The combined effects of continents and the 660 km-depth endothermic phase boundary on the thermal regime in the mantle

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ABSTRACT

We explore a wide range of parameter space to investigate the dynamical interaction between effects due to surface boundary conditions representing continental and oceanic lithosphere and the endothermic phase boundary at 660 km-depth in simple two-dimensional convection calculations. Phase boundary induced mantle layering is strongly affected by the wavelength of convective flows and mixed surface boundary conditions strongly increase the horizontal wavelength of convection. Our study shows that for mixed cases the effects of the surface boundary conditions dominate the effects of the phase boundary.

1. Introduction

Although both the effects of layering, caused by the spinel to perovskite and magnesiowüstite phase change at 660 km-depth, and continents have been widely studied separately, their combined effects on mantle dynamics have not been investigated previously. Many length scales are involved when modeling thermal convection within the mantle when incorporating continents and the endothermic phase boundary at 660 km-depth. These include the lateral extents (Lenardic and Moresi, 2003) and thicknesses (Lenardic and Moresi, 2001) of the continental lithosphere. In the case of a layered mantle, the depth to the phase boundary can play an important role in the flow dynamics by partially separating the mantle into two different convective regimes and decreasing the surface heat flow. The depth of the mantle is always important and the chosen aspect ratio of the box used for modeling can introduce further length scales.

In a laboratory tank experiment, Guillou and Jaupart (1995) showed that when the surface has variable thermal boundary conditions representing oceanic and continental lithospheric areas, the upwelling tends to be beneath the continent and the convection cells become elongated. Grigné et al. (2007b) investigated how the presence of a region with a lid of finite conductivity affects the wavelength of mantle convection and forces the zone of upwelling under the continent while the cold downwellings form at a distance from the continent. Their study also found that the presence of a partial lid produces longer wavelength convection cells with increasing Rayleigh numbers in contrast with Guillou and Jaupart (1995) who showed no effect of cell size with different values of the Rayleigh numbers. This difference is likely due to the difference in the mechanical boundary conditions.

Being less dense and therefore, more buoyant than the mantle, continents do not participate in mantle overturn. Both numerical- and laboratory-based models have been created to investigate the dynamic interaction between the continental lithosphere and the mantle (Gurnis, 1988; Zhong and Gurnis, 1993; Guillou and Jaupart, 1995; Lowman and Jarvis, 1995, 1996; Lenardic and Kaula, 1995, 1996; Lenardic and Moresi, 2001; Moresi et al., 2005).
Continents significantly affects the mantle cooling rate (Guillou and Labrosse, 2001) showed that the thermal blanketing effect of perovskite (Akaogi et al., 2007). The reaction has a negative Clapeyron viscosity, cooling could actually be enhanced by continents. Lenardic et al. (2005) showed that, in the presence of temperature-dependent mantle materials provides additional buoyancy enhancing mantle layering to be decreased in the presence of partial continental coverage (Grigné et al., 2007b). One might expect longer wavelength convection cells (Grigné et al., 2007b), one might expect longer wavelength convection cells. Since the presence of a partial lid causes longer wavelength convection cells and hence, stronger horizontal advection. As we will demonstrate, this causes the geothermal gradient to be subadiabatic despite the absence of internal heating in these cases.

In the following section, we describe our numerical models and subsequently we define our model diagnostics. Our calculations and their control parameters and calculated diagnostics have been listed in Table 1. We discuss the effects of the endothermic phase boundary in the presence of different surface boundary conditions in Section 4.1. The effects of different lengths of the continent and internal heating on layering will be presented in Section 4.2. Finally, in Section 4.3 we demonstrate that the mantle geotherm is substantially subadiabatic in the presence of mixed surface boundary conditions. Our final section summarizes the results and findings.

2. Numerical model description

We employed two numerical models, one finite-difference and one finite-element. The finite-element simulations employed the commercial software package COMSOL. The finite-element solver was found to be more stable for simulations with large Rayleigh numbers. A number of repeat simulations indicate that the results of the two methods are very similar. Finite-element simulations are indicated with footnote ‘a’ in Table 1. The infinite-Prandtl number Navier–Stokes equation for a Boussinesq fluid and an energy equation incorporating the buoyancy and the thermal effects of an endothermic phase boundary were solved in two-dimensional 1 × 1, and 8 × 1 boxes using 289 × 289, and 2305 × 289 grids, respectively, for the finite-difference calculations. All of our finite-element models were calculated in 8 × 1 boxes with 37,084 Lagrange–Quadratic elements and denser mesh was used along the horizontal and the endothermic phase boundaries. The dimensionless governing equations are

\[ T \frac{\partial k}{\partial t} - \beta \frac{\Delta p}{\rho \Delta T} \frac{\partial k}{\partial t} - \nabla p + \nabla^2 \mathbf{U} = 0, \]  

Table 1

Models, M; Rayleigh number, \(Ra\); non-dimensional internal heating rate, \(H\); Clapeyron slope, \(\gamma\) (\(\times 10^{	ext{6}}\) Pa/K); non-dimensional length and thickness of the continental lithosphere, \(L\) and \(h\), respectively; average temperature, \(T\); non-dimensional surface heat flow or the Nusselt number over the continent, \(Nu_l\) and over the ocean, \(Nu_o\); mass flux across the phase boundary, \(M_j\) (\(\times 10^{	ext{3}}\)); thermal layering parameter, \(\beta\); number of convection cells in the lower and upper mantle, \(C_{\text{lm}}\) and \(C_{\text{um}}\), respectively.

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<th>Model</th>
<th>M</th>
<th>(Ra)</th>
<th>(H)</th>
<th>(\gamma)</th>
<th>(\rho)</th>
<th>(\alpha)</th>
<th>(\Delta T)</th>
<th>(\gamma_l)</th>
<th>(\Delta T_l)</th>
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<td>-0.06</td>
<td>7.5</td>
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</tr>
</tbody>
</table>

\(a\) Computations were carried out using finite-element method.

\(b\) Surface contains two separate continents, located at the either end of the box covering 15% each.

\[ \mathbf{\nabla} \cdot \mathbf{U} = 0, \]

and

\[ \left[ 1 + 2\Gamma(1 - \Gamma) \gamma h \rho \phi \right] \left( \frac{\partial T}{\partial t} + \mathbf{U} \cdot \nabla T \right) = \frac{1}{Ra} \nabla^2 T - 2\Gamma(1 - \Gamma) \frac{h d T}{\partial \phi} \nabla H. \] (3)

In the above equations, \(T, \mathbf{U} = [u, v]\), \(p\) and \(t\) are the temperature, velocity, pressure and time, respectively, while \(Ra, H, g, \rho, \alpha, \Delta T, \gamma, c_p\) and \(d\) are the Rayleigh number, non-dimensional internal heating rate, acceleration due to gravity, density of the mantle, thermal expansion coefficient, total temperature difference across the mantle, specific heat capacity at constant pressure and the depth of the whole mantle, respectively. Time and length are scaled with \(d^2/(\kappa Ra)\) and \(d\), where \(\kappa\) is the thermal diffusivity. The symbols \(\gamma, L, \Delta \rho\) and \(h\) are the Clapeyron slope of the endothermic phase boundary, latent heat, density jump across the phase boundary and a thickness parameter, respectively, and

\[ \Gamma = \frac{1}{2}(1 + \tanh \theta), \] (4)

where

\[ \theta = \frac{h d}{\gamma} \left[ \int_{z_{660}}^{z} \frac{\gamma \Delta T}{\rho g d} (T - (T)_{660}) - 2 \right]. \] (5)

Here \(z\) is the height and \((T)_{660}\) is the average temperature at 660 km-depth. The height at which \(\theta = 0\) gives the height of the phase boundary, which we define as \(z_p\). In our calculations we use \(g = 10 \text{ ms}^{-1}, \rho = 4000 \text{ kg m}^{-3}, \alpha = 2.5 \times 10^{-5} \text{ K}^{-1}, \Delta T = 3700 \text{ K}, c_p = 1200 \text{ J kg}^{-1} \text{ K}^{-1}, d = 2880 \text{ km}, \Delta \rho = 440 \text{ kg m}^{-3}, z_{660} = 0.77 \text{ km} and \(h = 3.5 \times 10^{4} \text{ m}^{-1}\). We calculate \(t\) using the Clausius–Clapeyron relation.

In our finite-difference models, the Navier–Stokes equation was converted to the following stream-function (\(\psi\)) vorticity (\(\omega\)) pair of Poisson’s equations, which were solved using MUDPACK (Adams, 1991).

\[ \nabla^2 \omega = \frac{\partial}{\partial x} \left[ 1 + 2\Gamma(1 - \Gamma) \frac{\rho h \gamma}{\gamma g \alpha} \right], \] (6)

\[ \nabla^2 \psi = \omega. \] (7)

In models that include the effects of continental lithosphere, an insulating layer of thickness \(h_i = 0.04d\) (approximately 115 km) is...
placed on the top of the solution domain. It is assumed that the heat flow across the base of the continental lithosphere is the same as the heat flow at the surface and the mechanism of heat transport within the continent is entirely vertical conduction. If we take the mantle and the continental thermal conductivities to be equal, the heat flow balance at the base of the continental lithosphere can be written as

\[
\frac{T_s - T_b}{h_c} = \frac{\partial T}{\partial z}
\]

where \(T_s\) and \(T_b\) are the temperatures at the surface and the base of the continental lithosphere and we evaluate \(\partial T/\partial z\) at the top of the mantle. We solve for \(T_b\) at each time-step and at each horizontal position beneath the continent as it serves as the top boundary temperature for the subcontinental mantle. We define \(L\) as the length of the continent. Oceanic and continental regions are modeled as free-slip and no-slip.

All boundaries in all models have zero mass flux, while the side walls are reflecting. The bottom boundary (CMB) is free-slip and kept isothermal with a constant non-dimensional temperature, \(T = 1\), and the isothermal top surface is kept at a non-dimensional temperature of 0 in regions with free-slip. All of our models are isoviscous.

3. Diagnostics

To calculate \(\beta\), the amount of heat carried across 660 km-depth by conduction and advection are required. If the conductive heat flux is \(Q_{\text{cond}}\) and the advective heat flux is \(Q_{\text{adv}}\), then

\[
\beta = \frac{Q_{\text{cond}}}{Q_{\text{cond}} + Q_{\text{adv}}},
\]

where

\[
Q_{\text{cond}} = - \int_0^L \left( \frac{\partial T}{\partial z} \right) \left( \frac{\partial z_p}{\partial x} \right) dx,
\]

\[
Q_{\text{adv}} = - \int_0^L u(T) \left( \frac{\partial z_p}{\partial x} \right) - \nu T \left( \frac{\partial z_p}{\partial x} \right) dx.
\]

We define \(M_f\) as

\[
M_f = \int_0^L \left( \frac{|v - u(T)\partial z_p/\partial x|}{\sqrt{1 + (\partial z_p/\partial x)^2}} \right) dx,
\]

where \(z_p(x)\) is the location of the moving phase boundary and \(L\) is the length of the box. All of the above quantities are evaluated at height \(z_p(x)\).

The Nusselt number is calculated by averaging the vertical gradient of the temperature along the top boundary, while \(T\) gives the average temperature of the solution domain. The variables \(C_m\) and \(C_{um}\) are the number of convection cells calculated by counting the zero-crossings of \(\psi\) along the mid-depths of the lower and the upper mantle. The total mass flux is zero between two adjacent locations where \(\psi = 0\) along one upwelling and one downwelling indicating one complete convection cell. All models were run to a statistically steady state and then the above-mentioned quantities were time-averaged. All these diagnostics for different models are listed in Table 1.

We also calculate the critical thermal boundary layer Rayleigh number, \(R_{\alpha}\), as a measure of the thermal resistance of the boundary layer. From boundary layer theory (Schubert et al., 2001) it can be shown that

\[
Nu = \left( \frac{Ra}{Ra_0} \right)^{1/3} \delta T^{4/3}
\]

Fig. 1 shows temperature snapshots overlain by the stream function from models with free-slip (a), conducting lid (b) and mixed (c) surface boundary conditions. A Rayleigh number of \(10^6\) was used and the Clapeyron slope is 0 in these models (M1, M7 and M14). The black horizontal line indicates the fixed position of the endothermic phase boundary. When the surface is free-slip, \(R_{\alpha}\) values range between 13.4 and 35.6, which is in agreement with previous studies (Honda, 1996; Sotin and Labrosse, 1999; Butler and Peltier, 2000; Sinha and Butler, 2007).

4. Numerical model results

4.1. Effects of surface boundary conditions and aspect ratio on layering

Fig. 1 shows temperature snapshots overlain by the stream function from models with free-slip (a), conducting lid (b) and mixed (c) surface boundary conditions. A Rayleigh number of \(10^6\) and \(\gamma = 0\) MPa K\(^{-1}\) for a free-slip (M1), (b) conducting lid (M7) and (c) mixed (M14) surface boundary conditions. The black horizontal line represents the location of the phase boundary.

From Eq. (13),

\[
R_{\alpha} = \frac{\delta T^4 Ra}{Nu^4},
\]

where \(Ra\) and \(Nu\) are the Rayleigh number and the Nusselt number and \(\delta T\) is the temperature difference across the thermal boundary layer. When the surface is free-slip, \(R_{\alpha}\) values range between 13.4 and 35.6, which is in agreement with previous studies (Honda, 1996; Sotin and Labrosse, 1999; Butler and Peltier, 2000; Sinha and Butler, 2007).
Fig. 2. Temperature fields overlain by the streamlines for calculations with $Ra = 10^6$ and $\gamma = -9 \text{MPaK}^{-1}$ for (a) free-slip (M6), (b) conducting lid (M13) and (c) mixed (M30) surface boundary conditions. The black horizontal line represents the location of the phase boundary.

The layering diagnostics from calculations in $8 \times 1$ boxes are shown in Figs. 5–7. Here also the mixed surface boundary models of the Clapeyron slope have very large convection wavelength in the lower mantle, shown in Fig. 2 a–c, because the lower boundary always remains free-slip and there is some degree of mass flux across the phase boundary. Although calculations with mixed and complete free surfaces are very different in planform in the absence of strong layering (Fig. 1 a and c), they become quite similar when strongly layered as can be seen by comparing Fig. 2 a and c. Note the multiple weak cold downwellings in the upper mantle for free and mixed cases due to very strongly layered mantle convection. We say weak because they are unable to penetrate through the phase boundary and also they cause deflections but not zero-crossings of the stream function. Carried by the dominant long-wavelength mantle flow, the weak surface boundary layer instabilities migrate and accumulate together before penetrating through the phase boundary into the lower mantle. The deflection of the phase boundary depends on the temperature anomaly at the boundary only, however, the total thermal density anomaly depends on the temperature above and below the phase boundary as well. As a result, anomalies of significant vertical and horizontal extent are more likely to penetrate the phase boundary and the accumulation of the boundary layer instabilities partially explains the tendency towards long-wavelength flows seen when large magnitudes of the Clapeyron slope are used.

In order to examine the thermal degree of layering, we plot $\beta$ in Fig. 3 as a function of the Clapeyron slope in the presence of different surface boundary conditions for calculations with $Ra = 10^6$, performed in a unit aspect ratio box. Mixed surface boundary models have 30% continental coverage on one side of the box. All of the calculations in square boxes show a sudden change from slightly negative $\beta$, caused by a positive temperature gradient at 660 km-depth due to latent heating, to almost 1, indicating that almost all of the heat transport across the phase boundary is by conduction when the magnitude of the Clapeyron slope of the endothermic phase boundary is greater than 6 MPaK$^{-1}$. This result is consistent with the free-slip results of Christensen and Yuen (1985). A plot of the Nusselt number for the same calculations can be seen in Fig. 4 where a similar but downward jump is present, which also occurs in the mass flux across 660 km-depth (not shown). At $Ra = 10^7$, a similar phenomenon was observed (not shown) with the jump occurring between the Clapeyron slopes of $-3 \text{MPaK}^{-1}$ and $-4 \text{MPaK}^{-1}$ due to the increased degree of layering with increased Rayleigh number (Christensen and Yuen, 1985).

The layering diagnostics from calculations in $8 \times 1$ boxes are shown in Figs. 5–7. Here also the mixed surface boundary models...
have 30% coverage with the continent located on one side of the box (as shown in Figs. 1c and 2c). The plots show that the change in the degree of layering is much more gradual with increasing magnitude of the Clapeyron slope, for all of the different surface boundary conditions, than in the 1 × 1 case. The thermal layering (β) across 660 km-depth starts to increase when the magnitude of the Clapeyron slope is more than 3 MPa K⁻¹. However, at the largest magnitude of the Clapeyron slope β is less than in the case of square boxes, indicating that most of the heat is still being carried by advection across 660 km-depth and that the models are still only partially layered. This demonstrates that the degree of layering is strongly dependent on the aspect ratio of the box, which, for the case of a 1 × 1 calculation, strongly influences the wavelength of the convective rolls. This is in agreement with the results of Tackley (1995). In wider boxes the increased freedom allows for larger aspect ratio convection cells, causing a decreased degree of thermal layering.

As can be seen in Fig. 5, rigid-lid calculations have the largest thermal layering due to the short-wavelength mantle flows despite the decreased effective Rayleigh number associated with the thermal insulation and viscous drag of the continents. The fraction of heat carried by conduction, as parameterized by β, is very similar for all degrees of layering for the models with free and mixed surface boundary conditions, except for very large magnitudes of the Clapeyron slope (Fig. 5). This similarity is partly due to the fact that the total heat flow, as well as the total conduction of heat across 660 km-depth is less in the models with mixed surface boundary conditions. However, the surface heat flow and the mass flux across 660 km-depth are almost identical only for strongly layered models for these two types of calculations (Figs. 6 and 7), despite the significant difference in simulations with lesser Clapeyron slopes. The negative values of β in Fig. 5 are due to the inverted geotherms caused by the hotter upper mantle due to latent heat release.

Fig. 6 shows the variation in the average mass flux across the phase boundary as a function of the Clapeyron slope. Enhancement in convection due to the latent heating effect causes a very small increase in the mass flux for low magnitudes of the Clapeyron slope. At large magnitudes of the Clapeyron slope, models with free-slip surfaces show the most significant drop in mass flux due to layering. Both the continental region of the mixed cases and the models with total rigid lids have similar mass flux values and they are much less than the values under oceanic regions due to the drag at the surface. In the plot, mass flux for the mixed cases are the weighted average of the mass flux under the continental and oceanic regions from the corresponding models.

In Fig. 7, we plot the average surface heat flow with different magnitudes of the Clapeyron slope. Here also, we calculated continental and oceanic heat flows separately for models with mixed surface boundary conditions. The initial slight increase in the surface heat flow that is seen in many models for very low magnitudes of the Clapeyron slope is again due to the latent heating effect. Models with free-slip surface boundaries undergo a surface heat flow reduction of more than 30% or an absolute decrease of 5.9 with increased magnitudes of the Clapeyron slope. For total continental coverage it is close to 20% or 1.2 in absolute terms, while for mixed surface boundary calculations the absolute change is only 0.9 or 7%. We interpret this very small absolute change to arise because the mixed surface boundary conditions has a more significant effect on convective planform than the phase transition. As a result, the planform of convection is only weakly affected by the phase transition. In contrast, the models with free-slip surfaces show a significant change in planform with Clapeyron slope resulting in a large change in the surface heat flow. The heat flow over the continental regions for mixed cases show no dependence on Clapeyron slope and over the oceanic region the total decrease in heat flow is also very small. Since these models have no internal heating and are run to a statistical steady state, the average surface heat flow is the same as the CMB heat flow.

In order to isolate the effects of reduced mass flux on heat flow for different surface boundary conditions, we plot Nusselt number as a function of average mass flux across 660 km-depth boundary in Fig. 8. Note that the oceanic and the continental parts of the mixed cases plot along similar trends as the free-slip and the rigid-lid cases. However, the total variations in the surface heat flow as well as the mass flux under the oceanic region for mixed cases are much less than that for free-slip cases. Under the oceanic lithosphere the average surface heat flow shows significant correlation with the mass flux at 660 km-depth. However, under the continental lithosphere, the average mass flux is much smaller even without layering. In the rigid-lid case the slope of Ñμ vs. M̃f is quite shallow, indicating a large change in the mass flux results in only a moderate change in the heat flow. Because of the dominance of the effects of the boundary condition on flow planform, the mass flux in the oceanic region for the mixed case decreases significantly less than for the free-slip case, resulting in a smaller decrease in surface heat flow.

The critical upper boundary layer Rayleigh number (R̃aₜ) for the same calculations that are shown in Fig. 8, is plotted against average mass flux in Fig. 9. The Nusselt number, Ñμ, is decreased by layer-
ing both because of the decrease in $\Delta T$ at the surface due to the temperature drop caused by the internal thermal boundary layer at 660 km-depth and the different convection planform caused by the interruption of flow at that depth. The parameter $R_\alpha$ isolates the effect of planform and can be used as a measure of the thermal resistance across the upper boundary or the decrease in the effective upper mantle Rayleigh number caused by layering. In the plot, $R_\alpha$ is much less for the oceanic than the continental regions because of the drag induced by the continent. Models with free-slip surfaces show a decrease in $R_\alpha$ with mass flux, indicating lesser heat flow efficiency in the upper mantle for large magnitudes of the Clapeyron slope, which along with the decrease in $\Delta T$ is responsible for the large change in the Nusselt number shown in Fig. 7. The oceanic part of the mixed surface calculations does not show any significant change in the thermal resistance of the upper mantle flows indicating that there is no significant change in the planform. Under complete lids, the critical upper boundary layer Rayleigh number surprisingly increases with mass flux, which indicates that the upper boundary layer transports heat more efficiently in the presence of stronger layering. When not layered, the convective planform is affected by the free-slip lower boundary and the depth of the mantle, however, when layered, the upper mantle planform is affected by the depth to the phase boundary and hence has a shorter wavelength. This planform transports heat more efficiently under a rigid lid. This increased efficiency in continental regions explains the slight change in the average surface heat flow in the calculations with conducting lid surface boundary condition. Models with mixed surface boundary conditions also show an increase in the number of cold downwellings under the continent for large magnitudes of the Clapeyron slope (compare Figs. 1c and 2c) leading to a similar increase in the heat transport efficiency. The Nusselt number for complete lid models increases with mass flux because $\Delta T$ increases as the flow becomes less layered and the temperature drop at 660 km-depth decreases, which, in this case is more significant than the increase in the boundary layer efficiency. However, $\Delta T$ decreases much less for the continental part of the mixed cases with Clapeyron slope due to the weaker thermal layering. This combined with the increased efficiency in the upper boundary layer causes $Nu$ to be essentially independent of $M_f$ ($M_{14}, M_{19}, M_{20}, M_{28}, M_{29}$ and $M_{30}$).

In Fig. 10, the black and the grey bars correspond to the time-averaged number of convection cells in the lower ($C_{\text{lm}}$) and the upper mantle ($C_{\text{um}}$). Differences in the heights of the black and grey bars at a particular Clapeyron slope indicate separate convective regimes in the upper and the lower mantle, or decoupled mantle convection due to the endothermic phase boundary. We plot the results for all three different boundary conditions. Decoupling is evident in all calculations with large Clapeyron slopes.

In Fig. 10b, results of calculations where the surface is completely covered by a conducting lid are shown. A significant increase in the number of convection cells can be observed in the upper mantle for strongly layered cases. This indicates the presence of several short-wavelength convection cells driven by the surface boundary layer instabilities above the phase boundary. The short-wavelength flows in the upper mantle are easily blocked by the phase boundary resulting in a strong internal thermal boundary layer along the endothermic phase boundary producing large values of $\beta$.

The models with free-slip and mixed surfaces (Fig. 10a and c) show a lesser degree of decoupling between the upper and the lower mantle than the rigid-lid models. Although there is an increase in the upper mantle downwellings when strongly layered for the free-slip and the mixed cases, they are not sufficiently strong to cause zero-crossings of the stream function and are laterally swept by the dominant long-wavelength flows. The smallest number of convection cells, indicating long-wavelength
flows, are obtained for mixed cases, regardless of the value of the Clapeyron slope. For all of the surface boundary conditions used, the convection wavelength in the lower mantle is longest when large magnitudes of the Clapeyron slope are used because flow is driven by the long-wavelength convection cells that penetrate the phase boundary. Models with mixed surfaces show relatively little change in the number of rolls with Clapeyron slope. For all of the surface boundary conditions used, the oceanic region (see Nuo wavenumber circulation as seen by Lenardic et al. (2005). We changed the length of the continent starting from no lid to total coverage for every series. Our results are shown as grouped bar plots in Fig. 11 (similar to Fig. 10). The most noticeable feature in these plots is that the number of rolls decreases dramatically as the continental coverage is increased from 0% to only 30%. As soon as the surface is changed to a total lid, the mantle is dominated by short-wavelength flows resulting in a greater decoupling between the upper and the lower mantle. Note that all models with partial coverage have similar planform, but the models with 50% coverage produce the longest wavelength convection cells.

Comparing Fig. 11 a and b, it can be seen that the higher Rayleigh number causes a stronger decoupling between the upper and the lower mantle in the absence of any continent and in the case of a total lid, reflecting increased layering with increased Rayleigh number. This occurs because of the narrower convective features seen in the higher Rayleigh number calculations. Guillou and Jaupart (1995), in their tank experiment where all the horizontal and vertical boundaries were rigid, showed no change in the flow wavelength with increasing Rayleigh number, however, when modeled numerically in a box with all free-slip boundaries, Grigné et al. (2007b) observed a correlation between the two variables and they demonstrated that the long convection cell wavelength varies as $Ra^{1/4}$ and produced a scaling theory arguing for this dependence. We also observe that fewer rolls are produced in calculations with $Ra = 10^7$ than $Ra = 10^6$ even though the surface boundary condition is partly rigid.

Some of our models (M23, M24, M32, M34, M35, M36 and M37) contain two separate continents covering 15% at the extreme left and right sides of the box. Our goal was to compare models with a single continent with a surface area of 30% and two continents totalling a surface area of 30%. Two-continent models resulted in two rolls with upwellings at either end of the box, however, as can be seen in Table 1, the measured diagnostics are very similar. This is because the wavelengths are still sufficiently long as to be only weakly affected by the phase boundary.

When internal heating was included in our models (M22, M24, M34, M35 and M37), the average temperature and the surface heat flow increased as expected. We also see a significant decrease in the flow wavelength mostly in the upper mantle. In these models, much warmer mantle produces stronger cold downwellings from the surface due to a larger thermal buoyancy contrast. These downwellings force the flow in the upper mantle to contain a large number of short-wavelength convection cells. However, $\beta$ is similar to that of purely basally heated models. This is probably due to a larger total heat flow combined with a higher heat conduction across 660 km-depth in the presence of a stronger internal thermal boundary layer. The average temperature in cartesian geometry is higher than in spherical geometry (Vangenlov and Jarvis, 1994) and consequently, the presence of internal heating in cartesian geometry results in very high internal temperatures.

4.3. Effect of continents on the shape of the geotherm

The presence of a partial lid increases the convection wavelength, increasing the horizontal advection of heat. Sinha and Butler (2007) showed that subadiabaticity can occur in regions where the dominant thermal balance is between the horizontal and the vertical advection of heat. Consequently, the models with mixed surface boundary conditions, which have very large aspect ratio flows and a large amount of horizontal advection of heat, show subadiabatic geotherms (vertical profile of horizontally averaged temperature) even without the presence of internal heating. Our models are incompressible and the temperature can be considered to be an approximation of the potential temperature in a compressible model (Jarvis and McKenzie, 1980). As a result, if the geotherm, interior to the surface and the basal thermal boundaries, has a positive slope, the temperature gradient is subadiabatic. In Fig. 12, we plot the time-averaged geotherms from models with different continental coverage, $L, Ra = 10^7$ and $\gamma = -3$ MPa K$^{-1}$. In the absence of a continent, the geotherm is almost adiabatic inside the top and bottom boundary layers, with small overshoots resulting from the horizontal advection of heat near the boundaries (Jarvis and Peltier, 1982). Introducing a continent of 30% coverage results in a geotherm with positive slope, interior to the boundary layers. Although we show only one set of calculations here, all models with partial coverage show significant subadiabaticity, which can also be seen in Figs. 1 c and 2 c where temperature increases with height in the lower mantle above the cold, laterally advecting material near the CMB. When we introduce full continental coverage, the interior of the geotherm becomes adiabatic because the long-wavelength convection cells are no longer present. The small bump at 660 km-depth is due to the presence of the phase boundary. We also observed subadiabaticity in the lower mantle geotherm for strongly layered models (not shown) caused by the presence of long-wavelength flows induced by the phase boundary. The temperature drop at the CMB decreases with increasing continental
coverage because of the increase in the average mantle temperature. However, the models with mixed boundary conditions have larger CMB temperature drops than they would if they were adiabatic.

In Fig. 13, we plot the temperature drop due to the subadiabaticity in the geotherm as a function of the ratio of the temporally and spatially averaged magnitudes of horizontal to vertical advection for all of our calculations without internal heating. The models with two continents are also not included in this figure. To calculate the temperature drop we record the maximum and the minimum temperatures along the temporally averaged geotherm in the upper and the lower mantle, respectively, and take the difference. One can clearly see the increasing trend in the subadiabaticity as the ratio increases which is consistent with our previous finding that horizontal advection can cause subadiabaticity. Fig. 14 shows how the same ratio varies with the number of convection cells in the lower mantle ($C_{lm}$). The overall negative slope of the scatter plot indicates that increased horizontal advection is associated with the models with longer wavelength convection cells. This indicates that all calculations with mixed surface boundary conditions and also free-slip calculations with large Clapeyron slopes, which have long-wavelength convection cells, will have stronger horizontal advection and hence, are more likely to have significantly subadiabatic geotherms.

5. Discussion and conclusions

In this paper, the combined effects of different surface boundary conditions and the endothermic phase boundary were studied in detail. We measure the degree of layering in terms of its effect on the surface heat flow, mass flux, heat transport across 660 km-depth and the difference in the number of convection rolls in the upper and the lower mantle.

Our investigation shows a sudden increase in the degree of layering between Clapeyron slopes of $-6 \text{ MPa K}^{-1}$, as measured by the heat transport, mass flux and the surface heat flow in a unit aspect ratio box with $Ra = 10^7$ for all the different surface boundary conditions. However, a gradual increase in all of these measures of layering and a smaller maximum degree of layering is seen in wide aspect ratio boxes, which indicates a strong dependence of layering on the aspect ratio of the convection cell.

We have shown that the models with full continental coverage have the most strongly reduced advection of heat across 660 km-depth and the largest degree of decoupling between the upper and the lower mantle convection. This means that one-plate planets like Venus (Stevenson, 2003) might have stronger thermal layering and significantly decoupled mantle convection compared to Earth. Mixed surface boundary models, which are more Earth-like, show very little effect of change in Clapeyron slope on the surface heat flow. This occurs because in oceanic regions the mass flux is reduced to a lesser degree due to the dominance of the surface boundary condition and the efficiency of the boundary layer does not change significantly with mechanical layering while in continental regions the heat transport efficiency actually increases with layering. However, because of the gradual slope of the $Nu- M_f$ relationship, the mass flux across 660 km-depth can be somewhat impeded and the flow in the upper and the lower mantle can also be weakly decoupled. This indicates that even if mantle convection was more layered in the past due to higher Rayleigh numbers, there may have been relatively little effect on the surface heat flow while mechanical mixing between the upper and the lower mantle may have been reduced. This might affect the mantle composition and could be of significant importance for geochemical studies of mantle mixing (Kellogg et al., 2008; Peltier, 1996).
Although in the presence of a complete conducting lid the surface heat flow is somewhat affected by a decrease in the mass flux across 660 km-depth, no such effect is observed over the continental regions of the mixed boundary cases. The resistance to transport heat across the upper thermal boundary layer, as measured by $R_{ac}$, increases with increasing mass flux across 660 km-depth for cases where the surface is covered by rigid lid. This occurs because, in the presence of strong layering the convection planeform in the upper mantle is governed by the surface boundary condition and the depth to the phase boundary and a shorter wavelength convective planeform is chosen that is more favorable for efficient heat exchange at the surface.

The presence of a total lid results in significantly shorter wavelength mantle flows mostly in the upper mantle because of the drag on the horizontal flows along the surface. Layering usually increases with $Ra$ as the flows have narrower features. However, with a rigid lid, even though the effective $Ra$ is decreased compared with the free-slip cases, the wavelength is also decreased leading to stronger layering. The longest wavelength flows are seen in models with partial lids. Layering also increases the flow wavelength in the lower mantle as these flows are capable of penetrating the phase boundary. We have shown that increasing the width of the continent from 0 to 30% results in longer wavelength convection cells and that the largest rolls form when the width is 50%. This indicates that for approximately 30% continental coverage, we might expect long-wavelength convection in the Earth's mantle.

In our models, continents do not drift. However, in their study in 3D spherical geometry with mobile continents Phillips and Bunge (2005) found that partial continental coverage induces long-wavelength mantle flows. Zhong and Gurnis (1993) also used mobile continents in their cylindrical geometry calculations and demonstrated the presence of long-wavelength thermal structure. Numerical experiment by Lowman and Jarvis (1999) and laboratory tank experiment by Zhong and Zhang (2005) investigated the situations where continents were free to move and demonstrated periodic mantle flow behavior analogous to the Wilson cycle. All of these studies found upwellings beneath continents. We find that introducing partial continental coverage produces a subadiabatic thermal gradient in the mantle, even without the presence of internal heating. This occurs because of the increased importance of horizontal advection in the resulting long-wavelength flows. This results in a larger temperature drop at the CM than would obtain for a purely adiabatic internal gradient causing an increased estimate of the heat flow from the core into the lower mantle for a given mean temperature (Bunge, 2005). This would affect energy budget calculations for the core and the mantle and would indicate that the inner core is slightly younger than we might expect (Costin and Butler, 2006). A subadiabatic geotherm would also require us to revise our estimates of the composition of the mantle (Mattern et al., 2005) and its transport properties (Monnereau and Yuen, 2002).

We have explored a wide range of parameter space and identified the combined effect of layering and continents on mantle convection. We carried out all of our calculations with an isoviscous fluid in two-dimensional cartesian geometry. In future work, it will be interesting to investigate these effects in spherical geometry. Incorporating temperature-dependent viscosity and more realistic surface plates might also be of interest.

Acknowledgements

We would like to thank Mark Jellinek and an anonymous referee for their helpful comments in reviewing this manuscript. This work was supported financially by the Natural Sciences and Engineering Research Council of Canada.

References


