In the above paper [1], Behbahani et al. proposed a closed-form solution (in Section III-B1) for the optimal relay coefficients by minimizing the minimum mean-square error (MMSE) at the destination subject to a global power constraint at the relays. We provide corrections to this closed-form solution and a few comments.

In [1], a closed-form solution to problem (14) was given in (21). However, this result is not correct as \( \mathbf{f} \) was mistakenly designated as an eigenvector of \( \mathbf{L}^{-1} \mathbf{A} \mathbf{L}^{-1} \) (cf. [1], eq. (17)). In fact, \( \mathbf{L} \mathbf{f} \) should be assigned to the eigenvector of \( \mathbf{L}^{-1} \mathbf{A} \mathbf{L}^{-1} \). To clarify this fact, let \( \mathbf{f} = \mathbf{L} \mathbf{f} \). Applying the Rayleigh–Ritz theorem [2], the objective function in problem (16) is upper-bounded as

\[
\frac{\mathbf{f}^H \mathbf{A} \mathbf{f}}{\mathbf{f}^H \mathbf{f}} = \frac{\mathbf{f}^H \mathbf{L}^{-1} \mathbf{A} \mathbf{L}^{-1} \mathbf{f}}{\mathbf{f}^H \mathbf{f}} \leq \lambda_{\text{max}}
\]

where \( \lambda_{\text{max}} \) is the largest eigenvalue of \( \mathbf{L}^{-1} \mathbf{A} \mathbf{L}^{-1} \). The equality holds if \( \mathbf{f} = \chi \mathbf{e}_{\lambda_{\text{max}}} \), where \( \chi \) is a non-zero scaling factor and \( \mathbf{e}_{\lambda_{\text{max}}} \) is the principle eigenvector of \( \mathbf{L}^{-1} \mathbf{A} \mathbf{L}^{-1} \). Thus, \( \mathbf{f} = \mathbf{L} \mathbf{f} = \chi \mathbf{L} \mathbf{e}_{\lambda_{\text{max}}} \). As \( \mathbf{L}^{-1} \mathbf{A} \mathbf{L}^{-1} \) is rank-one, its principle eigenvector is the one corresponding to its only non-zero eigenvalue, \( \mathbf{L}^{-1} \mathbf{A} \mathbf{L}^{-1} \mathbf{D}_k \mathbf{h}_s \). As a result, \( \mathbf{f} = \chi \mathbf{B}^{-1} \mathbf{W}^{-1/2} \mathbf{D}_k \mathbf{h}_s \) maximizes the objective function in problem (16). The optimal \( \mathbf{f} \) can be expressed explicitly as

\[
f_k = \chi \frac{\mathbf{h}_s^H \mathbf{h}_k}{\mathbf{h}_s^H \mathbf{h}_k} \mathbf{h}_k
\]

The correct scaling coefficient is chosen to meet the sum relay power budget at \( p \) as

\[
\chi = \frac{\sqrt{p}}{\sqrt{\sum_{k=0}^{\kappa-1} \sigma_{x_k}^2 \sigma_{e_x}^2 + \frac{|h_{x_k}|^2}{\sigma_{h_k}^2} |h_{k}|^2 \sigma_{e_x}^2}}.
\]

Finally, the optimal amplifying coefficient at relay-\( k \) is

\[
f_{k} = \frac{\sqrt{p h_{x_k}^H h_{k}}}{\sqrt{\sum_{k=0}^{\kappa-1} \sigma_{x_k}^2 \sigma_{e_x}^2 + \frac{|h_{x_k}|^2}{\sigma_{h_k}^2} |h_{k}|^2 \sigma_{e_x}^2}}.
\]

Further Remarks: With the correct optimal amplifying coefficients, the maximum achievable signal-to-noise ratio (SNR) at the destination is \( \sigma_{x_k}^2 \lambda_{\text{max}} \). As \( \mathbf{L}^{-1} \mathbf{A} \mathbf{L}^{-1} \) is rank-one, \( \lambda_{\text{max}} \) is also its trace, i.e.,

\[
\lambda_{\text{max}} = \sum_{k=0}^{\kappa-1} \frac{|h_{x_k}|^2}{\sigma_{h_k}^2} |h_{k}|^2 \sigma_{e_x}^2 = \frac{\sigma_{e_x}^2}{\sigma_{h_k}^2} |h_{k}|^2.
\]

For notational simplicity, let \( a_k = \frac{\sigma_{x_k}^2 |h_{x_k}|^2}{\sigma_{h_k}^2} \), and \( b_k = \frac{\sigma_{x_k}^2 |h_{x_k}|^2}{\sigma_{h_k}^2} \). Then, the maximum achievable SNR with a sum relay power constraint of \( p \) is given by

\[
\text{SNR}_{\text{max}}(p) = \sum_{k=0}^{\kappa-1} \frac{p a_k}{p + b_k}
\]

which is a concave increasing function in \( p \), and is upper-bounded by \( \sum_{k=0}^{\kappa-1} a_k \). Since a sum relay power constraint is looser than an individual power constraint at each relay, this bound is also applicable to the latter case. Thus, \( \sum_{k=0}^{\kappa-1} a_k \) is the maximum achievable SNR in any amplify-and-forward relay network.

REFERENCES
