Successive Interference Mitigation in Multiuser MIMO Channels

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Abstract—Motivated by the work of Dahrouj and Yu in applying the Han-Kobayashi transmission strategy for mitigating the intercell interference in a multi-cell multi-user multiple-input single-output interference network (MISO IN), this paper considers splitting messages into private and common parts in a multi-cell multi-user MIMO IN. Specifically, the covariances of the private messages and common messages are designed to optimize either the sum rate or the minimal rate. The common messages and private messages are decoded in sequence using successive decoding. This paper shows how these difficult optimization problems can be adequately solved by means of d.c. (difference of concave functions) optimization over a simple convex set. Numerical and simulation results also reveal the great advantage of our proposed solutions for various types of INs. In particular, the proposed solutions are shown to outperform the algorithm developed by Dahrouj and Yu for the simpler case of the MISO IN.

Index Terms—Multi-user interference networks, multi-input multi-output systems, interference mitigation, nonsmooth optimization, d.c. programming.

I. INTRODUCTION

Due to the increasing demand for high data rates, wireless cellular systems are being increasingly designed with full frequency reuse. This leads to significant out-of-cell interference. Conventional wireless cellular systems, however, are typically designed to be interference-limited. This is because each base-station in a conventional cellular system transmits to the mobile terminals in its cell independently from other base-stations and the out-of-cell interference is simply treated as noise. Although there are new approaches to coordinate transmissions of base-stations in multi-cell downlink communications (see e.g., [1]–[6]), the residual signal interference is still treated as noise. A wireless multi-cell system can be modeled as an interference network (IN) with Nc users and K users (e.g., mobile terminals) in each cell. By treating residual interference as noise, the network capacity is achieved only at low interference regime for a general multi-user IN (see [7] and references therein) or at certain sufficient conditions in terms of matrix equations for two-user INs [8].

The achievable rate region for two-user INs with two cells, each serving one user in it, has been examined extensively in [9]–[15]. The Han-Kobayashi (H-K) strategy [9], [12] yields the best inner bound on the capacity region, while various outer bounds were also reported in [12], [14]–[16]. In the H-K strategy, the transmit signals are purposely designed so that they are partially decodable by both users’s receivers. In particular, the transmit signal of each user is split into two parts: (i) a private message that is decoded by the intended user’s receiver only, and (ii) a common message that is decoded by both users’ receivers for the purpose of interference mitigation. Under the H-K strategy for a two-user single-input single-output (SISO) IN, reference [12] allocates power to private and common messages in such a way that the power of the private message of each user is received at the level of the Gaussian noise at the other receiver. In this way, the interference caused by the private message has a small effect on the other link as compared to the impairments already caused by the noise. At the same time, a large amount of private information can still be conveyed in the user’s own link if the direct gain is appreciably larger than the cross gain. With such a power allocation strategy, reference [12] shows that a very simple H-K scheme can achieve the capacity region to within one bit. References [17], [18] extend [12] to the case of a two-user multiple-input multiple-output (MIMO) IN. They show that a H-K type coding scheme achieves the capacity region to within Nc bits (Nc is the number of receive antennas) or to within a constant gap. Reference [19] establishes the capacity region of the MIMO IN, but only for the so-called aligned strong interference regime, where the direct and cross link channel matrices satisfy a matrix equation.

Dahrouj and Yu [20] applies the H-K strategy for a multi-user multiple-input single-output (MISO) IN. The transmission scheme proposed in [20] has both the common and private messages beamformed at the base-stations and then sequentially decoded at the receivers. Different from the two-user IN in which user pairing is obvious, for the case of multi-user IN one also needs to consider selection of users for common message decoding (i.e., user pairing) in order to improve the IN capacity. Given a pre-defined user pairing protocol, the problem of splitting common and private rates and associated beamforming design is still difficult. Given that the achievable rate region is very complicated and no characterization is yet available, such a problem is approached in [20] by an ad-hoc intensive search at discrete points in the joint space of common and private message rates. The optimality of the worst user’s rate is not granted.
Also, the extensive search is not suitable for the problem of sum-rate maximization, which is a more popular metric for MIMO.

Inspired by the study in [20], the present paper is concerned with a multi-user MIMO IN that is more general than the multi-user MISO IN considered in [20]. Specifically, each message is split into a common and private part and the covariances of these message parts are designed to maximize the sum rate or the worst (minimal) user’s rate under the base-station power constraints. Like the approach proposed in [20], the common and private messages are sequentially decoded at the receivers. We show that such design problems for the MIMO IN can be formulated as nonsmooth function optimizations over a simple convex set. Furthermore, these nonsmooth functions are shown to be d.c. (difference of two concave functions/sets) [21],1 hence the optimization problems can be solved very efficiently by a sequence of convex programs, known as d.c. iterations (DCI) of d.c. programming (see e.g., [22]–[26] for the developments and successful applications of DCIs to various design problems in wireless communications). The DCI is a path-following procedure, which surely improves the solution at every iteration. As such, the DCI is guaranteed to converge to a local optimum. Intensive simulations for diverse design problems in [22]–[26] show that DCIs typically converge in a small number of iterations. The implementation of DCIs is simple and does not involve the control of step size.

To summarize, the contribution of the present paper is three-fold:

- Developing an efficient d.c. optimization framework for maximizing the sum rate and minimal user’s rate over the disconnected region of split private and common rates in multi-user MIMO INs;
- Numerically showing the benefit of rate splitting in mitigating multi-user interference;
- Comparing and contrasting the proposed inner bound with the existing inner and outer bounds for different cases of MIMO INs.

The rest of the paper is organized as follows. Section II gives problem formulations and discusses the challenges in finding their solutions. Section III and Section IV propose new solution methods for covariance and beamforming optimization problems, respectively. Section V provides simulation results. Section VI concludes the paper. The Appendix contains several formulas necessary for calculating the numerical solutions.

**Notation:** Variables are boldfaced. \( I_n \) is the identity matrix of dimension \( n \). The inner product \( (X, Y) \) between matrices \( X \) and \( Y \) is defined as trace \( (X^H Y) \), i.e., \( (X, Y) = \text{trace}(X^H Y) \). The notation \( A \) is used to refer to the trace of matrix \( A \) while \( |A| \) is its determinant. \( A > 0 \) (\( A \geq 0 \), resp.) for a Hermitian symmetric matrix \( A \) means it is positive definite (semi-definite, resp.). \( S_+^{N_t} \) denotes the cone of Hermitian symmetric positive semi-definite matrices of size \( N_t \times N_t \). \( \gamma \) is logical or operation, so \( \alpha = \beta \lor \gamma \) means \( \alpha \) is either \( \beta \) or \( \gamma \). \( \log \) is understood as the base-2 logarithm, i.e., \( \log_2 \).

1 Conventionally, the abbreviation “d.c.” stands for a difference of two convex functions [21], which is equivalent to a difference of two concave functions as we deal with in this paper. The term “d.c.” is still used for simplicity of presentation.

**II. Problem Formulations and Challenges**

Consider downlink transmission in a multi-cell multiuser cellular network consisting of \( N \) cells. Each cell has one base-station (BS) equipped with \( N_t \geq 1 \) antennas that serves its \( K \) mobile users, each of which is equipped with \( N_r \geq 1 \) antennas. Define \( I := \{1, 2, \ldots, N\} \) and \( J := \{1, 2, \ldots, K\} \). User \( j \) in the \( i \)-th cell is referred to as user \( (i,j) \). Transmit precoding is implemented at the base-station to separate signals from users within each cell. Thus, interference mitigation need only occur between users belonging to different cells. This paper is concerned with joint precoding and common message decoding approach [20] to mitigate intercell interference. Introduce a pairing operator \( a(i,j) \) that describes which other user, beside user \( (i,j) \), decodes the common message of user \( (i,j) \). When the user \( (i,j) \) has no common message, let \( a(i,j) = 0 \) be the empty set. Formally, it is a mapping \( a : I \times J \to (I \times J) \cup \emptyset \) with the restriction that \( a(i,j) = (i,j) \) always has \( i \neq j \) and \( a^{-1}(i,j) \) has cardinality no more than one. With \( \emptyset \neq a(i,j) = (i,j) \), \( i \neq j \), user \( (i,j) \) may split its data stream into two parts: private message \( x_{i,j} \in \mathbb{C}^{N_t} \) with covariance \( Q_{i,j} \) and a common message \( x_{i,j}^c \in \mathbb{C}^{N_r} \) with covariance \( Q_{i,j}^c \). The user \( (i,j) \)'s common message \( x_{i,j}^c \) is to be decoded by user \( (i,j) \)'s own receiver, and also by user \( (\hat{i},\hat{j}) \) in the different \( i \)-th cell. On the other hand, if \( (i,j) = (\hat{i},\hat{j}) \) for some \( i \neq j \), then user \( (i,j) \)'s receiver also decodes a common message from user \( (\hat{i},\hat{j}) \) in the different cell \( \hat{i} \neq i \). As in [20], each user \( (i,j) \) successively decodes the following messages (in a strict order as stated): (a) its common message \( x_{i,j}^c \) from its own transmitter; (b) the common message \( x_{\hat{i},\hat{j}}^c \) from user \( (\hat{i},\hat{j}) \)'s transmitter in the different cell \( \hat{i} \neq i \) for which \( a(\hat{i},\hat{j}) = (i,j) \); (c) the private message \( x_{i,j}^p \) from its own transmitter. Note that the decoded messages are also successively subtracted from the received signal for interference mitigation. Intuitively, one’s own common message is decoded first to help the decoding of common information from the other transmitter, while its own private message is decoded last to take advantage of the reduced interference due to common message decoding.

The number of user pairs selected for decoding the common message is the cardinality \( |a| \) of the set \( \{(i,j) \in I \times J : a(i,j) \neq \emptyset \} \). In what follows, we denote by \( A \) the set of such pairing operators \( a \) and \( A_L \subset A \) is the set of \( a \) with \( |a| \leq L \).

The multiple output \( y_{i,j} \in \mathbb{C}^{N_r} \) received at user \( (i,j) \) is the combination of intra-cell and inter-cell signals and noise, which is defined by (1), shown at the bottom of the next page. In (1), \( y_{i,j} \in \mathbb{C}^{N_r} \), and its entries are independent and identically distributed (i.i.d.) noise samples with zero-mean and variance \( \alpha^2 \), \( H_{i,j} \) and \( H_{m,i,j} \) for \( m \neq i \) are the channel gains of the direct and interfering channels with respect to user \( (i,j) \).

For simplicity, only BS power constraints are considered as defined by (2), shown at the bottom of the next page, although other power constraints can be easily incorporated. In (2), \( Q_{i,j}^p \in \mathbb{C}^{N_r \times N_t} \) and \( Q_{i,j}^c \in \mathbb{C}^{N_t \times N_t} \) are the covariances of messages \( x_{i,j}^c \) and \( x_{i,j}^c \), respectively. Note that \( x_{i,j}^p = 0 \) in (1) and thus \( Q_{i,j}^p = 0 \) in (2) whenever \( a(i,j) = \emptyset \), so there are only \( L \) variables \( Q_{i,j}^c \) in (2) for \( a \in A_L \).
The \( N_r \times N_r \) covariance matrix of the signal received at user \((i,j)\) is

\[
\mathcal{M}_{i,j}(\mathbf{Q}) := \sum_{(n,k) \in \mathbf{I} \times \mathcal{J}} H_{n,i,j} (\mathbf{Q}_{n,k}^p + \mathbf{Q}_{n,k}^c) H_{n,i,j}^H + \sigma^2 I_{N_r}
\]

Under the successive decoding and interference cancellation scheme described above, it follows that:

- User \((i,j)\) can decode its own common message \( x_{i,j}^c \) with the achievable rate
  \[
  r_{i,j}^c(\mathbf{Q}) = \log \left| I_{N_r} + H_{i,j} \mathbf{Q}_{i,j}^c H_{i,j}^H (\mathcal{M}_{i,j}(\mathbf{Q}))^{-1} \right|,
  \]
  where \( \mathcal{M}_{i,j}(\mathbf{Q}) := \mathcal{M}_{i,j}(\mathbf{Q}) - H_{i,j} \mathbf{Q}_{i,j}^c H_{i,j}^H \). Accordingly, \( \mathcal{Q}_{i,j}^c \equiv 0 \) if \((a(i,j)) \neq \emptyset \).

- User \((i,j)\) can decode the common message \( x_{i,j}^c \) from the interfering user \((\hat{i},\hat{j})\) with the achievable rate
  \[
  r_{i,j}^c(\mathbf{Q}) = \log \left| I_{N_r} + H_{i,j} \mathbf{Q}_{i,j}^c H_{i,j}^H (\mathcal{M}_{i,j}(\mathbf{Q}))^{-1} \right|,
  \]
  where \( \mathcal{M}_{i,j}(\mathbf{Q}) := \mathcal{M}_{i,j}(\mathbf{Q}) - H_{i,j} \mathbf{Q}_{i,j}^c H_{i,j}^H \). Accordingly, \( \mathcal{M}_{i,j}(\mathbf{Q}) := \mathcal{M}_{i,j}(\mathbf{Q}) \) if \( a(i,j) = \emptyset \).

- User \((i,j)\) can decode its private message \( x_{i,j}^p \) with the achievable rate
  \[
  r_{i,j}^p(\mathbf{Q}) = \log \left| I_{N_r} + H_{i,j} \mathbf{Q}_{i,j}^c H_{i,j}^H (\mathcal{M}_{i,j}(\mathbf{Q}))^{-1} \right|,
  \]
  for \( \mathcal{M}_{i,j}(\mathbf{Q}) := \mathcal{M}_{i,j}(\mathbf{Q}) - H_{i,j} \mathbf{Q}_{i,j}^p H_{i,j}^H \).

The achievable rate region under this successive decoding is thus given by

\[
\mathcal{R}(\mathbf{Q}) := \left\{ \mathbf{v} := [v_{i,j}]_{(i,j) \in \mathbf{I} \times \mathcal{J}} : v_{i,j}^p \leq r_{i,j}^p(\mathbf{Q}), v_{i,j}^c \leq r_{i,j}^c(\mathbf{Q}), v_{i,j} \leq r_{a(i,j)}(\mathbf{Q}) \right\} \subset \mathbb{R}^{N_r \times K}
\]

It is pointed out that \( r_{a(i,j)}(\mathbf{Q}) \) is the achievable rate of decoding the common message \( x_{i,j}^c \) of user \((i,j)\) by user \((\hat{i},\hat{j}) = a(i,j) \). The constraint \( v_{i,j}^c \leq r_{a(i,j)}(\mathbf{Q}) \) arises only when \( a(i,j) \neq \emptyset \).

With the pairing operator \( a \), the problem of rate splitting to maximize the sum rate under the BS power constraints is stated as

\[
\max_{\mathbf{Q}^p, \mathbf{Q}^c} \sum_{(i,j) \in \mathbf{I} \times \mathcal{J}} v_{i,j} : \mathbf{Q} \in \mathcal{W}_B, \mathbf{v} \in \mathcal{R}(\mathbf{Q}).
\]

On the other hand, the minimal rate maximization (maximin) rate problem is

\[
\max_{\mathbf{Q}^p, \mathbf{Q}^c} \min_{(i,j) \in \mathbf{I} \times \mathcal{J}} v_{i,j} : \mathbf{Q} \in \mathcal{W}_B, \mathbf{v} \in \mathcal{R}(\mathbf{Q}).
\]

Before introducing our methods to solve (8) and (9), it is worthwhile to discuss a simpler design problem for the MISO INs (when \( N_r = L = 1 \) as studied in [20]). The single output is a simplified version of the multiple output equation in (1), which is defined by (3), shown at the bottom of the page.

In (3), \( \mathbf{w}_{i,j}^p \in \mathbb{C} \) and \( \mathbf{w}_{i,j}^c \in \mathbb{C} \) are the private and common messages of user \((i,j)\), which are beamformed by vectors \( \mathbf{w}_{i,j}^p \in \mathbb{C}^{N_r} \) and \( \mathbf{w}_{i,j}^c \in \mathbb{C}^{N_r} \), respectively. This leads to (9) with additional rank-one constraints \( \text{rank}(\mathbf{Q}_{i,j}^p) = \text{rank}(\mathbf{Q}_{i,j}^c) = 1 \) on \( \mathbf{Q}_{i,j}^p = \mathbf{w}_{i,j}^p \mathbf{w}_{i,j}^p^H \) and \( \mathbf{Q}_{i,j}^c = \mathbf{w}_{i,j}^c \mathbf{w}_{i,j}^c^H \). The conventional maximin rate coordinated beamforming corresponds to the specific case \((a(i,j)) = \emptyset \forall (i,j) \in \mathbf{I} \times \mathcal{J} \). This case involves variables \( \mathbf{Q}_{i,j}^p \) only. Reference [20] solves this conventional maximin rate optimization by a sequence of SDP (semi-definite programming) programs in \( \mathbf{Q}_{i,j}^p \) by dropping (relaxing) the rank-one constraints on \( \mathbf{Q}_{i,j}^p \). Based on the obtained solution and maximin rate \( \lambda^p \), \( L \) user-pairs are selected for candidates in common-private message split. Then, the steps 3–5 of [20, Alg. 2] are to check the rate feasibility at points \([v_{i,j}^p, v_{i,j}^c]\) \((i,j) \in \mathbf{I} \times \mathcal{J} \) such that \( \min_{(i,j) \in \mathbf{I} \times \mathcal{J}} [v_{i,j}^p + v_{i,j}^c] > \lambda^p \). Each feasibility problem in \( \mathbf{Q} \) is seen as a nonconvex rank-one constrained problem, which is addressed in [20] again by SDP relaxation and randomization. Such an approach and procedure are not suitable for the sum rate optimization problem (8) in MISO networks, nor can they be extended to solve the maximin rate problem (9) in MIMO networks.

\[5\] The reader is also referred to [27] for efficiency analysis of such a SDP relaxation approach.
III. NEW EFFICIENT COVARIANCE SOLUTION

To solve (8), (9), we use the following three-step approach, an extension of the idea of [20]. First, solve for the optimal covariance without rate-splitting (i.e., treating interference as noise). Then, use that solution to choose a user pairing. Finally, with a user pairing chosen, solve for the optimal covariances with rate-splitting (i.e., a H-K rate-split). We now explain each of these steps in detail. The conventional coordinated sum rate and maximum rate optimizations correspond to the case $x_{i,j}^2 \equiv 0$ and $Q_{i,j}^P \equiv 0$, $(i, j) \in I \times J$ in (8), (9), which involves optimization in $Q_{i,j}^P$ only (see (16) below).\footnote{In essence, common messages are introduced in (8), (9) to improve the multiuser rate capacity of the coordinated transmissions.} Suppose $Q^{(0)}$ is the optimal solution of these conventional coordinated optimizations. Define the received interference-plus-noise covariance for user $(i, j)$ by

$$T_{i,j} = \sum_{(a,k) \in I \times J \setminus (i,j)} H_{n,i,j} Q_{n,k}^P H_{n,i,j}^H + \sigma^2 I_{N_r},$$

and the interference-to-noise covariance (INC) by

$$\text{INC}_{(i,j)\rightarrow(i,j)} = H_{i,i,j} Q_{i,j}^P H_{i,i,j}^H (T_{i,j} - H_{i,i,j} Q_{i,j}^P H_{i,i,j}^H)^{-1} \in \mathbb{C}^{N_r \times N_r}. \tag{11}$$

The relative strength of interference \( \text{INR}_{(i,j)\rightarrow(i,j)} \) is gauged by \( \text{DINC}_{(i,j)\rightarrow(i,j)} = \text{INC}_{(i,j)\rightarrow(i,j)} \). Similar to [20], $(N-1)K^2$ such DINC values, each representing a potential pairing option, are sorted and trimmed to create an effective list in the decreasing value order such that each $(i, j)$ (each $(i, j)$, resp.) appears at most once in $(i, j) \rightarrow (i, j)$. Then, $\mathcal{L}$ pairs with the highest DINC values are pre-selected for sharing common messages corresponding to pairing map $a \in \mathcal{A}_\mathcal{L}$.

Having chosen pairing map $a \in \mathcal{A}_\mathcal{L}$, we now combine the sum rate problem (8) and the maximin rate problem (9) by

$$\max_{Q,\rho} \psi(\rho) : Q \in \mathcal{W}_B, \rho \in \mathcal{R}(Q), \tag{12}$$

for $\psi(\rho) = \psi^+(\rho) \lor \psi^\wedge(\rho)$ with $\rho = [\rho_{i,j}]_{(i,j) \in I \times J}$, and 

$$\psi^+(\rho) := \sum_{(i,j) \in I \times J} \rho_{i,j}, \psi^\wedge(\rho) := \min_{(i,j) \in I \times J} \rho_{i,j}. \tag{13}$$

The objective function $\psi(\rho)$ in (12) is obviously concave so (12) is concave function maximization over nonconvex constraints. We now provide a novel approach to reformulate (12) as function maximization in $Q$ only over simple convex set $\mathcal{W}_B$ defined by (2). To this end, define the map

$$r(Q) := [r_{i,j}(Q)]_{(i,j) \in I \times J} = \left[ P_{i,j}^P(Q) + \min \left( r_{i,j}^C(Q), r_{a(i,j)}^a(Q) \right) \right]_{(i,j) \in I \times J}. \tag{14}$$

$\text{Theorem 1:}$ Problem (12) in variables $(Q, \rho)$ is equivalent to the following problem in variable $Q$ only

$$\max_Q \psi(r(Q)) : Q \in \mathcal{W}_B. \tag{15}$$

The equivalence is in the sense that the two problems share the same optimal value and optimal solution $Q^*$. Theorem 1: Each feasible $Q$ of (14) will result in $[P_{i,j}^P(Q)]_{(i,j)\in I \times J}$ and $[r_{i,j}^a(Q), r_{a(i,j)}^a(Q)]_{(i,j)\in I \times J}$ such that $[P_{i,j}^P(Q) + r_{i,j}^a(Q)]_{(i,j)\in I \times J} \in \mathcal{R}(Q)$, i.e., $(Q, [P_{i,j}^P(Q) + r_{i,j}^a(Q)])_{(i,j)\in I \times J}$ is feasible to (12). Therefore, max (14) $\leq$ max (12). Now, suppose $(Q, \rho)$ is feasible to (12). Then $\rho \in \mathcal{R}(Q)$ so there are $P_{i,j}^P \leq r_{i,j}^P(Q)$ and $r_{i,j}^a \leq r_{a(i,j)}^a(Q)$, $r_{i,j}^a \geq r_{a(i,j)}^a(Q)$ such that $v_{i,j}^P = v_{i,j}^P + v_{i,j}^a$. It follows that $r_{i,j} \leq r_{i,j}(Q)$ so $\psi(\rho) \leq \psi(r(Q))$ and max (12) $\leq$ max (14) yielding max (14) = max (12).

One can see that the constraints in (14) are convex with minimal number of variables involved but the objective is not concave. The next step is to represent the objective function $\psi(r(Q))$ as a d.c. (difference of two concave) function by exploiting the following two basic properties: each of the rate functions $r_{i,j}^P, r_{a(i,j)}^a$ is d.c. and then

$$\min\{a_1 - b_1, \ldots, a_k - b_k\} = \min_{j=1,2,\ldots,k} \left\{ a_j + \sum_{i \neq j} b_i \right\} - \sum_{i=1}^k b_i$$

for any real numbers $\{a_1, \ldots, a_k, b_1, \ldots, b_k\}$ [21]. Actually,

$$r_{i,j}^P(Q) = g_{i,j}^C(Q) - f_{i,j}^P(Q), \quad r_{a(i,j)}^a(Q) = g_{a(i,j)}^a(Q) - f_{a(i,j)}^a(Q), \quad i^P_{i,j}(Q) = g_{i,j}^P(Q) - f_{i,j}^P(Q), \tag{16}$$

with concave functions [28, p. 405]

$$g_{i,j}^C(Q) = \log |M_{i,j}^C(Q) + H_{i,i,j} Q_{i,j}^C H_{i,i,j}^H|, \quad f_{i,j}^P(Q) = \log |M_{i,j}^P(Q)|, \tag{17}$$

$$g_{a(i,j)}^a(Q) = \log |M_{a(i,j)}^a(Q) + H_{i,i,j} Q_{i,j}^C H_{i,i,j}^H|, \quad f_{a(i,j)}^a(Q) = \log |M_{a(i,j)}^a(Q)|, \tag{18}$$

$$g_{i,j}^P(Q) = \log |M_{i,j}^P(Q) + H_{i,i,j} Q_{i,j}^C H_{i,i,j}^H|, \quad f_{i,j}^P(Q) = \log |M_{i,j}^P(Q)|. \tag{19}$$

Next,

- $\min\{r_{i,j}^P(Q), r_{a(i,j)}^a(Q)\} = g_{i,j}^P(Q) - f_{i,j}^P(Q)$, where the functions

$$g_{i,j}^P(Q) := \min\{g_{i,j}^C(Q) + f_{a(i,j)}^a(Q), g_{a(i,j)}^a(Q) + f_{i,j}^C(Q)\}, \quad f_{i,j}^P(Q) := f_{i,j}^P(Q) + f_{a(i,j)}^a(Q) \tag{20}$$

are concave as they are minimum and sum of concave functions.

- $r_{i,j}(Q) = g_{i,j}(Q) - f_{i,j}(Q)$, with concave functions

$$g_{i,j}(Q) := g_{i,j}^P(Q) + g_{i,j}^C(Q), \quad f_{i,j}(Q) := f_{i,j}^P(Q) + f_{i,j}^C(Q). \tag{21}$$
• \( \varphi^+(\mathbf{Q}) = \sum_{(i,j) \in \mathcal{I} \times \mathcal{J}} r_{i,j}(\mathbf{Q}) = g^+(\mathbf{Q}) - f(\mathbf{Q}) \), with concave functions

\[
g^+(\mathbf{Q}) := \sum_{(i,j) \in \mathcal{I} \times \mathcal{J}} g_{i,j}(\mathbf{Q}),
\]

\[
f(\mathbf{Q}) := \sum_{(i,j) \in \mathcal{I} \times \mathcal{J}} f_{i,j}(\mathbf{Q})
\]

• \( \varphi^-(\mathbf{Q}) = \min_{(i,j) \in \mathcal{I} \times \mathcal{J}} r_{i,j}(\mathbf{Q}) = g^-(\mathbf{Q}) - f(\mathbf{Q}), \) where function

\[
g^-(\mathbf{Q}) = \min_{(i,j) \in \mathcal{I} \times \mathcal{J}} \left[ g_{i,j}(\mathbf{Q}) + \sum_{(n,k) \in \mathcal{I} \times \mathcal{J} \setminus (i,j)} f_{n,k}(\mathbf{Q}) \right]
\]

is concave as it is a minimum of concave functions [21].

Using the above facts, we obtain the following result:

**Theorem 2:** For \( g(\mathbf{Q}) = g^+(\mathbf{Q}) \lor g^-(\mathbf{Q}) \), problem (12) is the following convex constrained d.c. function maximization:

\[
\max_{\mathbf{Q}} \left[ g(\mathbf{Q}) - f(\mathbf{Q}) \right] : \mathbf{Q} \in \mathcal{W}_b.
\]

(15)

Analogously, for

\[
\Phi_{i,j}(\mathbf{P}) := \sum_{(n,k) \in \mathcal{I} \times \mathcal{J} \setminus (i,j)} H_{n,i} \mathbf{P}^n_{n,k} H_{n,i}^H + \sigma^2 I_{N_i},
\]

the conventional coordinated optimization problem is represented by the following convex constrained d.c. function maximization:

\[
\max_{\mathbf{Q}} \left[ g^p(\mathbf{Q}^p) - f^p(\mathbf{Q}^p) \right] : \mathbf{Q} \in \mathcal{W}_b,
\]

(16)

with concave functions

\[
f^p(\mathbf{Q}^p) = \sum_{(i,j) \in \mathcal{I} \times \mathcal{J}} \log |\Phi_{i,j}(\mathbf{Q}^P)|
\]

\[
g^p(\mathbf{Q}) = \sum_{(i,j) \in \mathcal{I} \times \mathcal{J}} \log |H_{i,i} \mathbf{I}^P_{i,i} H_{i,i}^H + \Phi_{i,j}(\mathbf{Q}^P)|
\]

\[
\lor \min_{(i,j) \in \mathcal{I} \times \mathcal{J}} |\log |H_{i,i} \mathbf{I}^P_{i,i} H_{i,i}^H + \Phi_{i,j}(\mathbf{Q}^P)| |
\]

\[
+ \sum_{(n,k) \in \mathcal{I} \times \mathcal{J} \setminus (i,j)} \log |\Phi_{n,k}(\mathbf{Q}^P)|.
\]

It is clear that both (15) and (16) belong to the following canonical d.c. programming [21]

\[
\max_{\mathbf{z} \in \mathcal{D}} G(\mathbf{z}) := G(\mathbf{z}) - F(\mathbf{z}),
\]

(17)

where \( \mathcal{D} \) is a compact convex set in a finite dimensional space, while \( G \) and \( F \) are concave functions with \( F \) smooth. Suppose that \( \mathbf{z}(\kappa) \) is feasible to (17). Since \( F \) is concave, its gradient \( \nabla F(\mathbf{z}(\kappa)) \) at \( \mathbf{z}(\kappa) \) is also a super-gradient [21]. Therefore,

\[
G(\mathbf{z}) - F(\mathbf{z}) \geq G(\mathbf{z}) - F(\mathbf{z}(\kappa)) - \left\langle \nabla F(\mathbf{z}(\kappa)), \mathbf{z} - \mathbf{z}(\kappa) \right\rangle \land \mathbf{z}.
\]

It follows that for any feasible \( \mathbf{z}(\kappa) \) to (17), the following convex program provides a global lower bound maximization for d.c. program (17):

\[
\max_{\mathbf{z} \in \mathcal{D}} \left[ G(\mathbf{z}) - F(\mathbf{z}(\kappa)) - \left\langle \nabla F(\mathbf{z}(\kappa)), \mathbf{z} - \mathbf{z}(\kappa) \right\rangle \right].
\]

(18)

Moreover, for the optimal solution \( \mathbf{z}(\kappa + 1) \) of (18), one has

\[
G(\mathbf{z}(\kappa + 1)) - F(\mathbf{z}(\kappa + 1)) \geq G(\mathbf{z}(\kappa)) - F(\mathbf{z}(\kappa)) - \left\langle \nabla F(\mathbf{z}(\kappa)), \mathbf{z}(\kappa + 1) - \mathbf{z}(\kappa) \right\rangle \geq G(\mathbf{z}(\kappa)) - F(\mathbf{z}(\kappa)) - \left\langle \nabla F(\mathbf{z}(\kappa)), \mathbf{z}(\kappa - \mathbf{z}(\kappa)) \right\rangle = G(\mathbf{z}(\kappa)) - F(\mathbf{z}(\kappa)),
\]

which means that \( G(\mathbf{z}(\kappa + 1)) > G(\mathbf{z}(\kappa)) \), i.e., \( \mathbf{z}(\kappa + 1) \) is better than \( \mathbf{z}(\kappa) \) toward optimizing (17) as long as \( \mathbf{z}(\kappa + 1) \neq \mathbf{z}(\kappa) \). Thus, initialized from a feasible \( \mathbf{z}(0) \in \mathcal{D} \), recursively generating \( \mathbf{z}(\kappa + 1) \) for \( \kappa = 0, 1, \ldots \), by the optimal solution of convex program (18) is a path-following procedure, which converges to at least a local optimal solution of (17) satisfying the first optimality principle.

To summarise, a d.c. procedure for the generic d.c. program (17) is sketched below.

**D.C. Iterations (DCI):**

- **Initialization:** Choose an initial feasible solution \( \mathbf{z}(0) \in \mathcal{D} \) of (17).
- **\( \kappa \)-th DC iteration (\( \kappa \)-DCI):** Solve convex program (18) to obtain the optimal solution \( \mathbf{z}(\kappa) \) and set \( \kappa \rightarrow \kappa + 1, \mathbf{z}(\kappa) \rightarrow \mathbf{z}(\kappa) \). Given a tolerance level \( \epsilon > 0 \), stop if

\[
\left| G(\mathbf{z}(\kappa)) - G(\mathbf{z}(\kappa - 1)) \right| / \left| G(\mathbf{z}(\kappa - 1)) \right| \leq \epsilon.
\]

(19)

Our previous works [22]–[26] demonstrated successful applications of the above DCIs to various nonconvex optimization problems. The reader is also referred to [29] for a new observation on its efficiency. In particular, the incumbent \( \mathbf{z}(\kappa + 1) \) is in fact the optimal solution of the following program:

\[
\max_{(\mathbf{z}, \mathbf{v}) \in \mathcal{D}} G_{\kappa}(\mathbf{z}) := G(\mathbf{z}) - \min_{v=0,1,\ldots,k} \left\{ F(\mathbf{z}(v)) \right\}
\]

\[
+ \left\langle \nabla F(\mathbf{z}(v)), \mathbf{z} - \mathbf{z}(v) \right\rangle \right\}.
\]

(20)

Now, it is obvious that \( G(\mathbf{z}) \geq G_{\kappa + 1}(\mathbf{z}) \geq G_{\kappa}(\mathbf{z}) \geq \ldots \geq G_0(\mathbf{z}) \), \( \forall \mathbf{z} \in \mathcal{D} \), so concave functions \( G_{\kappa} \) are iteratively better global approximations of d.c function \( G \). Consequently, DCIs by (18) not only generate improved solutions but also provide successively better convexifications for d.c. program (17).

Next, it is obvious that (15) is (17) with \( \mathbf{z} \to \mathbf{Q}^p, G(\mathbf{z}) \to g(\mathbf{Q}) \) and \( F(\mathbf{z}) \to f(\mathbf{Q}) \), while (16) is (17) with \( \mathbf{z} \to \mathbf{Q}^P, G(\mathbf{z}) \to g^P(\mathbf{Q}) \) and \( F(\mathbf{z}) \to f^P(\mathbf{Q}) \). Therefore both (15) and (16) are solved by the DCI described before. The gradients \( \nabla F(\mathbf{Q}(\kappa)), \mathbf{Q} - \mathbf{Q}(\kappa) \) or \( \nabla f^P(\mathbf{Q}(\kappa)), \mathbf{Q}^P - \mathbf{Q}(\kappa) \) in implementing \( \kappa \)-DCI (18) can be easily calculated based on Equation (32) given in Appendix I.
In summary, our approach toward finding the solution of (8), (9) is:

- Use DCIs to find optimized solution $Q^0(0)$ of the coordinated covariance problem (16);
- Use $Q^0(0)$ to determine the pairing strategy $a \in A_c$;
- Use DCIs to find optimized solution $Q$ of (12), (15).

### IV. New Beamforming Solutions

In this section we consider the beamforming problem for (3). The main disadvantage of using formulations (8) and (9) in the variables $Q^0_{ij} = w^p_{ij}(w^p_{ij})^H$ and $Q^c_{ij} = w^r_{ij}(w^r_{ij})^H$ is that their total dimensions become very large with the additional difficult rank-one constraints $\text{rank}(Q^0_{ij}) = \text{rank}(Q^c_{ij}) = 1$. As such, we shall focus on a direct formulation involving the original beamforming variable $w$ only. The BS power constraint (2) in terms of $w$ is

$$\mathcal{V}_B \equiv \left\{ w : \begin{array}{l}
[w^p_{ij}], w^c_{ij}, \in \mathcal{I} \times \mathcal{J} : \\
\sum_{i \in \mathcal{I}} \left( \|w^p_{ij}\|^2 + \|w^c_{ij}\|^2 \right) \leq P_B, i \in \mathcal{I} \end{array} \right\}. \quad (21)$$

The pairing $a \in A_c$ is chosen according to the previous section, which is based on the solution of the conventional coordinated beamforming. Under the selected pairing map $a$, recall from the definition (4)–(6) and Theorem 1 that the max sum rate and maximin rate problems is formulated by

$$\max_w \varphi (r(w)) : w \in \mathcal{V}_B, \quad (22)$$

where

$$r(w) := \left[ \tilde{r}_{ij}(w) \right]_{(i,j) \in \mathcal{I} \times \mathcal{J}}$$

with

$$\tilde{r}_{ij}(w) := \tilde{r}^p_{ij}(w) + \min \{ \tilde{r}^c_{ij}(w), \tilde{r}^a_{ij}(w) \}$$

and

$$\tilde{r}^p_{ij}(w) = r^p_{ij}(w),$$
$$\tilde{r}^c_{ij}(w) = r^c_{ij}(w),$$
$$\tilde{r}^a_{ij}(w) = r^a_{ij}(w),$$

$$W := \begin{bmatrix} w^p_{ij} & w^c_{ij} \end{bmatrix} \in \mathcal{I} \times \mathcal{J},$$

which are highly nonlinear functions in $w$.

Define the following convex quadratic functions in $w$:

$$\tilde{\mathcal{M}}^0_{ij}(w) = \mathcal{M}^0_{ij}(W),$$
$$\tilde{\mathcal{M}}^a_{ij}(w) = \mathcal{M}^a_{ij}(W),$$
$$\tilde{\mathcal{M}}^p_{ij}(w) = \mathcal{M}^p_{ij}(W).$$

For the above highly-nonlinear optimization problems, it is very important to classify convex and nonconvex variables [21]. The following result shall be used later to clarify all complex terms $\tilde{\mathcal{M}}^0_{ij}(w), \tilde{\mathcal{M}}^a_{ij}(w)$ and $\tilde{\mathcal{M}}^p_{ij}(w)$.

**Theorem 3**: Introduce variables $y_{ij} := [y^c_{ij}, y^a_{ij}, y^p_{ij}]_{(i,j) \in \mathcal{I} \times \mathcal{J}}$ and $y := [y_{ij}]_{(i,j) \in \mathcal{I} \times \mathcal{J}}$, which satisfy the convex inequality constraints:

$$\tilde{\mathcal{M}}^c_{ij}(w) \leq y^c_{ij},$$
$$\tilde{\mathcal{M}}^a_{ij}(w) \leq y^a_{ij},$$
$$\tilde{\mathcal{M}}^p_{ij}(w) \leq y^p_{ij}, (i, j) \in \mathcal{I} \times \mathcal{J}. \quad (23)$$

Then problem (22) is equivalent to the following convex constrained problem:

$$\max_{w,y} \varphi (h(w, y)) : w \in \mathcal{V}_B, \quad (24)$$

where $h(w, y) = [h_{ij}(w, y)]_{(i,j) \in \mathcal{I} \times \mathcal{J}}$ with

$$h_{ij}(w, y) := \tilde{\varphi}^p_{ij} (w^p_{ij}, y^p_{ij}) + \min \left\{ \tilde{\varphi}^c_{ij} (w^c_{ij}, y^c_{ij}), \tilde{\varphi}^a_{ij} (w^c_{ij}, y^a_{ij}) \right\},$$

and

$$\tilde{\varphi}^c_{ij} (w^c_{ij}, y^c_{ij}) := \log \left( 1 + \|w^c_{ij} - H_{ij} y^c_{ij} \|^2 \right) / y^c_{ij},$$
$$\tilde{\varphi}^a_{ij} (w^c_{ij}, y^a_{ij}) := \log \left( 1 + \|w^c_{ij} - H_{ij} y^a_{ij} \|^2 \right) / y^a_{ij},$$
$$\tilde{\varphi}^p_{ij} (w^p_{ij}, y^p_{ij}) := \log \left( 1 + \|w^p_{ij} - H_{ij} y^p_{ij} \|^2 \right) / y^p_{ij},$$

for the following concave functions

$$\tilde{g}^c_{ij} (w^c_{ij}, y^c_{ij}) = \log \left( 1 + \|w^c_{ij} - H_{ij} y^c_{ij} \|^2 \right) / y^c_{ij},$$
$$\tilde{f}^c_{ij} (w^c_{ij}, y^a_{ij}) = \log \left( 1 + \|w^c_{ij} - H_{ij} y^a_{ij} \|^2 \right) / y^a_{ij},$$
$$\tilde{g}^a_{ij} (w^c_{ij}, y^a_{ij}) = \log \left( 1 + \|w^c_{ij} - H_{ij} y^a_{ij} \|^2 \right) / y^a_{ij},$$
$$\tilde{f}^p_{ij} (w^p_{ij}, y^p_{ij}) = \log \left( 1 + \|w^p_{ij} - H_{ij} y^p_{ij} \|^2 \right) / y^p_{ij}. \quad (25)$$
Proof: The proof of the equivalence between (22) and (24) is similar to the proof of Theorem 1. The concavity of the concerned functions follow from [30, Lemma 1].

Next, one has the following sequential d.c. representations:

- \( \min \{ \tilde{g}_{i,j}^c(w_{i,j}^c, y_{i,j}^c), \tilde{g}_{a(i,j)}^a(w_{i,j}^a, y_{a(i,j)}^a) \} = \tilde{g}_{i,j}(w_{i,j}^c, y_{i,j}^c), y_{a(i,j)}^a \)
- \( f_{i,j}^c(w_{i,j}^c, y_{i,j}^c, y_{a(i,j)}^a) \), defined at the bottom of the page.
- \( \tilde{g}_{i,j}(w_{i,j}^c, y_{i,j}^c) = f_{i,j}(w_{i,j}^c, y_{i,j}^c) \) for \( \tilde{g}_{i,j}(w_{i,j}^c, y_{i,j}^c) \) and \( f_{i,j}(w_{i,j}^c, y_{i,j}^c) \), defined at the bottom of the page.
- \( \min_{i(j) \in I \times J} \tilde{g}_{i,j}(w_{i,j}^c, y_{i,j}^c) = \tilde{g}_{i,j}(w_{i,j}^c, y_{i,j}^c) \) for \( \tilde{g}_{i,j}(w_{i,j}^c, y_{i,j}^c) \), defined at the bottom of the page.
- \( \sum_{i(j) \in I \times J} \tilde{g}_{i,j}(w_{i,j}^c, y_{i,j}^c) = \tilde{g}_{i,j}(w_{i,j}^c, y_{i,j}^c) \) for \( \tilde{g}_{i,j}(w_{i,j}^c, y_{i,j}^c) \), defined at the bottom of the page.

Based on the above sequential d.c. representations we obtain the following result.

**Theorem 4:** For \( \tilde{g}(w, y) = \tilde{g}^+(w, y) \lor \tilde{g}^-(w, y) \), problem (22) is the following convex constrained d.c. function maximization

\[
\max_{w,y} \left[ \tilde{g}(w, y) - f(w, y) \right] : w \in \tilde{V}_B, \quad (26)
\]

where function \( f(w, y) \) is concave and smooth while function \( \tilde{g}(w, y) \) is concave but nonsmooth.

Obviously, program (26) is in form of (17) with \( z \to (w, y), D \to \{(w, y) : w \in \tilde{V}_B, (23)\} \), \( G(z) \to \tilde{g}(w, y) \) and \( F(z) \to \tilde{f}(w, y) \).

Accordingly, initialized from a feasible solution \((w(0), y(0))\) of (26), \( \kappa \)-iteration (18) for \( \kappa = 1, 2, \ldots \), generates a feasible solution \((w(k+1), y(k+1))\) by solving the convex program

\[
\max_{w,y} \left[ \tilde{g}(w, y) - \tilde{f}(w(k), y(k)) \right] - \left\{ \nabla \tilde{f}(w(k), y(k)), (w, y) - (w(k), y(k)) \right\}
\]

s.t. \( w \in \tilde{V}_B, \quad (23) \).

(27)

The gradient \( \nabla \tilde{f}(w(k), y(k)), (w, y) - (w(k), y(k)) \) can be easily calculated from Equation (33) in Appendix II.

Although (27) is a convex program, it is still not easily solved by existing convex solvers. Thus, for computationally-efficient implementation of (27) by the existing convex solvers, we next introduce slack variables \( t_{i,j} := (t_{i,j}^c, t_{a(i,j)}^a, t_{i,j}^p) \), \( t := [t_{i,j}]_{i(j) \in I \times J} \)

satisfying the following semi-definite constraints

\[
\begin{align*}
\left( \frac{w_{i,j}^c}{y_{i,j}^c} \right) & \leq t_{i,j}^c, \\
\left( \frac{w_{i,j}^c}{y_{a(i,j)}^a} \right) & \leq t_{a(i,j)}^a, \\
\left( \frac{w_{i,j}^p}{y_{i,j}^p} \right) & \leq t_{i,j}^p, \quad (i, j) \in I \times J. \quad (28)
\end{align*}
\]

For the self-concordant function

\[
\varphi(\tau) = \log(1 + \tau) - \log(e) \tau, \quad \chi(\tau) = -\log(e) \tau,
\]

and \( \varphi(t) \), defined at the bottom of the page, the convex program (27) is equivalent to the following convex program.
TABLE I
PROBLEM DATA AND NUMERICAL SOLUTION WITH $P_B = \sigma^2 = 0$ dB IN (2)

<table>
<thead>
<tr>
<th>Deterministic Example</th>
<th>weak IN 1 [12, (60)]</th>
<th>weak IN 2 [12, (64)]</th>
<th>mixed IN [12, (70)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(H_{1,1,1}^2, H_{2,1,2}^2)$ in dB</td>
<td>(38.799, 46.108)</td>
<td>(19.841, 17.037)</td>
<td>(10, 10)</td>
</tr>
<tr>
<td>$(H_{2,1,1}^2, H_{1,2,2}^2)$ in dB</td>
<td>(37.861, 18.039)</td>
<td>(−1.843, 11.879)</td>
<td>(15, 5)</td>
</tr>
<tr>
<td>sum rate/min rate (bps/Hz)</td>
<td>14.86/6.87</td>
<td>7.32/3.39</td>
<td>7.05/1.27</td>
</tr>
<tr>
<td>max sum split $(\alpha_1, \alpha_2)$</td>
<td>(0.03, 2.31)</td>
<td>(8.84, 0.65)</td>
<td>(0.39, 0)</td>
</tr>
<tr>
<td>max/min split $(\alpha_1, \alpha_2)$</td>
<td>(0.69, 1.81)</td>
<td>(2.81, 0.65)</td>
<td>(0.13, 0)</td>
</tr>
<tr>
<td>sum rate/min rate (bps/Hz)</td>
<td>16.05/7.64</td>
<td>7.92/3.83</td>
<td>7.05/2.85</td>
</tr>
<tr>
<td>max-sum/maximim improvement</td>
<td>4.75%/35.88%</td>
<td>same/almost same</td>
<td>5.90%/52.38%</td>
</tr>
<tr>
<td>max-sum/maximim DCI # iter.</td>
<td>11/15</td>
<td>8/5</td>
<td>5/4</td>
</tr>
</tbody>
</table>

computationally tractable program:

$$\max_{\omega, \zeta, \gamma} \left[ \varphi(t) - \tilde{f}(w^{(e)}, y^{(e)}) \right]$$

$$s.t. \quad \omega \in \hat{W}_B, \quad (23), \quad (28).$$

Likewise, the conventional coordinated beamforming optimization problems are easily represented by (17) with $z \rightarrow (w^0, y^p)$ and can be solved by the DCIs described before.

In summary, our approach toward finding the beamforming solution includes the following steps:

(i) Use DCIs to find optimized solution $w^{p(0)}$ of the corresponding coordinated beamforming problems;

(ii) Use $w^{p(0)}$ to determine the pairing strategy $\lambda_C$;

(iii) Use DCIs to find optimized solution $w$ of (22) and (26).

V. SIMULATION RESULTS

In this section, numerical results are presented to show the rate performances achieved by different schemes. For ease of presentation, the conventional coordinated transmission involving only private messages is referred to as “conventional scheme”, whereas the common-private successive decoding message scheme as “new scheme”. In the implementation, the number $L$ of common user pairs is equal to the total number $NK$ of users ($L= NK$). The pairing map $a \in A_L$ is predetermined according to the procedure described at the beginning of Section III to form problems (12) and (22). The computational tolerance in DCIs is set as $\epsilon = 10^{-5}$. Each plotted point for the Monte Carlo simulations is based on 200 random network realizations.

For convenience, set $H_{i,j} := \sqrt{\theta_{i,j}} H_{i,j}$ and $H_{m,i,j} := \sqrt{\theta_{m,i,j}} H_{m,i,j}$ for $m \neq i$, where $H_{m,i,j} \in C^{N_j \times N_i}$ represents the normalized MIMO channel from BS $m$ to user $(i,j)$. Its entries are independent and identically distributed complex Gaussian variables with zero mean and unit variance. The quantity $\theta_{i,j}$ defines the strength of the direct channel $H_{i,j}$, whereas $\theta_{m,i,j}$ (for $m \neq i$) is the strength of the interfering channel $H_{m,i,j}$. The IN can thus be fully parameterised by the set of $(\eta, P_B/\sigma^2, \theta)$, where $\eta$ is the set of all $\theta_{i,j}$ and $\theta_{m,i,j}$.

A. Two User SISO Networks Revisited

Consider three randomly generated examples corresponding to three scenarios [12, (60), (64), and (70)]. The corresponding numerical data is provided by the second and third of Table I. The reader is referred to [12, (61), or (66), or (70)] for inner bound, and [12, (36), or (48)] and [15, (1)–(3)] for outer bound on the capacity region. Following [12], we provide in Table I the numerical solution in terms of the ratios of private-message-induced interference to noise at each receiver $i$ defined by $\alpha_i := |H_{i,1,1}|^2 Q_{i,1}^p / \sigma^2, j \neq i$. At the solutions found by DCI (for successive decoding) the rates with successive decoding and joint decoding are the same though they are different at the solution by [12]. All the results in Table I are in terms of rates achievable by joint decoding. It is observed that the minimal rate improvement of the new scheme over the conventional scheme is very essential (35.88% and 52.38% in weak IN 1 and mixed IN). The computational performance of the proposed DCI was observed to not be sensitive to initial solutions. For these particular low dimension problems, the optimality of DCI is also confirmed by a heuristic search over $10^5 \times 10^5$ sample points $(\alpha_1, \alpha_2)$.

Consider another symmetric weak IN [31, (18)] with $|H_{1,1,1}|^2 = |H_{2,1,2}|^2 = 0$ dB, $|H_{1,2,1}|^2 = |H_{1,2,2}|^2 = -7.6955$ dB and $\sigma^2 = 0$ dB, where the sum rates achieved by joint and successive decoding schemes may be different. Fig. 1 provides the plot of sum rate versus the private message power to noise ratio $Q_p/\sigma^2$ under symmetric power split $(Q_{1,1}^p = Q_{2,1}^p = Q_p)$ [31], [32]. The obtained curves for $P_B = 30$ dB coincide with that in [31, Fig. 3]. In regard to the successive decoding scheme, the symmetric power split is not optimal for $P_B = 50$ dB. In fact, the DCI outputs multiple optimal solutions $(Q_{1,1}^p/\sigma^2, Q_{2,1}^p/\sigma^2) \in \{(32546.47, 0), (0, 34498.82), (97017.65, 0)\}$ with a sum rate of 16.84 bps/Hz, which is much better than the largest sum rate 11.03 bps/Hz achieved by the symmetric power split. The form of these optimal power splits by the DCI is
consistent with the conjecture [32, p. 1275] that for two-user symmetric INs, the maximal sum rate is achieved either using symmetric power splits or by constraining one of the users to symmetric INs, the maximal sum rate is achieved either using consistent with the conjecture [32, p. 1275] that for two-user strengths

\( h \) follows:

**B. Covariance Split for MIMO INs**

The initial feasible solutions for DCI are generated as follows:

- For \( a(i, j) = (i', j') \neq \emptyset \),
  \[
  Q_{i,j}^{(0)} = \beta \left[ \frac{\sigma^2 P_B}{KN_i} \left( I_{N_i} + H_{i,i,j}^H H_{j,j} \right)^{-1} + Z \right].
  \]

- For \( a(i, j) = \emptyset \), user \( (i, j) \)'s private covariance is defined by (30), shown at the bottom of the page.

This ensures that user \( (i, j) \)'s private message transmission at a certain power level while causing the minimal interference to other unintended receivers.

1) **Case Study I** [18, Example 1] and Its Variant: There are two transmitters each equipped with \( N_t = 2 \) antennas. The first receiver has 3 antennas and the second receiver has 2 antennas. \( P_B = \sigma^2 = 0 \) dB. The direct channel strengths are given by \( (\eta_{1,1,1}, \eta_{2,2,1}) = (20, 20) \) with the interfering channel strengths \( (\eta_{1,2,1}, \eta_{2,1,1}) = (8, 12) \) (in dB), while \( h_{1,1,1}, h_{2,1,1}, h_{1,2,1} \) and \( h_{2,2,1} \) are given in [18, p. 4789]. Numerical results are provided in Table II with the obtained solutions given by (34)–(36) in Appendix III. It reveals that the solution by [18, Th. 3] does not perform better than the conventional scheme (without common message) for this weak interference setting. The rates obtained by DCI for the sum-rate and maximin-rate problems exceed well the inner bound. In terms of the minimal rate, the advantage of the new scheme over the conventional is marginal, i.e., having common messages is not beneficial in this example. The numerical results of Table II are visualized in Fig. 2 together with the inner bound plotted by solving the linear inequalities [18, (52a)–(52i)] and the outer bound by [18, (11)–(17)] (also shown in [18, Fig. 6(b)]).

To see the benefit of the common message, we increase the interfering channel strength \( \eta_{2,1,1} \) to 19 dB. The achievable rates are provided in Table III with the obtained solutions given by (37)–(40) in Appendix III. The new scheme improves both the sum rate (by 2.09 bps/Hz) and the minimal rate (by 0.31 bps/Hz) from those of the conventional one. The solutions by [18, Th. 3] are less competitive. Moreover, when moving from joint decoding to successive decoding its achievable minimal rate drops significantly. On the contrary, the solutions found by DCI appear to be immune from such side-effects. These numerical results are also plotted in Fig. 3.

2) **Monte Carlo Simulation I** [33, Fig. 3]: Consider a two-user MIMO IN \( (N = 2, K = 1) \) with the antenna numbers \( N_t = N_r \) ranging from 1 to 8. The direct channel and inferring

\[
Q_{i,j}^{(0)} = \min_{Q_{i,j} \in \mathcal{S}_{i,j}^{(0)}} \sum_{(n,k) \in \mathcal{X} \cap \mathcal{J} \setminus (i,j)} \langle H_{i,n,k} Q_{i,j}^H H_{i,n,k} \rangle : \langle Q_{i,j}^p \rangle \geq \beta P_B / K
\]
channel strengths are given by \( \eta_{1,1,1} = \eta_{2,2,1} = 0 \) dB and \( \eta_{1,2,1} = \eta_{2,1,1} = -4.7712 \) dB [33, p. 4317]. For \( P_B = 0 \) dB, the reader is referred to [33, Fig. 3] for its relevant Monte Carlo simulation based on 20 sets of randomly generated channel generalisations, which is also confirmed in Fig. 4(a).

The authors in [33] adopted a two-stage scheme, where each receiver jointly decodes the two common messages in the first stage and then decodes the private message in the second stage. Therefore, its achievable capacity region should lie in between that achieved by joint decoding [9] and our used successive decoding schemes. The computational solution in [33] is based on nonconvex optimization in covariance matrices at sample power factors in \([0, 1] \times [0, 1]\). The computation implementation with 225 uniformly sample points taken consumed hours.

**Table III**

<table>
<thead>
<tr>
<th>Sum rate (in bps/Hz)</th>
<th>Minimal rate (in bps/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Private rates</td>
</tr>
<tr>
<td>Conv. scheme</td>
<td></td>
</tr>
<tr>
<td>New scheme</td>
<td>Joint</td>
</tr>
<tr>
<td></td>
<td>Succ.</td>
</tr>
</tbody>
</table>

Fig. 4(a) and (b) provide the plot of the sum rate performance vs. the number \( N_f/N_r \) of antennas, with \( P_B = 0 \) dB and \( P_B = 30 \) dB, respectively. Like [33, Fig. 3], the upper bound is obtained by setting \( \eta_{1,2,1} = \eta_{2,1,1} = 0 \) to have two parallel channels with interference free transmissions. With \( P_B = 0 \) dB, the sum rate obtained by the three scheme (two-stage, new and conventional) are all close to the upper bound. With \( P_B = 30 \) dB, according to Fig. 4(b), the benefit of common message is more evident with more antennas \( N_f/N_r \) equipped. The plot for the two-stage scheme is below that for our used successive decoding scheme simply because taking 225 sample points is still not dense enough for obtaining the optimal solution of the two-stage scheme.

3) Monte Carlo Simulation II \((N = 2, K = 1, N_f = 4, N_r = 2):\) The statistical performance of MIMO H-K networks depicted by Fig. 5(a) is analysed. Following [12], [34], the direct channel strengths are set \((\eta_{1,1,1}, \eta_{2,2,1}) = (10, 20) \) (in dB), while the interfering channel strengths \( \eta_{1,2,1} = \eta_{2,1,1} \) are increased from \(-5 \) dB to \( 20 \) dB. These values cover all channels effects such as path loss and shadowing which may be environment-sensitive. The simulation scenarios thus vary from weak MIMO INs to mixed MIMO INs. Fig. 6(a) and (b) show the sum rates and the minimal rates versus the interfering channel strength, respectively. For this simplest case only \((N = 2, K = 1)\), the upper and lower bounds on the sum or minimal rates can be obtained by [18, Lemma 1] and [18, Theorem 3], respectively. It is obvious that DCI is able to effectively realise the benefits of the new scheme as its performance is well above the lower bound, while the performance of the conventional scheme deteriorates significantly as interference is increased.

4) Monte Carlo Simulation III \((N = 3, K = 1, N_f = 4, N_r = 2):\) Fig. 5(b) depicts a 3-BS network where each BS serves only one user. The interfering channel strengths \( \eta_{1,2,1}, \eta_{1,3,1}, \eta_{2,3,1} \) and \( \eta_{3,1,1} \) are virtually disabled by setting all of them to very low value \(-50 \) dB, while the direct channel strengths \((\eta_{1,1,1}, \eta_{2,1,1}, \eta_{3,1,1}) \) are fixed at \((10, 20, 5) \) (in dB). The interfering channel strengths \( \eta_{2,1,1} = \eta_{3,2,1} \) are increased from \(-10 \) dB to \( 30 \) dB for testing different scenarios of relative interference strength. Under this setting, User (1, 1) and user (2, 1) suffer increasingly from inter-cell interference. Fig. 7(a) and (b) show that the rate capacity of the conventional scheme drops. In contrast, the new scheme gains the rate slightly. Again, the benefit of the new scheme over the conventional emerges as the level of interference increases.

5) Monte Carlo Simulation IV \((N = 2, K = 2, N_f = 4, N_r = 2):\) As shown in Fig. 5(c), the direct channel strengths \( \eta_{1,1,1} = \eta_{2,2,1} \) and \( \eta_{1,1,2} = \eta_{2,1,2} \) are, respectively, fixed at \( 10 \) dB and \( 15 \) dB, and the interfering channel strengths \( \eta_{2,1,1} \) and \( \eta_{2,1,2} \)
Fig. 5. Different network configurations considered in simulation. The number shown on each link indicates the channel gain in decibels, whereas a link with $x$ dB means that its channel gain is varied as discussed in the main text of the paper. (a) $N = 2$, $K = 1$, $N_t = 4$, $N_r = 2$. (b) $N = 3$, $K = 1$, $N_t = 4$, $N_r = 2$. (c) $N = 2$, $K = 2$, $N_t = 4$, $N_r = 2$.

Fig. 6. Rate performance versus interfering channel strength $\eta_{2,1,1} = \eta_{1,2,1}$ for $N = 2$, $K = 1$, $N_t = 4$, $N_r = 2$. (a) Sum rates. (b) Min-rates.

Fig. 7. Rate performance versus interfering channel strength $\eta_{2,1,1} = \eta_{3,2,1}$ for $N = 3$, $K = 1$, $N_t = 4$, $N_r = 2$. (a) Sum rates. (b) Min-rates.

are disabled by setting them to $-50$ dB. The interfering channel strengths $\eta_{1,2,1} = \eta_{1,2,2}$ are increased from $-10$ dB to 50 dB. In Fig. 8(a) and (b), the new scheme is able to attenuate the rate deterioration as inter-cell interference increases.

6) Computational Experience: The first column block in Table IV provides the number of d.c. iterations for all the MIMO networks investigated in this section. A typical convergence behavior for sum rate optimization ($N = 2$, $K = 2$, $N_t = 4$, $N_r = 2$).
is predetermined, it should be emphasized that $a(i,j) \neq \emptyset$ does not mean that $Q_{i,j}^{0} \neq 0$ at the optimality of (12), so the actual number of active common message user pairs still varies from 0 to 9. Our investigation indicates that not more than 5 common user pairs are actually selected. Due to uniform user distribution and path fading/shadowing effects, the data $(\eta, P_B/\sigma^2)$ represents quite a complicated and unpredictable interference situation. It has been reported in [7] that the conventional scheme still achieves the rate capacity for certain channel realizations. We call a channel realization “effective” if the rate achieved by the new scheme outperforms that by the conventional scheme by more than 1%. Otherwise, it is said to be “ineffective”. Fig. 11(a) and (b) show the minimal user’s rates and sum rates versus BS power budgets, respectively. With $P_B = 30$ dBm, effective channels account for 21.00% and 22.50% in the two problems, respectively. With the power budget increased to $P_B = 70$ dBm, the percentages grow to 66.75% and 78.25%, accordingly. Clearly, the new scheme is more effective for higher-SNR regimes.

3) Computational Experience: The second column block in Table IV provides the numbers of DCIs (29) for all the MISO networks considered in this section. The numbers of SOCPs (second-order cone programs) for solving the minimal rate maximization for the MISO case are also included. The typical convergence behavior of DCI for beamforming design is similar to that presented in Fig. 9. Table V provides the number of SDP needed for feasibility check by the algorithm of Reference [20]. It is also noted that the performance of the algorithm in [20] strongly depends on the order of pairs picked for private message decoding.

VI. CONCLUSION

This paper studied the optimized transmission strategies for interference mitigation in multi-cell multi-user MIMO networks, which are commonly modeled as multi-user MIMO interference networks. The ability of splitting user messages into private and common messages to increase the achievable rate region of a multi-user MIMO interference network has been well understood, but its optimization has never been adequately addressed.
As an important contribution to addressing this problem, this paper formulated the optimal rate splitting problem as a nonsmooth d.c. objective function minimization subject to convex constraints in the reduced space of the designed covariance variables only. Then tailored DCI algorithms were provided to obtain the solutions, which guarantee rate improvement after each iteration. In the presence of mild-to-strong interferences, comprehensive simulation results demonstrated significant rate gains obtained by the proposed message-splitting scheme, for both MIMO and MISO interference networks. The results also showed that our proposed DCIs outperform other existing methods and converge within relatively small numbers of iterations.

**TABLE IV**

<table>
<thead>
<tr>
<th></th>
<th>MIMO IN</th>
<th>MISO IN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>two-user</td>
<td>three-user</td>
</tr>
<tr>
<td>Conventional Sum-Rate</td>
<td>11.53</td>
<td>14.26</td>
</tr>
<tr>
<td>New Sum-Rate</td>
<td>19.34</td>
<td>24.62</td>
</tr>
<tr>
<td>Conventional Min-Rate</td>
<td>12.92</td>
<td>17.76</td>
</tr>
<tr>
<td>New Min-Rate</td>
<td>22.01</td>
<td>27.17</td>
</tr>
<tr>
<td>Conventional Min-Rate by bisection</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**TABLE V**

Complexity Analysis: Total Number of SDP Calls by the Procedure in [20]

<table>
<thead>
<tr>
<th>Designed Number of Common User Pairs (L)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>250.5</td>
<td>270.1</td>
<td>319.2</td>
<td>326.2</td>
<td>340.9</td>
<td>340.2</td>
<td>342.4</td>
<td>342.2</td>
<td>342.2</td>
</tr>
</tbody>
</table>

**APPENDIX I**

For function $\Theta(\mathbf{Q}) = \log\left( \sum_{(i,j) \in \mathcal{I} \times \mathcal{J}} H_{ij}^H \mathbf{Q}_{ij} H_{ij}^H + \sigma^2 I_{N_r} \right)$ it is true that

$$
\nabla \Theta\left(\mathbf{Q}^{(e)}\right), \mathbf{Q} - \mathbf{Q}^{(e)}
= \left\{ \sum_{(i,j) \in \mathcal{I} \times \mathcal{J}} H_{ij} \mathbf{Q}_{ij}^{(e)} H_{ij}^H + \sigma^2 I_{N_r} \right\}^{-1}
\times \sum_{(i,j) \in \mathcal{I} \times \mathcal{J}} H_{ij} \left( \mathbf{Q}_{ij} - \mathbf{Q}_{ij}^{(e)} \right) H_{ij}^H
$$

(32)
For function \( \theta(w, y) = |(w, H)|^2(y + \sigma^2) \) it is true that
\[
\nabla \theta \left( w^{(k)}, y^{(k)} \right), (w, y) - \left( w^{(k)}, y^{(k)} \right) = 2\Re \left\{ \frac{|(w^{(k)}, H)|^2}{y^{(k)} + \sigma^2} (w - w^{(k)}, H) - \frac{|(w^{(k)}, H)|^2}{y^{(k)} + \sigma^2} (y - y^{(k)}) \right\}.
\]

\[ (33) \]

**APPENDIX III**

**NUMERICAL SOLUTION FOR [18, EXAMPLE 1] AND ITS VARIANT IN SECTION V-B1**

- For sum rate optimization under the conventional scheme:
  \[
  Q_{1,1}^c = \begin{bmatrix}
  0.7018 \\
  0.0404 + 0.4575j \\
  0.0040 - 0.4575j \\
  0.2982
  \end{bmatrix},
  \]
  \[
  Q_{2,1}^c = \begin{bmatrix}
  0.7099 \\
  -0.0507 - 0.1009j \\
  0.2091
  \end{bmatrix}.
  \]

- For sum rate optimization under the new scheme: it is the same as above with \( Q^c = 0 \).

- For minimal rate optimization under the conventional scheme:
  \[
  Q_{1,1}^m = \begin{bmatrix}
  0.6508 \\
  -0.2651 + 0.3962j \\
  0.3492
  \end{bmatrix},
  \]
  \[
  Q_{2,1}^m = \begin{bmatrix}
  0.6177 \\
  -0.1103 + 0.4733j \\
  0.3823
  \end{bmatrix}.
  \]

- For minimal rate optimization under the new scheme:
  \[
  Q_{1,1}^n = \begin{bmatrix}
  0.2057 \\
  -0.0839 - 0.1738j \\
  0.1811
  \end{bmatrix},
  \]
  \[
  Q_{2,1}^n = \begin{bmatrix}
  0.6099 \\
  -0.0409 - 0.0194j \\
  0.0034
  \end{bmatrix},
  \]
  \[
  Q_{1,1}^n = \begin{bmatrix}
  0.4621 \\
  -0.1206 - 0.2705j \\
  0.2550
  \end{bmatrix},
  \]
  \[
  Q_{2,1}^n = \begin{bmatrix}
  0.8274 \\
  -0.0401 - 0.1202j \\
  0.1726
  \end{bmatrix}.
  \]
  \[ (37) \]

For the variant of [18, Example 1] in Section V-B1, the results are:

- For sum rate optimization under the conventional scheme:
  \[
  Q_{1,1}^c = \begin{bmatrix}
  0.5886 \\
  0.1326 + 0.1173j \\
  0.0532
  \end{bmatrix},
  \]
  \[
  Q_{2,1}^c = \begin{bmatrix}
  0.6015 \\
  0.0134 + 0.1352j \\
  0.3985
  \end{bmatrix}.
  \]
  \[ (38) \]

- For minimal rate optimization under the conventional scheme:
  \[
  Q_{1,1}^m = \begin{bmatrix}
  0.2135 \\
  -0.1285 - 0.1420j \\
  0.1718
  \end{bmatrix},
  \]
  \[
  Q_{2,1}^m = \begin{bmatrix}
  0.6192 \\
  -0.0285 - 0.0159j \\
  0.0017
  \end{bmatrix},
  \]
  \[
  Q_{1,1}^n = \begin{bmatrix}
  0.6392 \\
  0.0484 + 0.3008j \\
  0.2497
  \end{bmatrix},
  \]
  \[
  Q_{2,1}^n = \begin{bmatrix}
  0.4848 - 0.3008j \\
  0.2497
  \end{bmatrix}.
  \]
  \[ (39, 40) \]

**REFERENCES**


