Relay Beamforming Designs in Multi-User Wireless Relay Networks Based on Throughput Maximin Optimization

U. Rashid, H. D. Tuan, and H. H. Nguyen

Abstract—Beamforming design for multi-user wireless relay networks under the criterion of maximin information throughput is an important but also very hard optimization problem due to its nonconvex nature. The existing approach to reformulate the design as a matrix rank-one constrained optimization problem is highly inefficient. This paper exploits the d.c. (difference of two convex functions) structure of the objective function and the convex structure of the constraints in such a global optimization problem to develop efficient iterative algorithms of very low complexity to find the solutions. Both cases of concurrent and orthogonal transmissions from sources to relays are considered. Numerical results indicate that the proposed algorithms provide solutions that are very close to the upper bound on the solution of the non-orthogonal source transmissions case and are almost equal to the optimal solution of the orthogonal source transmissions case. This demonstrates the ability of the developed algorithms to locate approximations close to the global optimal solutions in a few iterations. Moreover, the proposed methods are superior to other methods in both performance and computation complexity.

Index Terms—Multi-user relay networks, amplify-and-forward relaying, beamforming, network throughput, maximin optimization, d.c. (difference of two convex functions) programming, d.c. decomposition.

I. INTRODUCTION

RELAY-ASSISTED wireless communication is currently one of the most active research topics (see e.g. [1]–[3]). The key advantage of relay-assisted communication is that spatial diversity can be exploited via cooperation of relays/nodes to improve the link reliability, and to extend the communication coverage area [4]–[6]. In a multi-user relaying framework, distributed relay nodes are employed to assist the communication among a number of multiple sources and destinations [7]. Relay-assisted communication schemes are generally classified into three main categories: decode-and-forward (DF), compress-and-forward (CF) and amplify-and-forward (AF) [2], [8]. Due to its simplicity in mathematical modeling and low cost in implementation, AF relaying scheme has been extensively studied. In AF relaying, the relays simply amplify the signals received from the sources and then forward the amplified versions to all the destinations. In a multi-user relay network, beamforming is implemented at relay nodes such that the desired signal of each user at the destination can be constructively combined, while the interferences and noise are efficiently mitigated [9]–[15].

A large body of research works considered beamforming design in the context of relay power minimization subject to the signal-to-interference-plus-noise ratio (SINR) constraints. While the objective function of beamforming power is quadratic convex in the complex beamforming vector, the SINR constraints are indefinite (nonconvex), which makes the overall program nonconvex quadratic. Such a nonconvex quadratic program can be trivially rewritten as a rank-one constrained matrix program with the variable dimension substantially increased. Without individual relay power constraints, this nonconvex program can be transformed into a relaxed semi-definite (convex) program (SDP) by dropping the rank-one constraint because the relaxed SDP often outputs its rank-one optimal solution [9], [10]. However, as shown both theoretically and numerically in [12]–[14], the presence of the individual relay power constraints (which are necessary to reflect practical limitations of relay hardware [4]) would make the optimized solution of the corresponding relaxed SDP just low-rank, but no longer rank-one. Not having rank-one with the optimal solution of the SDP relaxation means that it is not possible to locate even feasible beamforming solutions. On the other hand, the low-rank of its optimal solution means that a further randomization [10], which in fact tries to randomly generate feasible solutions in the low-dimensional subspace of eigenvectors corresponding to nonzero eigenvalues of its optimal solutions, is highly inefficient. It can be easily shown that a nontrivial randomization would simply perform the same. Our previous works [12]–[14] developed efficient algorithms to address this beamforming power minimization, where the original matrix rank-one (nonconvexity) constrained optimization is exactly reformulated as minimization of nonsmooth matrix spectral objective functions under convex constraints. Additionally, the objective functions of the reformulated optimizations have been shown to be d.c. (difference of two convex) functions in the matrix variables [16]. The d.c. path-following procedures [17], [18] have been shown to effectively locate the global optimal solutions.

One of the main objectives of emerging wireless technologies is to provide reliable services at high throughput under limited power resources. In the context of multi-user relay-assisted wireless networks, the goal is to maximize the information throughput among multiple sources and destinations in a fair manner. Accordingly, the beamforming design problem is to maximize the minimum information throughput among source-destination pairs. In other words, the problem...
is maximin throughput optimization, which is related to the maximization problem of the minimum SINR under limited beamforming power constraints. However, in contrast to the above mentioned power beamforming problem, the objective function of maximin information throughput optimization is neither smooth nor concave. This means that the program is maximization of nonconcave and nonsmooth objective functions subject to (nonconvex) rank-one constraints. Such a program belongs to the most challenging optimization class [16].

Specifically, even by dropping the rank-one constraints, the relaxed maximization problem is still not computationally tractable as the objective function is still not concave. The conventional rank-one dropping relaxation simply does not work. This problem has been considered by [19] for a particular case of a total relay power constraint only. A standard bisection procedure to iteratively update feasible SINR thresholds was employed. As mentioned above, without individual relay power constraints, the feasibility/infeasibility of a SINR threshold can be accurately solved by the SDP relaxation. Under the individual relay power constraints, this nonconvex program cannot be addressed by SDP relaxation and so the bisection method is no longer a good solution procedure. Again, a trivial randomization would perform the same. In the end, this maximin throughput optimization is convex constrained optimization in the beamforming vector. Such a matrix rank-one constrained re-formulation not only increases the problem dimension substantially but also unnecessarily invites the nonconvex hard rank-one constraint that destroys the tractable convexity of the original constraints, while the new objective function is still nonconvex. In other words, the existing matrix optimization approach reformulates the original hard convex constrained minimization of a nonconvex objective function to a much harder and much larger dimension nonconvex constrained minimization of a nonconvex objective function.

In this paper, we follow [17], [18], [20] and adopt d.c. programming [16], [21] to directly address this nonconvex maximin throughput optimization problem, bypassing the above mentioned matrix rank-one constrained optimization. The main issue is how to recognize and then explore hidden partial convex structures of the problem at hand in order to develop an effective algorithm to find the solutions. As both dimension and nonconvexity rank of this optimization problem are surely high for applicability of global optimization algorithms [16], we pursue alternative iterations of the local developments are presented in Section V. Conclusions are drawn in Section VI. The Appendix provides technical proofs for the theoretical results of Section III.

Notations: Matrices and column vectors are denoted by boldfaced uppercase and lowercase characters, respectively. \(0_N\) and \(I_N\) are zero and identity matrices of dimension \(N \times N\), respectively. The notation \(A \geq 0\) means \(A\) is a (Hermitian) positive semi-definite matrix. We denote \(\langle A, B \rangle = \text{trace}(AB)\) for matrices \(A\) and \(B\) of appropriate sizes, while \((a, b) = a^T b\) (their dot product) and \(|a|^2 = \langle a, a \rangle\), where \(\bar{a}\) is the conjugate of \(a\). The notation \(\lambda_{\max}(X)\) stands for the maximal eigenvalue of \(X\), while \(\rho(X) := \max_{i=1,2,...} |\lambda_i(X)|\) with its eigenvalues \(\lambda_i(X)\) is its spectral radius. For \(x = (x_1, x_2, \ldots, x_N) \in \mathbb{C}^N\), \(|x|^2\) stands for \(|x_1|^2, |x_2|^2, \ldots, |x_N|^2\) \(\in \mathbb{R}^N\) while \(\text{diag}(x)\) stands for a diagonal matrix with diagonal entries \(x_i\), \(i = 1, 2, \ldots, N\). \(a \circ b\) is the element-wise Hadamard product of two vectors \(a\) and \(b\), which is obviously commutative, i.e., \(a \circ b \circ c\) does not depend on the order of \(a, b\) and \(c\). It is also obvious that \((a, b \circ c) = (a \circ b, c)\).

II. Maximin Throughput Optimization: Problem Formulations and Challenges

Figure 1 illustrates a wireless relay network, in which \(M\) pairs of source-destination communicates with the help of \(N\) relays. All relay and user nodes are restricted to have a single antenna and to operate in half-duplex mode. In the first timeslot, the sources simultaneously send their signals to the relays. The relays “amplify” their received signals by multiplying with certain weights and then simply forward these processed signals to all destinations. The case that sources communicate with the relays over orthogonal channels shall be treated in Section IV.

Let \(s = (s_1, s_2, \ldots, s_M)^T \in \mathbb{C}^M\) be the vector of signals sent by \(M\) sources, which is assumed to be zero mean and component-wise independent with variance \(\sigma_i^2 = \mathbb{E}|s_i|^2\). Let \(\tilde{h}_m = (\tilde{h}_{m1}, \tilde{h}_{m2}, \ldots, \tilde{h}_{mN})^T \in \mathbb{C}^N\), \(m = 1, 2, \ldots, M\), be the vector of backward channel coefficients between the \(m\)th source and all the relays. Likewise, let \(\tilde{e}_i = (\tilde{e}_{i1}, \tilde{e}_{i2}, \ldots, \tilde{e}_{iN})^T \in \mathbb{C}^N\), \(i = 1, 2, \ldots, M\), be the vector of forward channel coefficients between all the relays and the \(i\)th destination. The received signals at all the relays can be collectively written as

\[
y_{up} = \sum_{m=1}^{M} \tilde{h}_m s_m + n_R, \tag{1}
\]
where \( \mathbf{n}_n = (n_{R,1}, \ldots, r_{R,N})^T \in \mathbb{C}^N \) represents the additive noises at the relay receivers, which is modeled as zero-mean white Gaussian random variables of variance \( \sigma_R^2 = \mathbb{E}[|n_{R,n}|^2] \), \( n = 1, 2, \ldots, N \).

Let \( \mathbf{x} = (x_1, x_2, \ldots, x_N)^T \) be the vector of beamforming weights. Then the relays send the following signals to the destinations:

\[
y_{amp} = \mathbf{x} \odot \mathbf{y}_{up} = \sum_{m=1}^{M} \mathbf{x} \odot \tilde{h}_m s_m + \mathbf{x} \odot \mathbf{n}_R. \tag{2}
\]

Accordingly, the received signal at the \( i \)th destination is

\[
y_{D,i} = \langle \ell_i, y_{amp} \rangle + n_{D,i} = \sum_{m=1}^{M} \langle \tilde{c}_{mi}, x \rangle s_m + \langle \tilde{c}_{mi} \odot n_R, x \rangle + n_{D,i}, \tag{3}
\]

where \( n_{D,i} \) is the additive Gaussian noise at the \( i \)th destination with variance \( \sigma_D^2 \). The vector \( \tilde{c}_{mi} = \ell_i \odot \tilde{h}_m \) represents the compound channel coefficient from source \( m \) to destination \( i \).

To incorporate the channel uncertainties in the beamforming design, assume that

\[
\begin{align*}
\tilde{h}_m \ell_i^H & = h_m \ell_i^H + \Delta \mathbf{H}_m, \quad m = 1, 2, \ldots, M, \\
\tilde{c}_{mi} \ell_i^H & = c_{mi} \ell_i^H + \Delta \mathbf{C}_{mi}, \quad m, i = 1, 2, \ldots, M,
\end{align*}
\tag{4}
\]

where

- \( h_m, \ell_i, c_{mi} \) are nominal values that can be obtained through channel estimation (see e.g. [26], where the compound channel coefficients \( c_{mi} \) can be directly obtained at the destination side);
- \( \Delta \mathbf{H}_m \in \mathbb{C}^{N \times N}, \Delta \mathbf{L}_i \in \mathbb{C}^{N \times N} \) and \( \Delta \mathbf{C}_{mi} \in \mathbb{C}^{N \times N} \) are Hermitian symmetric full block uncertainty matrices [27], which satisfy the following spectral constraints for \( m, i = 1, 2, \ldots, M \):

\[
\rho(\Delta \mathbf{H}_m) \leq \zeta^2, \quad \rho(\Delta \mathbf{L}_i) \leq \zeta^2, \quad \rho(\Delta \mathbf{C}_{mi}) \leq \zeta^2. \tag{5}
\]

The uncertainties in the form of (4) are called unstructured (see e.g. [27]). On the other hand, the structured uncertainties have the following forms:

\[
\begin{align*}
\tilde{h}_m & = h_m + \delta h_m, \quad \tilde{c}_{mi} = c_{mi} + \delta c_{mi}, \tag{6}
\end{align*}
\]

with

\[
||\delta h_m|| \leq \zeta, \quad ||\delta \ell_i|| \leq \zeta, \quad ||\delta c_{mi}|| \leq \zeta. \tag{7}
\]

In fact, (6) can be rewritten in the form of (4) with

\[
\begin{align*}
\Delta \mathbf{H}_m & = h_m (\delta h_m)^H + (\delta h_m) h_m^H + \delta h_m (\delta h_m)^H, \\
\Delta \mathbf{L}_i & = \ell_i (\delta \ell_i)^H + (\delta \ell_i) \ell_i^H + \delta \ell_i (\delta \ell_i)^H, \\
\Delta \mathbf{C}_{mi} & = c_{mi} (\delta c_{mi})^H + (\delta c_{mi}) c_{mi}^H + \delta c_{mi} (\delta c_{mi})^H,
\end{align*}
\tag{8}
\]

and

\[
\zeta^2 := \zeta^2 + 2 \max_{m = 1, 2, \ldots, M} \max_{i = 1, 2, \ldots, M} \frac{||h_m||}{||h_m||}, \quad \max_{m = 1, 2, \ldots, M} \frac{||\ell_i||}{||\ell_i||}, \quad \max_{m = 1, 2, \ldots, M} \frac{||c_{mi}||}{||c_{mi}||}. \tag{9}
\]

For the received signal given in (3), since only the signal component \( \langle c_{mi}, x \rangle s_i \) is of interest, its “robust” power is defined by

\[
S_i(x) = \sigma_s^2 \inf_{\rho(\Delta \mathbf{C}_{mi}) \leq \zeta^2} ||\langle x, \tilde{c}_{mi} \rangle||^2
\]

\[
\begin{align*}
&= \sigma_s^2 \inf_{\rho(\Delta \mathbf{C}_{mi}) \leq \zeta^2} ||\langle x, c_{mi} \rangle||^2 + \sigma_R^2 ||\langle x, \ell_i \rangle||^2 \\
&= \sigma_s^2 ||\langle x, c_{mi} \rangle||^2 - \zeta^2 ||x||^2. \tag{10}
\end{align*}
\]

Analogously, the “robust” interference power in (3) is defined by

\[
\begin{align*}
\text{INT}_i(x) & = \sigma_s^2 \sup_{\rho(\Delta \mathbf{L}_i) \leq \zeta^2} \sigma_R^2 ||\langle x, \tilde{c}_{mi} \rangle||^2 \\
&= \sigma_s^2 ||\langle x, c_{mi} \rangle||^2 + \zeta^2 ||x||^2 \\
&+ \sigma_R^2 ||\langle x, \ell_i \rangle||^2 + \zeta^2 ||x||^2. \tag{11}
\end{align*}
\]

It follows that the “robust” signal-to-interference-plus-noise ratio (SINR) at destination \( i \) can be expressed as

\[
\text{SINR}_i(x) = \frac{S_i(x)}{\text{INT}_i(x) + \sigma_D^2}. \tag{12}
\]

Note that by (2) the total beamforming power across all the relays is

\[
P_T(x) = \mathbb{E}[||y_{amp}||^2] \]

\[
\begin{align*}
&= \sigma_s^2 \sup_{\rho(\Delta \mathbf{H}_m) \leq \zeta^2} \sum_{m=1}^{M} ||\langle x, \tilde{h}_m \rangle||^2 + \sigma_R^2 ||x||^2 \\
&= \langle xx^H, R \rangle,
\end{align*}
\]

with

\[
R := \text{diag}(r), \quad r := (r_1, \ldots, r_N)^T,
\]

\[
r_n = \sigma_s^2 \sum_{m=1}^{M} (||h_{mn}||^2 + \zeta^2) + \sigma_R^2, \quad n = 1, 2, \ldots, N.
\]

On the other hand, the individual beamforming power at relay \( n \) is

\[
P_n(x_n) = r_n ||x_n||^2.
\]

In order to quantify quality of service (QoS) under multi-user communication framework, we use the metric of information throughput computed for each source-destination pair. This quantity is expressed for the \( i \)th source-destination pair as

\[
I_i(x) = \log_2(1 + \text{SINR}_i(x)). \tag{13}
\]

Fig. 1. A multi-user amplify-and-forward wireless relay network.
In particular, the program of maximin information throughput under the individual relay power constraints, \( P_n(x_n) \leq \gamma_n, \ n = 1, 2, \ldots, N, \) is formulated as
\[
\max_{\mathbf{x} \in \mathbb{C}^N} \min_{i=1,2,\ldots,M} \log_2(1 + \text{SINR}_i(\mathbf{x})) : \ r_n|x_n|^2 \leq \gamma_n, \ n = 1, 2, \ldots, N. \tag{14}
\]

The equivalent program in terms of SINR threshold maximin optimization is
\[
\max_{\mathbf{x} \in \mathbb{C}^N} \min_{i=1,2,\ldots,M} \text{SINR}_i(\mathbf{x}) : \ r_n|x_n|^2 \leq \gamma_n, \ n = 1, 2, \ldots, N. \tag{15}
\]

While the constraints in maximin programs (14) and (15) are convex, their objective functions are not concave nor convex. To the best of our knowledge, effective methods to solve this program were not known.

Obviously, the optimal value of the maximin program (15) is not less than \( \alpha \) if and only if the following constraints for \( i = 1, 2, \ldots, M \) in \( \mathbf{x} \) are feasible:
\[
\text{SINR}_i(\mathbf{x}) \geq \alpha, \ r_n|x_n|^2/\gamma_n \leq 1, \ n = 1, 2, \ldots, N,
\]
or equivalently, the optimal value of the following minimax program is not more than one,
\[
\min_{\mathbf{x}} \max_{n=1,2,\ldots,N} \left[ r_n|x_n|^2/\gamma_n \right] : \ \text{SINR}_i(\mathbf{x}) \geq \alpha. \tag{16}
\]

In other words, the optimal value of the maximin program (15) is the maximum of those \( \alpha \) such that the optimal value of the minimax program (16) is not more than one. Under the variable change \( \mathbf{X} = \mathbf{xx}^H \), the above minimax program (16) is equivalently reformulated as the matrix rank-one constrained optimization:
\[
\min_{\mathbf{X} \in \mathbb{C}^{N \times N}} \max_{n=1,2,\ldots,N} \left[ r_n\mathbf{X}(n,n)/\gamma_n \right] : \ \text{rank}(\mathbf{X}) = 1, \mathbf{X} \succeq 0, \ 
\sigma^2_2(\mathbf{X}, \mathbf{c}_i^H) - \zeta(\mathbf{X}, \mathbf{J}_n) \geq 0, \ 
\alpha\sigma^2_2 \sum_{m \neq i} \langle \mathbf{X}, \mathbf{c}_m^H \mathbf{c}_m \rangle + \sigma^2_R(\mathbf{diag}[|\mathbf{e}_i|^2], \mathbf{X}) + \sigma^2_D. \tag{17}
\]

Similarly to [9], [10], this program can be relaxed to the following SDP by dropping the only nonconvex rank-one constraint \( \text{rank}(\mathbf{X}) = 1 \), also known as semidefinite relaxation (SDR):
\[
\min_{\mathbf{X} \in \mathbb{C}^{N \times N}} \max_{n=1,2,\ldots,N} \left[ r_n\mathbf{X}(n,n)/\gamma_n \right] : \ \mathbf{X} \succeq 0, \ 
\sigma^2_2(\mathbf{X}, \mathbf{c}_i^H) - \zeta(\mathbf{X}, \mathbf{J}_n) \geq 0, \ 
\alpha\sigma^2_2 \sum_{m \neq i} \langle \mathbf{X}, \mathbf{c}_m^H \mathbf{c}_m \rangle + \sigma^2_R(\mathbf{diag}[|\mathbf{e}_i|^2], \mathbf{X}) + \sigma^2_D. \tag{18}
\]

Appropriately, one can use the bisection method to find the maximum \( \alpha_{\text{opt}} \) of those \( \alpha \) such that the optimal value of SDP (18) is less than one. Such \( \alpha_{\text{opt}} \) provides an upper bound for the maximin program (15). If SDP (18) at \( \alpha = \alpha_{\text{opt}} \) has rank-one optimization solution \( \mathbf{X}_{\text{opt}} = \mathbf{xx}^H_{\text{opt}} \) then such \( \mathbf{x}_{\text{opt}} \) is surely optimal solution of the nonconvex program (16) and then of maximin program (15). However, \( \mathbf{X}_{\text{opt}} \) is often not rank-one [12], [14], so it admits an eigenvalue decomposition (EVD) \( \mathbf{X}_{\text{opt}} = \mathbf{USU}^H \) with unitary matrix \( \mathbf{U} = (\mathbf{u}_1, \ldots, \mathbf{u}_N) \) and diagonal matrix \( \mathbf{\Sigma} \), whose diagonal entries are arranged in decreasing order and so \( \Sigma(i,i) = 0 \) for \( i \geq i_{\text{opt}} := \text{rank}(\mathbf{X}_{\text{opt}}) \geq 2 \). The SDR-based randomization generates
\[
\mathbf{z}^{(v)} = \mathbf{U}\Sigma^{1/2}v = \sum_{i=1}^{i_{\text{opt}}} \Sigma^{1/2}(i,i)v_i \mathbf{u}_i, \tag{19}
\]
where \( v = (v_1, \ldots, v_N) \) is generated as unit-variance complex Gaussian vector with uncorrelated components. Hence, \( \mathbf{z}^{(v)} \) always belongs to \( \text{span}\{\mathbf{u}_1, \ldots, \mathbf{u}_{i_{\text{opt}}}\} \) of low dimension \( i_{\text{opt}} \), i.e., randomization is performed in a low-dimensional sub-space. Each randomly generated vector \( \mathbf{z}^{(v)} \) is further scaled by \( v_v := \min_{n=1,2,\ldots,N} \frac{\sqrt{\gamma_n/r_n|\mathbf{z}^{(v)}|^2}}{\gamma_n/\gamma_n} \) to satisfy the power constraint in maximin program (15) and then \( \min_{n=1,2,\ldots,N} \text{SINR}_n(\mathbf{t}_v, \mathbf{z}^{(v)}) \) is calculated to update its current best value. Obviously, the optimized solution of maximin program (15) hardly resides in subspace \( \text{span}\{\mathbf{u}_1, \ldots, \mathbf{u}_{i_{\text{opt}}}\} \subset \mathbb{C}^N \). As such, no matter how many \( \mathbf{z}^{(v)} \) are randomly generated, the value of maximin program (15) is not much improved. As a matter of fact, the simulation results in Section 6 will show that a more trivial randomization will simply work quicker because it calls only one SDP solver.

From the view point of finding solutions, the above standard bisection method is backward because the original maximin program (15) as maximization of a nonconcave objective function over convex constraints is in fact easier than the minimax program (16) used for the bisection routine, which is minimization of a convex objective function over nonconvex constraints [16]. Moreover, variable \( \mathbf{X} \) in matrix rank-one constrained formulation (17) for program (16) is of dimension \( N(N+1)/2 \), which is \( (N+1)/2 \) times larger than that of variable \( \mathbf{z} \) in the maximin program (15). The computational complexity of each SDR in (18) is \( O((N(N+1)/2)^3(N+M)) \), which is fairly high.

Although both the minimax program (16) and maximin program (15) in nonrobust scenarios (\( \zeta = 0 \)) can be solved by our nonsmooth matrix spectral optimization algorithm with the matrix variable setting \( \mathbf{X} = \mathbf{xx}^H \) of dimension \( N(N+1)/2 \) [14], [28], the next section develops and presents direct approaches for obtaining solutions of maximin programs (14) and (15), hence bypassing any bisection approach and computationally demanding matrix optimization.

Before closing this section, it should be mentioned that the present paper does not touch on relay beamforming design for limited feedback half-duplex relay networks, where the forward channels coefficients \( \mathbf{t}_v \) are estimated (exactly) at the destinations but only a limited number of bits representing them are fed back to the relays. Additional statistical assumptions on both backward and forward channel coefficients are also necessary. In fact, such a design was considered only for single-user networks, where the beamforming design can be expressed by a Grassmannian line packing problem of computational intractability (see e.g. [29]). Its extension to multi-user networks appears to be still open.

III. MAXIMIN THROUGHOUT OPTIMIZATION BY D.C. PROGRAMMING

Our strategy is to express the maximin program (15) in the following canonical form of d.c. optimization [16]:
\[
\min_{\mathbf{z}} f(\mathbf{z}) - g(\mathbf{z}): \ \mathbf{z} \in \mathcal{K}, \tag{20}
\]
where $z$ is a matrix and/or vector variable, $\mathcal{K}$ is a compact and convex set, $f(\cdot)$ is a quasi-convex function (i.e., for each $t$ the level set $\{z: f(z) \leq t\}$ is either empty or convex) and $g(\cdot)$ is a convex and smooth function. Suppose that $z^{(k)} \in \mathcal{K}$ and $\nabla g(z^{(k)})$ is the gradient of $g(\cdot)$ at $z^{(k)}$. Then [16]

$$f(z) - g(z) \leq f(z) - g(z^{(k)}) - \langle \nabla g(z^{(k)}), z - z^{(k)} \rangle \quad \forall z \in \mathcal{K}$$

It then follows that the following convex program provides a global upper bound minimization for d.c. program (20):

$$\min_{z} [f(z) - g(z^{(k)}) - \langle \nabla g(z^{(k)}), z - z^{(k)} \rangle : z \in \mathcal{K}],$$

where $z^{(k)}$ is also its feasible solution. Moreover, for the optimal solution $z^{(k+1)}$ of (21),

$$f(z^{(k+1)}) - g(z^{(k+1)}) \leq f(z^{(k)}) - g(z^{(k)}) - \langle \nabla g(z^{(k)}), z^{(k+1)} - z^{(k)} \rangle \leq f(z^{(k)}) - g(z^{(k)}),$$

which means that $g^{(k+1)}$ is better than $z^{(k)}$ toward (20). Thus, initialized from a feasible $z^{(0)} \in \mathcal{K}$, for $\kappa = 0, 1, \ldots$, generating $z^{(k)}$ by the optimal solution of convex program (21) is a path-following algorithm, which converges to an optimal solution.

It can be seen that there are infinite number of d.c. representations for the same nonconvex problems and it is clear from (21) that the efficiency of the d.c. path-following procedure critically depends on the choice of d.c. representation. The next section shall examine this issue in more details. Unlike the approach of [30], which attempts to locate the global optimal solution of d.c. program (20) by combining iterations (21) with customized branch-and-bound of high computational complexity, here we develop effective equivalent d.c. decompositions that make iterations (21) converge very close to the upper bound (as our numerical results in Section V will clearly show).

**Proposition 1:** The maximin program (15) is equivalent to

$$\max_{x \in \mathbb{C}^N, y \in \mathbb{R}^N} \min_{i=1,2,\ldots,M} \varphi_i(x, y_i) :$$

$$\sum_{m \neq i} p_i^m \left( |\langle x, c_{mi} \rangle|^2 + \frac{\epsilon^2}{\sigma^2} \right) + \frac{\epsilon^2}{\sigma^2} \sum \left[ \text{diag} \left( \left| \ell_i \right|^2 \right), |x|^2 + \frac{\epsilon^2}{\sigma^2} \right] \leq y_i, \quad i = 1, \ldots, M;$$

$$r_n |x_n|^2 \leq \gamma_n, \quad n = 1, 2, \ldots, N.$$  

(22b)

(22c)

where $\varphi_i(x, y_i) := \frac{|\langle x, c_{i1} \rangle|^2 - \epsilon^2}{y_i + \frac{\epsilon^2}{\sigma^2}}$. Obviously, (22b)-(22c) are convex quadratic constraints, which can also be represented by the linear matrix inequality (LMI):

$$\begin{bmatrix} Q_1 & \bar{Q} x \\ \bar{x}^H Q_1 & y_i \end{bmatrix} \geq 0; \quad \begin{bmatrix} 2a_x \\ x \end{bmatrix} \geq 0, \quad Q_1 := \sum_{m \neq i} c_{mi} c_{mi}^H + \epsilon^2 I_N + \frac{\epsilon^2}{\sigma^2} \sum \left[ \text{diag} \left( \left| \ell_i \right|^2 \right) + \epsilon^2 I_N \right].$$

(23)

**Proposition 2:** Each fractional function $|\langle x, c_{i1} \rangle|^2/(y_i + \frac{\epsilon^2}{\sigma^2})$ is convex on $\mathbb{C}^N \times R_+$. Also from [16] one has

$$\min_{i=1,2,\ldots,M} \left[ \frac{|\langle x, c_{i1} \rangle|^2 - \epsilon^2}{y_i + \frac{\epsilon^2}{\sigma^2}} \right] =$$

$$\min_{i=1,2,\ldots,M} \left[ \frac{M}{M} \sum_{j=1}^M \frac{\epsilon^2}{y_j + \frac{\epsilon^2}{\sigma^2}} \right] =$$

$$\frac{M}{M} \sum_{j=1}^M \frac{\epsilon^2}{y_j + \frac{\epsilon^2}{\sigma^2}}.$$
In order to use the existing convex program software such as SeDuMi [31] for solution of (27), we express it by the following SDP:

$$\begin{align*}
\min_{x \in \mathcal{C}^N, \ y \in \mathbb{R}^M, \ t, \ t_0, \ t_1, \ i = 1, 2, \ldots, M} & \quad t - f_{02}(x^{(k)}, y^{(k)}) - \\
+ & \sum_{i=1}^{M} \left( 2\text{Re}(\langle x^{(k)}, c_i \rangle \langle c_i, x - x^{(k)} \rangle) - \frac{|\langle c_i, x \rangle|^2}{\langle y_i, y_i \rangle + \sigma_D^2/\sigma_s^2} \right) : (23), (46b), (47).
\end{align*}$$

(28)

IV. BEAMFORMING DESIGN WITH ORTHOGONAL SOURCE TRANSMISSIONS

Our previous studies (see e.g. [12], [14]) show that it is not practical to aim for a very high SINR threshold under nonorthogonal (concurrent) transmissions of the source-destination pairs if there are more than five pairs in the network. This motivates us to consider a system model where the backward channels \( h_m, m = 1, 2, \ldots, M \) are orthogonal [13], [32], [33]. This allows the beamforming to be applied individually on the received signal from each source before combining them for forwarding to the destinations.

Let \( x_m = (x_{m1}, x_{m2}, \ldots, x_{mN})^T \in \mathcal{C}^N \) be the beamforming weight applied by the relays to the signals received from source \( m \), namely \( h_m s_m + n_R \). After beamforming is applied, the signal received from source \( m \) becomes

$$y^{(m)} = x_m \odot (h_m s_m + n_R).$$

Thus, the signals forwarded by the relays to the destination are given in the following vector:

$$y_{\text{amp}} = \sum_{m=1}^{M} x_m \odot (h_m s_m + n_R).$$

(30)

The received signal at destination \( i \) is thus

$$y_{D,i} = \langle \ell_i, y_{\text{amp}} \rangle + n_D,i = \sum_{m=1}^{M} \langle c_{mi}, x_m \rangle s_m + \langle \ell_i \odot n_R, x_m \rangle + n_D,i.$$

(31)

As in Section II, all the concerned channels gains \( h_m, c_{mi} \) and \( \ell_i \) are subject to uncertainties as described in (4)-(5). Similarly to (10) and (11), the robust power of the desired signal component at destination \( i \) is

$$S_i(x_i) = \sigma_s^2(|\langle c_{ii}, x_i \rangle|^2 - \zeta^2||x_i||^2),$$

while the robust interference power is

$$\text{INT}_i(x_1, \ldots, x_M) = \sigma_s^2 \sum_{m \neq i}^{M} |\langle c_{mi}, x_m \rangle|^2 + \zeta^2||x_m||^2 + \sigma_R^2 \sum_{m=1}^{M} (|\text{diag}(\ell_i)\langle x_m h_m^H \rangle + \zeta||x_m||^2).$$

The total beamforming power can be computed as

$$P_T(x_1, \ldots, x_M) = \sum_{p(\Delta H_i) \leq \zeta} \mathbb{E} \{ |y_{\text{amp}}|^2 \} = \sum_{i=1}^{M} (\sigma_2^2 \text{diag}(h_i \odot h_i^H) + (\sigma_s^2 \zeta^2 + \sigma_R^2) I_N, x_i h_i^H),$$

(32)

while the individual beamforming power at relay \( n \) is

$$P_n(x_1, \ldots, x_M) = \sum_{m=1}^{M} \sigma_s^2 |h_{mn}|^2 + \sigma_R^2 \zeta^2 + \sigma_R^2 |x_{mn}|^2.$$

(33)

The maximin optimization of the information throughput can now be expressed as

$$\begin{align*}
\max_{x_m \in \mathcal{C}^N, \ m=1, 2, \ldots, M, \ i=1, 2, \ldots, M} & \quad \min_{\theta \in \mathbb{R}^M} \log_2 \left( \frac{S_i(x_i)}{\text{INT}_i(x_1, \ldots, x_M)} \right), \quad \text{s.t.} \quad \sum_{m=1}^{M} (\sigma_s^2 |h_{mn}|^2 + \sigma_s^2 \zeta^2 + \sigma_R^2 |x_{mn}|^2) \leq \gamma_n \quad \text{(34b)}
\end{align*}$$

where (34b) is individual relay power constraint for \( n = 1, \ldots, N \). Like the maximin program (15), (34) is equivalent to

$$\begin{align*}
\text{max} & \quad \min_{x_i, y_i \in \mathbb{R}^N} \varphi_i(x_i, y_i) := \frac{|\langle c_{ii}, x_i \rangle|^2 - \zeta^2||x_i||^2}{y_i + \sigma_s^2/\sigma_s^2} : (35a) \\
\text{s.t.} & \quad \sum_{m \neq i}^{M} (|\langle c_{mi}, x_m \rangle|^2 + \zeta^2||x_m||^2) + \sigma_R^2 \sum_{m=1}^{M} (|\text{diag}(\ell_i)\langle x_m h_m^H \rangle + \zeta||x_m||^2) \leq y_i, \quad \text{(35b)}
\end{align*}$$

Define \( x = (x_1^T, x_2^T, \ldots, x_M^T) \in \mathcal{C}^{NM} \), \( \tilde{C}_i = \text{blkdiag}(c_{i1}, \ldots, c_{i,N-M+1}), \in \mathcal{C}^{M \times (N-M)} \), \( J_i = \text{blkdiag}(I_{N-M}, 0, \ldots, I_{N-M}) \), \( \tilde{L}_i = \text{blkdiag}(\ell_1^2, \ldots, \ell_N^2) \in \mathcal{C}^{N \times N} \). It then follows that

$$\sum_{m \neq i}^{M} (|\langle c_{mi}, x_m \rangle|^2 + \zeta^2||x_m||^2) + \sigma_R^2 \sum_{m=1}^{M} (|\text{diag}(\ell_i)\langle x_m h_m^H \rangle + \zeta||x_m||^2) =
$$

$$x^H (\tilde{C}_i^H \tilde{C}_i + \zeta^2 J_i + \sigma_s^2 \tilde{L}_i + \zeta^2 I_{MN}) x$$

and the convex constraints (35b) are expressed by the following LMIs:

$$\begin{bmatrix}
Q_i & y_i
g^H Q_i & y_i
\end{bmatrix} \succeq 0,$$

$$Q_i := \tilde{C}_i^H \tilde{C}_i + \zeta^2 J_i + \sigma_s^2 \tilde{L}_i + \zeta^2 I_{MN}.$$
is quasi-convex while the function \( f_{02}(\mathbf{x}, \mathbf{y}) := \sum_{i=1}^{M} |\langle \mathbf{c}_i, \mathbf{x}_i \rangle|^2/(y_i + \sigma_i^2/\sigma_0^2) \) is convex and smooth.

Obviously, DCI is also applicable to find the solutions of (38) by initializing from a feasible solution \((\mathbf{x}^{(0)}, \mathbf{y}^{(0)})\) of (38). In particular, the following convex program is required at \( \kappa \)-th iteration for generating \((\mathbf{x}_{1}^{(\kappa+1)}, \cdots, \mathbf{x}_{M}^{(\kappa+1)}, \mathbf{y}^{(\kappa+1)})\) instead of (27):

\[
\begin{align*}
\min_{\mathbf{x} \in \mathbb{C}^{NM}, \mathbf{y} \in \mathbb{R}^{M}} & \left[ f_{01}(\mathbf{x}, \mathbf{y}) - f_{02}(\mathbf{x}^{(\kappa)}, \mathbf{y}^{(\kappa)}) - \sum_{i=1}^{M} 2\text{Re}(\langle \mathbf{c}_i, \mathbf{x}_i \rangle \cdot \langle \mathbf{c}_i, \mathbf{x}_i - \mathbf{z}_i^{(\kappa)} \rangle) \right. \\
& \left. \quad - \frac{|\langle \mathbf{x}_i^{(\kappa)}, \mathbf{c}_i \rangle|^2}{(y_i^{(\kappa)} + \sigma_i^2/\sigma_0^2)^2} - \frac{|\langle \mathbf{y}_i^{(\kappa)} + \sigma_i^2/\sigma_0^2 \rangle|^2}{(y_i^{(\kappa)} + \sigma_i^2/\sigma_0^2)^2} \right] : (34b), (36)
\end{align*}
\]

Similarly to (28), the above convex program can be easily converted to a SDP format so that the existing SDP solvers can be readily used.

V. Numerical Results

In all the simulations carried out in this section, the power of AWGN at relays and destination nodes is normalized to \( \sigma_R^2 = \sigma_D^2 = 1 \), while the signal power of all the sources is set at \( \sigma_s^2 = 100 \). Both backward and forward channel gains are randomly generated according to a circularly symmetric complex Gaussian distribution (i.e., Rayleigh distribution of their magnitudes). All results are averaged over 100 Monte-Carlo simulation runs to capture the variation of the channel coefficients. The tolerance \( \varepsilon \) of DCIs is set equal to 0.01 where the tolerance of SeDuMi [31] for iterations (28) and (39) is default \((10^{-5})\). It should be pointed out that the number of relays, \( N \), has been selected such that both programs (14) and (34) achieve the targeted ranges of the information throughput.

A. Non-Orthogonal (Concurrent) Source Transmissions

First, consider the case of no channel uncertainty, i.e., \( \zeta = 0 \) in (5). This means that

\[
\mathbf{h}_m \equiv \mathbf{h}_m, \mathbf{e}_i \equiv \mathbf{e}_i, \mathbf{c}_{mi} \equiv \mathbf{c}_{mi} \quad (40)
\]

in (4). Note that one can easily find initial feasible solution of (26) by solving convex feasibility problem defined by the set of convex constraints of (26). It should be emphasized, however, that successful implementation of DCI requires a good initial feasible point to guarantee convergence towards right optimized solution. Such a good initial point is obtained by solving the following convex program for any \( \alpha_i \geq 0 \) with \( i = 1, 2, \ldots, M \),

\[
\min_{\mathbf{X} \in \mathbb{C}^{N \times N}} \max_{n=1,2,\ldots,N} \mathbf{X}(n, n) \quad \text{s.t.} \quad \mathbf{X} \geq 0,
\]

\[
\sigma_s^2(\mathbf{C}_{ii}, \mathbf{X}) \geq \alpha_i \left( \sigma_s^2 \sum_{m \neq i} \langle \mathbf{C}_{mi}, \mathbf{X} \rangle + \sigma_R^2 \langle \mathbf{L}_i, \mathbf{X} \rangle + \sigma_D^2 \langle \mathbf{c}_{mi}, \mathbf{X} \rangle \right),
\]

(41)

We take as an initial \((\mathbf{x}^{(0)}, \mathbf{y}^{(0)})\) for the DCI the eigenvector \( \mathbf{x}^{(0)} \) corresponding to the eigenvalue \( \lambda_{\text{max}}(\mathbf{X}^{(0)}) \) of the optimal solution \( \mathbf{X}^{(0)} \) of (41) such that \(|\mathbf{x}^{(0)}|^2 = \lambda_{\text{max}}(\mathbf{X}^{(0)})\), and accordingly \( y_i^{(0)} = \sum_{m \neq i} \langle \mathbf{x}^{(0)}, \mathbf{c}_{mi} \rangle^2 + \frac{\sigma_R^2}{\sigma_s^2} |\text{diag} \mathbf{e}_i| \cdot |\mathbf{x}^{(0)} \mathbf{H}^{(0)}(i)|^2 \). Note that the value of \( \alpha_i \) in (41) can be chosen arbitrarily very small to ensure that the solution of (41) always remains feasible. It is also important to note that such a solution of (41) is always feasible to (26) and thus satisfies relay power constraints. Consequently there is no need to further scale the initial point obtained by (41) which reflects the flexibility of our canonical d.c. formulation (26) for which initial feasible point can easily be obtained.

Table I lists the numbers of iterations used, while Figures 2, 3, 4 and 5 present simulation results for different numbers of users and relay nodes. Specifically, plotted in these figures are the minimum information throughput among all users achieved by our proposed DCI approach given by (26). Since the DCI method achieves the balanced throughputs for all users, this minimum throughput is the actual information throughput for all users as well.

![Fig. 2. Information throughput versus total relaying power for \( M = 3 \) and \( N = 10 \) under non-orthogonal source transmissions.](image1)

![Fig. 3. Information throughput versus total relaying power for \( M = 4 \) and \( N = 12 \) under non-orthogonal source transmissions.](image2)
to be very close to the upper bound curve, which establishes the ability for locating approximately global solutions of our proposed method.

It is naturally expected that an increase in throughput would happen with an increased value of the total relay power. In this paper the total relay power budget is equally divided among the relay nodes. We start with a scenario consisting of \( M = 3 \) users and \( N = 10 \) relays. It can be observed in Figure 2 that with our proposed d.c. programming based approach, a throughput of \( 2.5 \) bps/Hz is achieved for each user when the total power budget is \( 9.03 \) dB, while the SDR-based randomization approach only gives \( 1.7 \) bps/Hz for the same amount of total relay power. The randomization techniques fail to distribute the information throughput fairly among all users.

The effect of the number of relay nodes on throughput performance can be illustrated by comparing results of Figure 3 and 4 where, as expected, an increase in the number of nodes boosts the corresponding throughput for the same amount of total power. Figure 4 also illustrates the impact on spectral efficiency by increasing number of users for fixed number of relays. Furthermore, the results in Fig. 5 with \( M = 5, N = 20 \) show that performance of the randomization techniques further deteriorates with when the number of users increases (to \( M = 5 \) in this case) due to its inability to handle excessive amount of interference.

Table II provides the averaged rank of the optimal solution \( \mathbf{X}_\text{opt} \) of SDP (18) at \( \alpha_\text{opt} \) for different choices of users and relay nodes. It reveals that vectors \( \mathbf{z}_i(\cdot) \) in (19) are generated in 2 or 3 dimensional subspace of 10, 16 and 20-dimensional space. This explains why such SDR based randomization performs poorly as Figures 2-5 show, even 5,000 such \( \mathbf{z}_i(\cdot) \) have been generated for each case. This randomization is only as good as a more trivial randomization, which needs only one SDP solver for a feasible solution \( \mathbf{X} \) of SDP (18) at some \( \alpha \) to generate \( \mathbf{z}_i(\cdot) \) according to (19) with \( i_\text{opt} \) replaced by the rank of \( \mathbf{X} \) and unitary \( \mathbf{U} \) and diagonal \( \Sigma \) in EVD \( \mathbf{X} = \mathbf{U} \Sigma \mathbf{U}^H \). Figures 2-5 show that the performance of this simple randomization is still poor although it is comparable to that of the more computationally-intensive SDR based randomization (which requires tens of SDP solvers for solution of SDP (18) in the bisection procedure to locate the optimal \( \alpha_\text{opt} \)).

Next, Figure 6 presents throughput for the scenario that \( \zeta \neq 0 \) in (5), whose value can be set to represent varying degrees of channel uncertainties. For each user, throughput curves are plotted versus the total relay power for \( \zeta = 0.1, 0.3, 0.6, 0.8 \). Except for the fifth user in \( \zeta = 0.3 \) case, achieves a slightly higher throughput, all users achieve approximately the same throughput when beamforming is performed according to our proposed DCI method.

### Table I

<table>
<thead>
<tr>
<th>( M = 3, N = 10 )</th>
<th>( M = 4, N = 16 )</th>
<th>( M = 5, N = 20 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_T ) (dB)</td>
<td>Iterations</td>
<td>( P_T ) (dB)</td>
</tr>
<tr>
<td>0.04</td>
<td>13.57</td>
<td>0.04</td>
</tr>
<tr>
<td>4.08</td>
<td>13.25</td>
<td>7.27</td>
</tr>
<tr>
<td>7.53</td>
<td>12.11</td>
<td>11.46</td>
</tr>
<tr>
<td>8.58</td>
<td>12.23</td>
<td>12.63</td>
</tr>
<tr>
<td>9.43</td>
<td>11.33</td>
<td>13.55</td>
</tr>
<tr>
<td>10.14</td>
<td>10.15</td>
<td>14.31</td>
</tr>
<tr>
<td>10.75</td>
<td>9.16</td>
<td>14.96</td>
</tr>
<tr>
<td>11.28</td>
<td>7.95</td>
<td>15.52</td>
</tr>
<tr>
<td>11.76</td>
<td>6.11</td>
<td>16.02</td>
</tr>
</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th>( M = 4, N = 10 )</th>
<th>( M = 4, N = 16 )</th>
<th>( M = 5, N = 20 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_T ) (dB)</td>
<td>Avg. ( i_\text{opt} )</td>
<td>( P_T ) (dB)</td>
</tr>
<tr>
<td>0.04</td>
<td>2.18</td>
<td>0.04</td>
</tr>
<tr>
<td>4.08</td>
<td>2.09</td>
<td>7.27</td>
</tr>
<tr>
<td>6.14</td>
<td>2.11</td>
<td>9.85</td>
</tr>
<tr>
<td>7.53</td>
<td>2.11</td>
<td>11.46</td>
</tr>
<tr>
<td>8.58</td>
<td>2.11</td>
<td>12.63</td>
</tr>
<tr>
<td>9.43</td>
<td>2.14</td>
<td>13.55</td>
</tr>
<tr>
<td>10.14</td>
<td>2.11</td>
<td>14.31</td>
</tr>
<tr>
<td>10.75</td>
<td>2.14</td>
<td>14.96</td>
</tr>
<tr>
<td>11.28</td>
<td>2.15</td>
<td>15.52</td>
</tr>
<tr>
<td>11.76</td>
<td>2.14</td>
<td>16.02</td>
</tr>
</tbody>
</table>

---

**Fig. 4.** Information throughput versus total relaying power for \( M = 4, N = 16 \) and \( M = 5, N = 16 \) under non-orthogonal source transmissions.

**Fig. 5.** Information throughput versus total relaying power for \( M = 5 \) and \( N = 20 \) under non-orthogonal source transmissions.
such that the optimal value of the following SDP is less than one:

\[
\begin{align*}
\mathbf{x}_m^{(0)} &\in \mathbb{C}^{N \times N}, \quad m = 1, 2, \ldots, M = 4, 6, \quad n = 1, 2, \ldots, N = 10
\end{align*}
\]

where \( \mathbf{C}_{m} = \mathbf{e}_m \mathbf{e}_m^H \) and \( \mathbf{L}_i = \mathbf{e}_i \mathbf{e}_i^H \). However, in contrast to the SDP (18), the SDP (42) always admits the optimal rank-one solution \( \mathbf{x}_m^{(i)} = \mathbf{z}_m^{(i)} \mathbf{z}_m^{(i)H} \), \( i = 1, 2, \ldots, M \). This means that \( \log_2(1 + \alpha_{\text{opt}}) \) is actually the global solution of the maximin program (34) and \( \mathbf{z}_m^{(i)} \), \( i = 1, 2, \ldots, M \) form the optimal solution. This also means that SDR is able to provide the optimal solution for the maximin program (34).

Table III provides the iteration numbers of DCI and also of the above mentioned SDR bisection for finding the solution of the nonconvex program (34). These numbers correspond to the numbers of required SDP (39) to implement the DCI and the SDR-based bisection procedure, respectively. One can see that the required number of SDP (39) is always less than that of the required number of SDP (42). Moreover, the variable dimension of (39) is \((N + 1)M\) while that of (42) is \(MN(N + 1)/2\). This clearly indicates that the DCI approach is much more computationally efficient than the SDR-based bisection approach. We solve (42) for some arbitrary value of \( \alpha \) (e.g., \( \alpha \geq 0.01 \)) to obtain \( \{\mathbf{x}_m^{(0)}, \ldots, \mathbf{x}_M^{(0)}\} \), whose eigenvectors \( \mathbf{z}_m^{(0)} \) corresponding to eigenvalues \( \lambda_{\text{max}}(\mathbf{x}_m^{(0)}) \) constitute the initial \( \{\mathbf{x}_1^{(0)}, \ldots, \mathbf{x}_M^{(0)}\} \) of DCI. The simulation results presented in Figure 7 illustrate throughput performance of the DCI algorithm. We consider typical scenarios for orthogonal transmission with \( M = 4, 5, 6 \) while the number of relay nodes \( N = 10 \) is fixed. The rank-one solution obtained by the aforementioned SDR (42) is also plotted to establish an upper bound. It is noted that the DCI method achieves approximately the same throughput curve as that of the SDR bisection method despite the lower dimensions of the DCI variables.
Therefore,\[
\max (22) \leq \max (15). \quad (43)
\]

On the other hand, for the optimal solution \( \mathbf{x}_{\text{opt}} \) of the maximin program (15), it is also obvious that \((\mathbf{x}_{\text{opt}}, \mathbf{y}_{\text{opt}})\) with \( y_{\text{opt},i} = \sum_{m \neq i} (|\langle \mathbf{x}_{\text{opt}}, \mathbf{c}_m \rangle|^2 + \zeta^2||\mathbf{x}_{\text{opt}}||^2 + \frac{\sigma^2}{\sigma^2_z} ||\mathbf{x}_{\text{opt}}||^2)^{1/2} \) is feasible to program (22). Then

\[
\max (15) = \min_{i=1,2,\ldots,M} \varphi_i (\mathbf{x}_{\text{opt}}, y_{\text{opt},i}) \leq \max (22). \quad (44)
\]

It is concluded from (43) and (44) that \((\mathbf{x}_{\text{opt}}, \mathbf{y}_{\text{opt}})\) must be the optimal solution of (22).

\(\square\)

**Proof of Proposition 2.** First, each function \( \phi_i(s, y_i) := |s|^2/(y_i + \sigma^2_D/\sigma^2_z) \) is convex in \((s, y_i) \in C \times R^+_1\) because its Hessian, which is defined by

\[
H_{s, y_i} := \begin{bmatrix}
\frac{\partial^2 \phi_i(s, y_i)}{\partial s^2} & \frac{\partial^2 \phi_i(s, y_i)}{\partial s \partial y_i} \\
\frac{\partial^2 \phi_i(s, y_i)}{\partial s \partial y_i} & \frac{\partial^2 \phi_i(s, y_i)}{\partial y_i^2}
\end{bmatrix}
\]

is positive definite [16]:

\[
H_{s, y_i}(1, 1) = 2 > 0, \quad H_{s, y_i}(2, 2) = 2|s|^2/(y_i + \sigma^2_D/\sigma^2_z)^3 \geq 0, \quad \det (H_{s, y_i}) = 4(|s|^2 - \text{Re}(s)^2)/(y_i + \sigma^2_D/\sigma^2_z)^3 \geq 0.
\]

This means that, for any \( 0 \leq \theta \leq 1 \) and \((s, y_i), (s', y_i') \in C \times R^+_1\), one has

\[
\frac{|\theta s + (1 - \theta)s'|^2}{y_i + \sigma^2_D/\sigma^2_z} \leq \frac{\theta|s|^2}{y_i + \sigma^2_D/\sigma^2_z} + \frac{(1 - \theta)|s'|^2}{y_i' + \sigma^2_D/\sigma^2_z}.
\]

Therefore,

\[
\frac{|\theta \mathbf{x} + (1 - \theta)\mathbf{x}', \mathbf{c}_i|}{y_i + \sigma^2_D/\sigma^2_z} \leq \frac{|\theta \mathbf{x}, \mathbf{c}_i| + (1 - \theta)|\mathbf{x}', \mathbf{c}_i|}{y_i + \sigma^2_D/\sigma^2_z},
\]

\[
\leq \frac{|\mathbf{x}, \mathbf{c}_i|^2}{y_i + \sigma^2_D/\sigma^2_z} + (1 - \theta)\frac{|\mathbf{x}', \mathbf{c}_i|^2}{y_i' + \sigma^2_D/\sigma^2_z},
\]

which shows that \(|\mathbf{x}, \mathbf{c}_i|^2/(y_i + \sigma^2_D/\sigma^2_z)\) is convex. \(\square\)

**Proof of Proposition 3.** For every \( t \geq 0 \), the level set\[
\{(x, y) : f_{01}(x, y) \leq t\} \quad (45)
\]

is fully described by the following constraints for \( m, i = 1, 2, \ldots, M \):

\[
\frac{\zeta^2||\mathbf{x}||^2}{y_i + \sigma^2_D/\sigma^2_z} \leq t_{0i}, \quad \frac{|\mathbf{x}, \mathbf{c}_{mm}|^2}{y_m + \sigma^2_D/\sigma^2_R} \leq t_{1m}, \quad (46a)
\]

\[
t_{0i} + \sum_{m \neq i} t_{1m} \leq t, \quad (46b)
\]

On the other hand, the LMIs representations for convex constraints (46a) are

\[
\begin{bmatrix}
t_{0i} \\
\mathbf{x} \\
y_i + \sigma^2_D/\sigma^2_z I_N
\end{bmatrix} \succeq 0, \quad \begin{bmatrix}
t_{1m} \\
|\mathbf{x}, \mathbf{c}_{mm}| \\
y_m + \sigma^2_D/\sigma^2_R
\end{bmatrix} \succeq 0.
\]

Thus, the level set (45) is fully described by LMIs (46b) and (47), so it is convex, proving that \( f_{01} \) is quasi-convex. \(\square\)
REFERENCES


Umar Rashid (S’10) received the B. Sc. degree in electrical engineering from University of Engineering and Technology, Lahore, Pakistan in 2007. He has been working as Lecturer/Lab Engineer in the same university since 2008. Currently, he is on study leave to pursue his PhD degree at University of Technology, Sydney, Australia under supervision of Prof. H.D. Tuan. His research interests include multi-user communications, statistical signal processing and Bayesian filtering.

Ha H. Nguyen (M’01-SM’05) received the B.Eng. degree from the Hanoi University of Technology (HUT), Hanoi, Vietnam, in 1995, the M.Eng. degree from the Asian Institute of Technology (AIT), Bangkok, Thailand, in 1997, and the Ph.D. degree from the University of Manitoba, Winnipeg, MB, Canada, in 2001, all in electrical engineering. He joined the Department of Electrical and Computer Engineering, University of Saskatchewan, Saskatoon, SK, Canada, in 2001, and became a full Professor in 2007. He holds adjunct appointments at the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB, Canada, and TRLabs, Saskatoon, SK, Canada, and was a Senior Visiting Fellow in the School of Electrical Engineering and Telecommunications, University of Technology, Sydney. He has been involved in research with the areas of optimization, control, signal processing, wireless communication and bioinformatics for 20 years.

Hoang Duong Tuan (M’94) received the diploma (Hon.) and the PhD degrees, both in applied mathematics, from Odessa State University, Ukraine, in 1987 and 1991, respectively. He spent nine academic years in Japan as an Assistant Professor in the Department of Electronic-Mechanical Engineering, Nagoya University from 1994 to 1999, and then as an Associate Professor in the Department of Electrical and Computer Engineering, Toyota Technological Institute, Nagoya from 1999 to 2003. He has been a Professor in the School of Electrical Engineering and Telecommunications, from 2003 to 2011. He is currently a Professor of Centre for Health Technologies, University of Technology, Sydney. He has been involved in research with the areas of optimization, control, signal processing, wireless communication and bioinformatics for 20 years.

Ha H. Nguyen (M’01-SM’05) received the B.Eng. degree from the Hanoi University of Technology (HUT), Hanoi, Vietnam, in 1995, the M.Eng. degree from the Asian Institute of Technology (AIT), Bangkok, Thailand, in 1997, and the Ph.D. degree from the University of Manitoba, Winnipeg, MB, Canada, in 2001, all in electrical engineering. He joined the Department of Electrical and Computer Engineering, University of Saskatchewan, Saskatoon, SK, Canada, in 2001, and became a full Professor in 2007. He holds adjunct appointments at the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB, Canada, and TRLabs, Saskatoon, SK, Canada, and was a Senior Visiting Fellow in the School of Electrical Engineering and Telecommunications, University of New South Wales, Sydney, Australia during October 2007-June 2008. His research interests include multi-user communications, statistical signal processing and Bayesian filtering.

Umar Rashid (S’10) received the B.Sc. degree in electrical engineering from University of Engineering and Technology, Lahore, Pakistan in 2007. He has been working as Lecturer/Lab Engineer in the same university since 2008. Currently, he is on study leave to pursue his PhD degree at University of Technology, Sydney, Australia under supervision of Prof. H.D. Tuan. His research interests include multi-user communications, statistical signal processing and Bayesian filtering.