = 0 \) decreases with the magnitude of the vertical velocity for a given value of \( H \). For this reason, in fast moving slabs and plumes, the temperature gradient predicted by this model is close to adiabatic. In internally heated convection, the effects of subadiabatic upwellings and superadiabatic downwellings will not cancel out when the temperature is averaged horizontally due to the asymmetry between upwelling and downwelling in internally heated convection. In the presence of strong internal heating, downwelling occurs in narrow, high-velocity regions while upwellings travel at a much lower speed and occur over a much wider horizontal distance [e.g., Jarvis and Peltier, 1982]. As a result of the greater area that the upwellings cover and because they are strongly subadiabatic, while the downwellings are only weakly superadiabatic, the mean temperature variation with depth will be subadiabatic.

[9] In order to quantify the magnitude of this effect for the Earth, we can consider the potential temperature in a rising parcel of fluid where the energy balance is given by equation (2). For the Earth, the total surface heat flow is roughly 44 TW [Pollack et al., 1993], of which roughly 8 TW comes from radioactivity in the continental crust [Hart and Zindler, 1986] resulting in 36 TW of power coming from convection in the mantle. If all of this is attributed to the combined effects of internal heating and secular cooling within the mantle (i.e., there is no flux of heat from the core), we can calculate an upper bound for \( H_{\text{eff}} = (\chi_{\text{tot}} \alpha \delta \rho)(M \Delta T \kappa) = 30.5 \). Here we have combined secular cooling and internal heating to give an effective internal heating, \( H_{\text{eff}} \) and \( \chi_{\text{tot}} = 36 \) TW and \( M \) are the total internal heating rate and total mass of the mantle \((4 \times 10^{24} \text{ kg})\), and we have used typical mantle values for \( \rho = 4500 \text{ kg/m}^3 \), \( \Delta T = 3700 \text{ K} \) and \( k = 3 \text{ W/mK} \) [Schubert et al.,
Using equation (2), we can derive an expression for the total subadiabatic temperature drop for a fluid parcel rising at a constant speed, \( w_0 \), with an energy addition given at a rate controlled by \( H \) and inverting this for \( w_0 = \frac{H}{\Delta T_{\text{sub}}} \). In order to have a subadiabatic temperature drop that is at least 10% of the total temperature drop across the mantle, we require a nondimensional mean vertical velocity of no more than \( w_0 = 305 \) or 0.18 cm/yr after dimension-alizing. This value is roughly less by a factor of 20 than the mean plate velocity of 4 cm/yr. As a result, the slow upward return flow must be significantly slower than the flows in mantle plumes or descending slabs for this mechanism to be significant. We will show it is only large in calculations with large values of \( H \) and we will refer to this mechanism hereafter as \( B_1 \). Also, using our numerical results, we will demonstrate that the balance given in equation (2) is not the only mechanism that causes mantle subadiabaticity.

Another situation where regions of subadiabatic temperature gradients are observed, occur even in purely basally heated convection. Temperature overshoots often occur just inside the top and bottom thermal boundary layers where subadiabatic gradients occur on the interior of these overshoots. They occur because vertical flows are forced to turn due to of the presence of the top and bottom boundaries. In the subadiabatic portions of the overshoot the dominant energy balance is between the vertical and horizontal contributions to advection [Jarvis and Peltier, 1982]. McKenzie et al. [1974] showed examples of increased overshoot at the bottom boundary and decreased overshoot at the top boundary when internal heating was increased. In their study of convection with mixed internal and basal heating, Sotin and Labrosse [1999] observed that a small degree of internal heating resulted in the disappearance of the overshoot at the upper thermal boundary layer. We further investigate the increasing asymmetry between the top and bottom overshoots and their cause in section 5. This overshoot mechanism will be referred to hereafter as \( B_2 \).

When the calculation is time-dependent another mechanism, the balance between local secular cooling and the other terms in the energy equation, becomes significant. This mechanism remains significant in the energy balance at a given location, even after the model is run to a statistical steady state. However, it does not make large changes to the time-averaged geotherm. This mechanism only replaces \( B_1 \) and \( B_2 \) and as a result the geotherm is not significantly affected by it. As an example, if the subadiabatic gradient due to the overshoot is displaced up or down by transient flow, the local secular cooling will be one of the dominant terms in the energy equation. From now on we will refer to this mechanism as \( B_3 \).

## 3. Numerical Model Description

[12] We solve equation (1) and the infinite Prandtl number Navier-Stokes equations using the finite difference method on \( 289 \times 289, 1153 \times 289 \) and \( 2305 \times 289 \) grids in \( 1 \times 1, 4 \times 1 \) and \( 8 \times 1 \) boxes, respectively. The calculations for \( Ra = 10^8 \) were carried out with a resolution of \( 1153 \times 1153 \). The Navier-Stokes equations were converted to a stream function vorticity pair of Poisson’s equations, which were solved using MUDPACK [Adams, 1991].

[13] Some of our models included the effect of a jump in viscosity at a nondimensional height of 0.77, which is appropriate to 670 km depth in the mantle. We have plotted the vertical viscosity profile with depth in Figure 1. We use the following equation to describe the vertical variation in viscosity, \( \mu \):

\[
\mu(z) = \left( \frac{\mu_j - 1}{2} \right) \tanh[\lambda(z_{670} - z)] + \left( \frac{\mu_j + 1}{2} \right),
\]

where \( \mu_j \) is the total jump in the viscosity and \( \lambda \) is a width parameter which we take to have a value of 50.

[14] In models that include the effects of continental lithosphere, an insulating layer of thickness \( h_c \) is placed on the top of the solution domain. It is assumed that the heat flow at the base of the continental lithosphere is the same as the heat flow at the surface and the mechanism of heat transport in the continent is purely conductive. If we take the mantle and the continental thermal conductivities to be equal:

\[
\frac{T_b - T_s}{h_c} = \frac{\partial T}{\partial z} |_{z=1},
\]

where \( T_s \) and \( T_b \) are the temperatures at the surface and the base of the continental lithosphere and we evaluate \( \partial T/\partial z \) at the base of the continent. We solve for \( T_b \) at each time step and at each horizontal position beneath the continent as it serves as the top boundary temperature for that part of the mantle that is covered by a continent. Oceanic and continental regions are modeled as free slip and no slip, respectively.

[15] In order to implement mixed surface dynamical boundary conditions, we iterate the solution using the following expression for the surface vorticity:

\[
\omega_{n+1} = \{0.5[\tanh(a(x - x_1)) + \tanh(a(x_2 - x_1))]\}(\omega_n - b \ u_n),
\]

where \( n \) is an iteration index, \( a \) describes the thickness of the transition between the free-slip region and the no-slip
region, $x_1$ and $x_2$ define the horizontal extent of the continent, $b$ is empirically chosen so as to give rapid convergence and $u_n$ is the surface velocity as of the $n$th iteration step.

[16] All of the boundaries have zero mass flux, while the sidewalls are reflecting and free slip. The bottom boundary (CMB) is also free slip and kept isothermal with a constant nondimensional temperature $T = 1$ and except for the models with a conducting lid, the isothermal top surface is kept at a nondimensional temperature of $T = 0$.

4. Diagnostics

[17] For each calculation we list the minimum and maximum temperatures along the geotherm outside the thermal boundary layers, $T_{\text{min}}$ and $T_{\text{max}}$, average tempera-
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<th>$H$</th>
<th>$\mu_2$</th>
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Notes: $M$: aspect ratio, $A_t$: Rayleigh number, $Ra$: nondimensional internal heating, $H$: total jump in viscosity, $\mu_2$: nondimensional length and thickness of the continental lithosphere, $L$ and $h$: respectively; energy balance between local secular cooling and the other terms, $B_s$, $B_c$, and $B_t$ and $C_H$ are the energy balances between vertical advection-internal heating vertical-horizontal advection, and conduction-internal heating, respectively; percentage of the points with positive vertical temperature gradient, $A_{sub}$.

Table 2. Model Energy Balance Diagnostics

- $\Delta T$: and the surface and core-mantle boundary heat flows, $Q_s$ and $Q_{sub}$ (Table 1). The difference between $T_{max}$ and $T_{min}$ gives the magnitude of the subadiabatic temperature drop, $\Delta T_{sub}$ (Table 1). We look for the terms in the energy equation with the highest positive and lowest negative value in order to determine the dominant balance at a point (Table 2). We only consider the points where the vertical temperature gradient is positive. The value of $B_s$ indicates the percentage of the total subadiabatic temperature variation arising from volumes where local secular cooling is one of the dominant terms in the energy balance. The remaining columns $B_s$, $B_c$ and $C_H$ indicate the percentage of the total subadiabatic temperature change arising from volumes where there is a dominant balance between vertical advec-
tion and internal heating, vertical and horizontal advection, and conduction and internal heating, respectively. We only list the most important energy balances in Table 2, so the numbers do not sum to 100%. The quantity $A_{sub}$ indicates the area of the domain with a positive temperature gradient. These terms are time-averaged when the solutions are time-dependent.

In section 5 we analyze a number of simple calculations to determine where subadiabatic temperature gradients occur and the exact balance in the energy equation where they do. The effects of depth-dependent viscosity are discussed in section 6. We then consider the effects of subadiabaticity on the CMB heat flow, and in section 10 we discuss our results in the context of thermal convection in the Earth’s mantle.

5. Numerical Model Results

In Figure 2 we show results of calculation M13, run with $Ra = 10^5$ and $H = 10$, where the ratio of internal heating to total heat flow (Urey ratio) is 0.77. We first consider this calculation with a relatively low Rayleigh number and unit aspect ratio because of the clarity afforded by displaying results that consist of only one convective roll in the absence of boundary layer instabilities. Figure 2a displays the steady state temperature field, while in Figure 2b we display a filled color contour plot indicating...
the dominant balance between different terms of the energy equation at each point within the box. The filled contours are overlaid by a line contour plot of the positive part of the vertical derivative of the temperature. Every integer value in the balance plot indicates a particular dominant balance between two different terms in the energy equation (described in Figure 2 caption). In order to maintain a consistent color scheme we use the same color bar for all plots of this type. As can be seen, and as is characteristic of strongly internally heated convection, the flow field is strongly asymmetric with a narrow region of rapid downwelling on the left-hand side of the box and slow upward flow occurring over the rest of the domain. The geotherm for this calculation can be seen in Figure 3a. The broad bottom overshoot in the geotherm is caused by the large area of positive vertical temperature gradient associated with the dominant balance between vertical and horizontal advection, $B_2$ (shown as green in Figure 2b). In models without internal heating, symmetric overshoots are seen adjacent to the top and bottom thermal boundary layers.

[20] Matyska and Yuen [2002] reported similar subadiabaticity above regions where cold, avalanche material had ponded at the base of a simulation with phase transitions and temperature-dependent viscosity. In calculation M13 the upwelling at the top boundary is very broad and the ambient temperature is essentially the same as the temperature in the upwelling, resulting in very little horizontal advection of heat and hence no overshoot. The cross-like pattern in Figure 2b, which can be seen in the regions where diffusion and internal heating are balancing one another, occurs at the center of the convection roll, where vertical and horizontal velocities are 0. Within much of the slow upward return flow the dominant balance in the energy equation is between internal heating and vertical advection ($B_1$), that leads to the broad area of low-amplitude positive thermal gradient (shown as orange in Figure 2b). Although the positive gradient associated with the overshoot at the base of this calculation appears to make a larger contribution to the subadiabatic gradient, the data in Table 2 indicate that mechanisms $B_1$ and $B_3$ make similar contributions to the total subadiabatic temperature variation.

[21] In Figures 3a, 3b, and 3c we show time-averaged geotherms calculated from simulations with $Ra = 10^5$ (M5, M7, M9, M11 and M13) and $10^7$ (M49, M50, M51, M52, M53, M54 and M55) in a $1 \times 1$ box and $Ra = 10^6$ (M16, M18, M23, M42, M45 and M47) in a $4 \times 1$ box, respectively, with different degrees of internal heating. The time-averaged geotherms for $Ra = 10^5$ with different $H$ in a $4 \times 1$ box looked very similar to the ones in a $1 \times 1$ box. Because of the mechanism described in the previous paragraph, as the degree of internal heating is increased, the surface overshoot decreases in magnitude and then disappears as the lower overshoot increases. It can also be seen that at $Ra = 10^7$ the geotherm interior to the overshoot and top thermal boundary is close to adiabatic for values of $H$ up to roughly 20 whereupon $B_1$ and $B_3$ mechanisms start to cause significantly subadiabatic interior gradients. This phenomenon can be observed for calculations with values of $H = 10$ and higher when $Ra = 10^6$. $B_2$ makes up a decreasing fraction of the subadiabatic gradient in the models as internal heating is increased. This phenomenon can be seen in Table 2 for the set of calculations with $Ra = 10^5$
and increasing $H$ (M5, M7, M9, M11 and M13), where the value for $B_2$ decreases from 71% to 44% and $B_1$ increases from 0 to 40% as the amount of internal heating is increased. Also, the scaled value of $T_{\text{sub}}$ increases from 296 K to 592 K, assuming a temperature drop $\Delta T = 3700$ K [Boehler, 2000]. The models with $Ra = 10^6$ show time dependence, indicated by the higher values of $B_3$, but overall, the percentage of the total subadiabatic gradient due to mechanism $B_2$ decreases with increasing $H$ while $B_1$ increases. All of the models for $Ra = 10^7$ (M49, M50, M51, M52, M53, M54 and M55) are strongly time-dependent and as a result, the most significant energy balance is due to mechanism $B_3$. Mechanisms $B_1$ and $B_2$ show similar trends as those seen for lower Rayleigh number calculations, but these are not as clear due to the dominance of $B_3$. For higher Rayleigh numbers or when the calculation is time-dependent, $B_3$ acts very much like $B_1$ and $B_2$, concealing their effects.

Figures 4a and 4b show the temperature snapshot and the energy balance plot corresponding to the same time instant, for a Rayleigh number of $10^6$ and $H = 3$ (M18). The blue color in the balance plots indicates time dependence ($B_3$). In Table 2 the time-averaged value for mechanism $B_3$ shows that it is responsible for 60% of the subadiabatic gradient for calculation M18, while $B_2$ accounts for 35% and corresponds to the green color near the CMB, in the regions of high subadiabaticity. $B_1$ (shown as orange in the balance plots) is almost insignificant in M18 because of the low internal heating, but it accounts for 16% of the subadiabatic gradient in model M47, which is almost entirely internally heated. We have not shown a similar plot for calculations with $Ra = 10^7$ or $10^8$ because the complex, short-wavelength patterns make these plots very difficult to interpret visually.

Figures 5a and 5b show the magnitude of the subadiabatic temperature drop, $T_{\text{sub}}$, and the area of the domain with positive temperature gradient, $A_{\text{sub}}$, with increasing Urey ratio (ratio of the internal heating to the surface heat flow) for $Ra = 10^5$ (M6, M8, M10, M12 and M14) and $10^6$ (M16, M18, M21, M23, M42, M45 and M47) in a $4 \times 1$ box, and $10^7$ (M49, M50, M51, M52, M53, M54 and M55) in a $1 \times 1$ box. $T_{\text{sub}}$ clearly increases with increasing Urey ratio and decreases as the Rayleigh number is increased, however, no scaling could be found. As can be seen in Figure 5, models with $Ra = 10^5$, $10^6$ and $10^7$ have values of $A_{\text{sub}}$ close to 50% up to a Urey ratio of roughly 60% whereupon $A_{\text{sub}}$ increases suddenly.

6. Effects of Depth-Dependent Viscosity

[24] We ran simulations with a surface Rayleigh number of $Ra = 10^6$ with $H = 0$ and 10 and specified total increases of $\mu_j = 10$ and 100 in viscosity (M1, M3, M25 and M27). Because of the very low effective Rayleigh number when we use $\mu_j = 100$ and $H = 0$ (M1) the interior of the geotherm was subadiabatic (not shown). Jarvis and Peltier [1982] showed in their basally heated constant viscosity calculations that the geotherm is subadiabatic for Rayleigh numbers between 5 to almost 100 times the critical value. Our analysis shows that the subadiabaticity in these cases is caused by mechanism $B_2$. We ran another set of models with increased surface Rayleigh numbers such that the surface heat flow was close to the same for the models with depth-dependent and constant viscosity in order to be able to compare models with the same effective Rayleigh number.

[25] In Figure 6 we plot the time-averaged geotherms from calculations with almost identical effective Rayleigh numbers, $H = 10$ and viscosity increases of $\mu_j = 1$, 10 and 100 (M22, M26 and M28). The average temperature decreases as we increase the total viscosity jump, because a larger temperature drop is required at the CMB when the basal thermal boundary layer becomes thicker due to the increased viscosity [Butler and Peltier, 2000]. Note that as...
the viscosity jump increases, the surface overshoot reappears and becomes more pronounced while the lower overshoot becomes broader. The reappearance of the surface overshoot in the presence of internal heating is caused by the lower average temperature in the case of depth-dependent viscosity and the greater mobility near the surface. Both of these factors enhance the horizontal advection of heat leading to a surface overshoot due to the mechanism $B_2$.

However, if the effective Rayleigh number is kept the same, depth-dependent viscosity does not lead to a significant change in the total subadiabaticity.

7. Effects of Surface Boundary Conditions

Figure 7 shows the geotherms from models with a Rayleigh number of $10^5$ and $10^6$ calculated with different surface boundary conditions including free slip, no slip, and no slip with a conducting lid of thickness 0.04 (dimensionally 116 km), as well as models with mixed free slip and conducting lid boundary conditions in a 1 x 1 box (M22, M29, M32, M35 and M38). We also conducted similar calculations in higher aspect ratio (4 and 8) boxes and the time-averaged geotherms looked very similar. The models with a stagnant lid on the top (e.g., no slip or no slip with a conducting lid) may be especially applicable to Venus and Mars [e.g., Reese et al., 1998; Solomatov and Moresi, 1996; Stevenson, 2003]. In models where the continental length is 0.4 (M32, M33 and M34) the continent is in the middle and for models with continental length 0.3 (M29, M30 and M31) it is at the left of the box. In the case when the surface is entirely covered by a conducting lid the mantle transports heat into the core because of its very high internal temperature, otherwise, all the geotherms show subadiabaticity with an overshoot at the bottom. Note that putting the conducting lid on the left or in the middle of the box does not make much difference as shown by the geotherm plots in Figure 7. The values in Table 2 show that when the surface is changed to no slip from free slip the subadiabatic contribution due to $B_2$ decreases from 62% to 42% and further decreases to only 2% when the entire surface is covered by a conducting lid. On the other hand the total contribution due to the mechanism $B_1$ increases from 4% to 36% indicating that mechanism $B_1$ becomes more important when a stagnant lid is in place. Although the mechanism causing the subadiabaticity changes, the data in Table 1 show that the total magnitude of the subadiabatic temperature drop, $T_{\text{sub}}$, is not significantly affected by the surface boundary condition in a unit aspect ratio box. In section 8 we discuss the effects due to larger aspect ratio boxes.

8. Effects of Aspect Ratio

In larger aspect ratio boxes, solutions become more time-dependent and as a result, $B_3$ becomes increasingly important as a mechanism causing subadiabaticity as can be seen in the results listed in Table 2. The data in Table 1 show that in almost all of the models with a free or mixed surface boundary, $T_{\text{sub}}$ stays the same or increases slightly as the viscosity jump increases, the surface overshoot reappears and becomes more pronounced while the lower overshoot becomes broader. The reappearance of the surface overshoot in the presence of internal heating is caused by the lower average temperature in the case of depth-dependent viscosity and the greater mobility near the surface. Both of these factors enhance the horizontal advection of heat leading to a surface overshoot due to the mechanism $B_2$. However, if the effective Rayleigh number is kept the same, depth-dependent viscosity does not lead to a significant change in the total subadiabaticity.

Figure 6. Geotherms from calculations M22, M26, and M28 with $H = 10$ and viscosity jumps of $\mu_j = 1, 10$, and 100 with depth, respectively, using the same effective Rayleigh number.
the aspect ratio of the box is increased. When complete or partial free-slip surface boundary conditions are used, wider boxes result in longer wavelength convection cells and enhanced horizontal advection of heat near the lower boundary. Consequently, $B_2$ becomes more important causing a greater bottom overshoot and hence increased subadiabaticity.

[25] If the surface is no slip or is completely covered by continental lithosphere, the wavelength of the convection cells decreases when the aspect ratio of the box is increased. With this boundary condition, more downwellings occur and they are caused by closely spaced surface thermal boundary layer instabilities. Because of the shorter wavelength, $B_2$ is less significant and hence the total subadiabaticity is decreased. The results from $4 \times 1$ and $8 \times 1$ boxes were essentially identical, indicating that an aspect ratio of 4 is sufficient to analyze the energy balances causing the subadiabaticity.

9. Effects of Subadiabatic Temperature Gradients on Heat Flow at the CMB

[29] In Figure 8 we present the time-averaged geotherms for calculations carried out with the Rayleigh numbers indicated on Figure 8 and with an internal heating rate of $H = 10$ (M13, M22, M50 and M57). As has been pointed out by Sotin and Labrosse [1999] and Butler and Peltier [2000] in calculations with mixed basal and internal heating, the mean internal temperature is a decreasing function of the thermal Rayleigh number for a fixed internal heating rate. Sotin and Labrosse [1999] presented scaling results for the average temperature and for the surface heat flow based on their three-dimensional numerical results, and our two-dimensional results are in excellent agreement with their scaling. Of interest is that as the thermal Rayleigh number increases, the surface boundary layer thickness, $\delta_s$, decreases as roughly $Ra^{-1/3}$ while the internal temperature, and hence the temperature drop across the surface thermal boundary layer, $\Delta T_s$, decreases. The surface heat flow can be calculated from $Q = \Delta T_s / \delta_s$. As a result, the effects offset one another, but the boundary layer thickness decreases more rapidly leading to an increase in the surface heat flow with increasing thermal Rayleigh number, but the increase is much slower than in the purely basally heated case. At the base of the convecting system, however, the decrease in the average internal temperature increases the temperature drop, $\Delta T_b$, and the basal boundary layer thickness, $\delta_b$, decreases as roughly $Ra^{-1/3}$ so that both effects should lead to an increase in the basal heat flow. In an equilibrium state, however, the heat flow at the base is linked to the heat flow at the surface by $Q_s = Q_b = H$ where $Q_s$ and $Q_b$ represent the total heat flows at the CMB and surface, respectively. The effect of the subadiabatic gradient is to increase the temperature drop at the CMB and it decreases with increasing thermal Rayleigh number for fixed $H$ as can be seen from the results in Table 1. As we will show, the decrease in the subadiabaticity allows for the energy balance at the CMB.

[30] That the subadiabatic gradient is important can be easily verified by considering the run with $Ra = 10^8$ and $H = 10$ (M13) for which there would be practically no temperature drop at the CMB in the absence of a subadiabatic gradient while in this case the CMB heat flow accounts for roughly 23% of the surface heat flow. The scaling results for the surface heat flow of Sotin and Labrosse [1999] used the average internal temperature, $\langle T_i \rangle$, in order to calculate the temperature drop at the surface and this was related to the heat flow at the surface by a relationship roughly of the form $Q_s = (Ra/Ra_{crit})^{1/3} \langle T_i \rangle^{4/3}$, as would be expected from boundary layer theory where $Ra_{crit}$ is the critical boundary layer Rayleigh number [e.g., Butler and Peltier, 2000]. In Figure 9 we present the surface heat flow $Q_{meas} = (\partial T/\partial z)|_{z=1}$, calculated from numerical models run with $Ra \geq 10^6$, $H > 0$, constant viscosity and free-slip boundaries, plotted versus $Q_{pred} = (Ra/Ra_{crit})^{1/3} \langle T_i \rangle^{4/3}$ for $T = T_{max}$ (+ signs) and $T = \langle T_i \rangle$ (squares). For both $T = T_{max}$ and $T = \langle T_i \rangle$ we have chosen $Ra_{crit}$ in order to give the smallest least squares misfit between the predicted and measured value of $Q_s$. The values obtained for $Ra_{crit}$ in this way were 23.8 and 28.2, respectively. We have plotted a straight line with a slope of

Figure 7. Geotherms from the calculations M22, M29, M32, M35, and M38 with $H = 10$ and different surface boundary conditions for $Ra = 10^8$.

Figure 8. Geotherms calculated for simple Boussinesq calculations M13, M22, M50, and M57 in a $1 \times 1$ box with $H = 10$ and different thermal Rayleigh numbers.
was thought to be caused by the balance between vertical advection and internal heating. However, our study shows that there are three different mechanisms responsible for the subadiabaticity. We find that $B_2$ (balance between horizontal and vertical advection) is significant for calculations with lower degrees of internal heating, whereas $B_1$ (balance between vertical advection and internal heating) becomes increasingly important with increasing internal heating. $B_3$ (balance between local secular cooling and the other terms in the energy equation) can play the role of either $B_1$ or $B_2$ in a temporally averaged geotherm. When the model is not time-dependent, $B_1$ is mostly responsible for the subadiabaticity within the bulk of the mantle, whereas $B_2$ mostly produces the bottom geotherm overshoot. Our results suggest that the bottom overshoot makes the largest contribution to the subadiabaticity for values of $H$ smaller than roughly $Q_{o0}$, where $Q_{o0}$ is the surface heat flow in the absence of internal heating. The total subadiabaticity and the area with a subadiabatic gradient increase with increasing Urey ratio.

[34] Our models with depth-dependent viscosity and mixed heating indicate that the surface overshoot reappears as we increase the total jump in viscosity. The surface overshoot may be responsible for the presence of the seismic low-velocity zone (LVZ) [Dziewonski and Anderson, 1981]. It is interesting that a low-velocity channel, such as the asthenosphere, may cause a larger thermal overshoot near the surface which may in turn reinforce the low-velocity channel if viscosity is temperature dependent.

[35] In the case of mixed surface boundary conditions, the shape of the geotherm does not depend on the position of the continent; however, the presence of a rigid conducting lid increases the subadiabatic gradient in the bulk, implying that Mars and Venus may have significantly subadiabatic interiors.

[36] If we consider an Earth-like surface heat flow, then calculations M41 to M45 are most comparable to our planet, and have Urey ratios of 72% to 87%. This suggests that the mantle subadiabaticity may be as large as 450 K and 55% to 60% of the volume of the mantle may have a subadiabatic

\begin{align*}
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temperature gradient. Here we are taking the effective internal heating rate for the Earth to be the sum of the secular cooling and actual internal heating. In these calculations all three balance mechanisms are active.

[37] We have shown that the effects of subadiabaticity are important for parameterizing the CMB heat flow. As an example, if we consider M38, as Earth-like, this suggests a mantle subadiabaticity of $T_{\text{sub}} = 407$ K. This would change the total CMB heat flow by 3.8 TW, if we assume a core-mantle boundary layer thickness equal to average thickness of D$^\ast$ layer, which is 260 km [Kendall and Shearer, 1994] and a thermal conductivity of 16 W/mK [Brown, 1986].

[38] We have identified and quantified the various effects by which subadiabaticity is produced in simple models of internally heated infinite Prandtl number convection. In future work, it will be interesting to explore the effects of compressibility, temperature-dependent viscosity, sphericity, better representations of surface plates and the effects of mantle phase transitions and how these affect the mechanisms, that we have identified here, responsible for the subadiabatic gradients.

[39] Acknowledgments. We would like to thank Citirad Matyska, David Yuen, and an anonymous referee for their helpful comments in reviewing this manuscript. Thanks go also to Simona Costin and Chad Glemsner for proofreading the paper. This work was supported financially by the Natural Sciences and Engineering Research Council of Canada.

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