Signature and Chip Waveform Designs for Asynchronous CDMA Systems

SUMMARY In this paper, the design of signature waveforms for asynchronous CDMA systems equipped with a correlation receiver is first considered. Optimal signature waveforms that minimize the average multiple access interference (MAI) at the output of a correlation receiver are found, while satisfying the constraint on available transmission bandwidth. Comparison to signature waveforms previously obtained for synchronous systems is also made to justify the superior performance of the designed signature waveforms in asynchronous systems. Furthermore, for direct-sequence CDMA (DS-CDMA) systems with random signature sequences, the use of multiple chip waveforms is also proposed as a means of suppressing MAI. Bandwidth constrained multiple chip waveforms that maximize the signal-to-interference ratio (SIR) at the output of each correlation receiver are found. Numerical results show that by using double chip waveforms instead of a single chip waveform, it is possible to reduce the MAI by 10% for a fixed transmission bandwidth (or equivalently, to save about 10% of transmission bandwidth for a given SIR requirement). The advantage of using double chip waveforms is also demonstrated in terms of the bit error rate (BER), whose calculation is based on our extension to Holtzman’s approximation in [1].

key words: CDMA, signature waveform, chip waveform, bandwidth constraint, interference cancellation

1. Introduction

Code-division multiple access (CDMA) is a multiplexing technique where a number of users simultaneously and asynchronously access a channel by modulating and spreading their information-bearing signals with distinct signature waveforms. Since users transmit over the same channel at the same time, their signals interfere, causing multiple access (or multiuser) interference. Loosely speaking, there are three different methods to suppress multiple access interference (MAI) in CDMA systems: i) to design signature waveforms with MAI-suppression capability for a given type of receiver; ii) to design efficient multiuser receivers for a given set of signature waveforms and iii) to jointly design signature waveforms and multiuser receivers.

There has been a strong research interest in the topic of multiuser detection in the past ten years and many types of multiuser receiver have been proposed (see [2] and the references therein). By contrast, signature waveform design for CDMA systems, especially for asynchronous systems, has received less attention. This paper is a contribution to this important area. Specifically, signature waveform design is considered for asynchronous CDMA systems equipped with correlation receivers. The correlation receiver is preferred to other multiuser receivers in this paper because the complexity of multiuser detection is usually prohibitive in systems with a large number of users and therefore a correlation receiver is still the only practical solution in many CDMA systems. On the other hand, allowing the users to be asynchronous (i.e., users can start their transmissions at any time) is always a desirable property in any practical CDMA systems since it makes the systems simpler. Asynchronous model for CDMA systems has also been adopted in numerous research work on CDMA (e.g., see [1], [3]–[11]). The asynchronous transmission mode is also offered in practical systems such as CDMA-I and CDMA-II [12].

A common and important performance measure for the correlation receiver is the signal-to-interference ratio (SIR) at its output. In order to maximize the SIR, it is necessary to minimize the variance of MAI at the output of each correlation receiver. Ideally, the signature waveforms should be designed so that the MAI is zero. However this is usually impossible due to the limitation of the transmission bandwidth as well as the asynchronous nature of the users’ signals. Nevertheless, for a given transmission bandwidth, the sets of signature waveforms that produce minimum MAI are desired. Finding such signature waveforms is precisely one goal of this paper. To quantify the transmission bandwidth, both the fractional out-of-band power (FOBP) and root-mean-square (RMS) bandwidth criteria [13] can be considered. If the FOBP bandwidth is considered, then the Fourier transforms of the optimal signature waveforms are found by a series expansion in prolate spheroidal wave functions as in [3]. On the other hand, if the RMS bandwidth criterion is used, then the signature waveforms are given by a series expansion in sinusoids.

The signature waveforms obtained as described above essentially may admit any shape as long as they are limited to the symbol duration and have a specified energy. There is, however, a popular form of CDMA known as direct-sequence CDMA (DS-CDMA) where more structure is imposed on the signature waveforms. In particular, each user’s signature waveform is constructed by modulating a given time-limited chip waveform with the user’s signature sequence. Clearly the SIR at the output of each correlation receiver in DS-CDMA systems depends on both the signature sequences and the shape of the chip waveform. To maximize the SIR in this case, one needs to jointly optimize the signa-
signature sequences and the chip waveform. Recently, the use of random signature sequences has been widely adopted to analyze the performance of DS-CDMA systems [3]–[6]. With random signature sequences, the average SIR depends only on the chip pulse shape. Common chip waveforms found in CDMA research literature are rectangular and half-sine [7], [8]. Other chip waveforms are introduced and evaluated in [6], [9], [10].

It is important to note that when a single chip waveform is used and when the delays between the desired user and the interfering users are exactly multiples of the chip duration, the chip pulse shape has no effect on the amount of MAI. In such situations the MAI depends only on the cross correlations of the signature sequences, which can be large if the signature sequences are chosen randomly. Instead of using a single chip waveform, suppose that two orthogonal chip pulses are alternatively employed for the construction of signature waveforms. Now if the delays of the interfering users are exactly odd multiples of the chip duration, the signals from interfering users will be orthogonal to that of the desired user, hence the MAI will be zero, no matter what the signature sequences are. Motivated by this observation, in this paper we also introduce the use of multiple chip waveforms as a means of reducing MAI in asynchronous DS-CDMA systems. Again, the series expansion method is used to obtain the optimal chip waveforms. Numerical results show that a significant gain can be achieved by using multiple chip waveforms instead of a single chip waveform.

The paper is organized as follows. In Sect. 2, the SIR at the output of the correlation receiver is evaluated for both asynchronous CDMA systems and DS-CDMA systems using random signature sequences and multiple chip waveforms. In both cases, the SIRs are expressed in terms of the Fourier transforms of the signature waveforms and the chip waveforms respectively. These expressions suggest a method to obtain the signature and chip waveforms through series expansion. Bandwidth constraint and problems under consideration are discussed in Sect. 3. The series expansion method is used to obtain optimal signature waveforms and multiple chip waveforms in Sect. 4 and Sect. 5 respectively. Finally, conclusions are drawn in Sect. 6.

2. System Model and SIR Evaluation

2.1 Asynchronous CDMA Systems

The model for asynchronous CDMA systems under consideration is similar to the one in [7], [8]. There is, however, a major difference in the way the signature waveforms are constructed. Here, each signature waveform is not constrained to consist of a sequence of rectangular pulses as in [7], [8], but rather it can be of any shape. There are $K$ users sharing the same channel of bandwidth $W$. The received signal component corresponds to the transmitted signal of the $k$th user is:

$$y_k(t - \tau_k) = \sum_{j=0}^{\infty} \sqrt{2P_k b_j(i)} s_k(t - iT - \tau_k) \cos(2\pi f_c t + \varphi_k)$$

(1)

In (1), $P_k$ is the signal power; $f_c$ is the carrier frequency; $T$ is the symbol duration; $\tau_k$ and $\varphi_k$ are the phase shift and delay of the $k$th user, which can be modeled as random variables uniformly distributed over $[0, T]$ and $[0, 2\pi]$ respectively. The data sequence $(b_j(i))$ of user $k$ is modeled as a sequence of binary ($\pm 1$) independent and identically distributed (i.i.d) random variables. The signature waveforms $s_k(t)$ are time-limited to $T$ whose spectrum can occupy the entire bandwidth $W$. Furthermore $s_k(t)$ is normalized so that

$$\int_0^T s_k^2(t) dt = T, \quad k = 1, 2, \ldots, K$$

(2)

The received powers $P_k$’s are, in general, different from user to user since they may include the effects of both propagation loss and slow fading variation. In this paper, all the received powers $P_k$’s are assumed to be known, i.e., the propagation loss can be accurately measured and the fading variation is very slow (which can be considered to be constant over many symbol intervals). The received signal is

$$y(t) = \sum_{k=1}^{K} y_k(t - \tau_k) + n(t), \quad \text{where } n(t) \text{ is additive white Gaussian noise (AWGN) with two-sided power spectral density of } N_0/2.$$ 

Without loss of generality, consider the detection of the first information symbol of the $i$th user, i.e., $b_i(0)$. Also, since only relative delays and phases are important one can set $\tau_k = 0$ and $\varphi_k = 0$ and the delays and phase shifts of all other users are interpreted with reference to the $k$th user. Ignoring the double frequency component at $2f_c$, the output of the $k$th correlation receiver is

$$Z_k = \int_0^T y(t) s_k(t) \cos(2\pi f_c t) dt$$

$$= \sqrt{P_k/2b_k(0)T} + \sum_{i=1, i \neq k}^{K} \sqrt{P_i/2I_{k,i}} + n_k$$

(3)

where $n_k$ is a Gaussian random variable with zero mean and variance $N_0T/4$ and $I_{k,i}$ is the interference caused by the $i$th user, given by

$$I_{k,i} = |b_i(-1)R_{k,i}(\tau_i) + b_i(0)\tilde{R}_{k,i}(\tau_i)| \cos \varphi_i$$

(4)

Note that, since the delay of user $i$ is modeled as a uniform random variable over $[0, T]$, only the two bits $b_i(0)$ and $b_i(1)$ of user $i$ cause interference to bit $b_k(0)$ of the desired user. This is illustrated in Fig. 1.

The functions $R_{k,i}(\tau)$ and $\tilde{R}_{k,i}(\tau)$ in (4) are the continuous-time partial cross-correlation functions between the $k$th and the $i$th signature waveforms. These functions were originally introduced in [7] and can be written here as

$$R_{k,i}(\tau) = \int_0^T s_k(t)s_i(t + \tau - T) dt$$

and

$$\tilde{R}_{k,i}(\tau) = \int_0^T s_k(t)s_i(t - \tau + T) dt$$

for $0 \leq \tau \leq T$. If $k = i$, then denote $R_k(\tau) = R_{k,k}(\tau)$ and $\tilde{R}_k(\tau) = \tilde{R}_{k,k}(\tau)$. 

In (1), $P_k$ is the signal power; $f_c$ is the carrier frequency; $T$ is the symbol duration; $\tau_k$ and $\varphi_k$ are the phase shift and delay of the $k$th user, which can be modeled as random variables uniformly distributed over $[0, T]$ and $[0, 2\pi]$ respectively. The data sequence $(b_j(i))$ of user $k$ is modeled as a sequence of binary ($\pm 1$) independent and identically distributed (i.i.d) random variables. The signature waveforms $s_k(t)$ are time-limited to $T$ whose spectrum can occupy the entire bandwidth $W$. Furthermore $s_k(t)$ is normalized so that

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$$y(t) = \sum_{k=1}^{K} y_k(t - \tau_k) + n(t), \quad \text{where } n(t) \text{ is additive white Gaussian noise (AWGN) with two-sided power spectral density of } N_0/2.$$ 

Without loss of generality, consider the detection of the first information symbol of the $i$th user, i.e., $b_i(0)$. Also, since only relative delays and phases are important one can set $\tau_k = 0$ and $\varphi_k = 0$ and the delays and phase shifts of all other users are interpreted with reference to the $k$th user. Ignoring the double frequency component at $2f_c$, the output of the $k$th correlation receiver is [8]

$$Z_k = \int_0^T y(t) s_k(t) \cos(2\pi f_c t) dt$$

$$= \sqrt{P_k/2b_k(0)T} + \sum_{i=1, i \neq k}^{K} \sqrt{P_i/2I_{k,i}} + n_k$$

(3)

where $n_k$ is a Gaussian random variable with zero mean and variance $N_0T/4$ and $I_{k,i}$ is the interference caused by the $i$th user, given by

$$I_{k,i} = |b_i(-1)R_{k,i}(\tau_i) + b_i(0)\tilde{R}_{k,i}(\tau_i)| \cos \varphi_i$$

(4)

Note that, since the delay of user $i$ is modeled as a uniform random variable over $[0, T]$, only the two bits $b_i(0)$ and $b_i(1)$ of user $i$ cause interference to bit $b_k(0)$ of the desired user. This is illustrated in Fig. 1.

The functions $R_{k,i}(\tau)$ and $\tilde{R}_{k,i}(\tau)$ in (4) are the continuous-time partial cross-correlation functions between the $k$th and the $i$th signature waveforms. These functions were originally introduced in [7] and can be written here as

$$R_{k,i}(\tau) = \int_0^T s_k(t)s_i(t + \tau - T) dt$$

and

$$\tilde{R}_{k,i}(\tau) = \int_0^T s_k(t)s_i(t - \tau + T) dt$$

for $0 \leq \tau \leq T$. If $k = i$, then denote $R_k(\tau) = R_{k,k}(\tau)$ and $\tilde{R}_k(\tau) = \tilde{R}_{k,k}(\tau)$. 

$$y_k(t - \tau_k) = \sum_{j=0}^{\infty} \sqrt{2P_k b_j(i)} s_k(t - iT - \tau_k) \cos(2\pi f_c t + \varphi_k)$$

(1)
The random variables $I_{k,i}$ and $n_k$ are uncorrelated and have zero mean. Furthermore, it can be shown that the SIR at the output of the $k$th correlator receiver is given by

$$\text{SIR}_k = \left( \frac{2E_k}{N_0} \right)^{-1} + \frac{1}{T^2} \sum_{i=1,i \neq k}^{K} \frac{P_i}{P_k} \text{var}(I_{k,i})$$

(5)

where $E_k = P_k T$ is the energy per symbol of user $k$. Note that $I_{k,i}$ depends on the random variables $b_i = [b_i(-1), b_i(0), b_i(1)]$, $\varphi_i$, and $\tau_i$. As usual, these random variables are assumed to be mutually independent, hence the variance of $I_{k,i}$ can be computed as follows:

$$\text{var}(I_{k,i}) = E_{\tau_i} \left[ E_{\varphi_i} \left( E_{b_i} (I_{k,i}^2 | b_i, \varphi_i, \tau_i) \right) \right]$$

$$= \frac{1}{2T} \int_0^T \left[ R_{k,i}^2(\tau) + R_{k,i}^*(\tau) \right] d\tau$$

(6)

Though (6) is useful to evaluate the variance of MAI for a given set of signature waveforms, it is not convenient to use when finding the optimal signature waveforms. In what follows, it is shown that $\text{var}(I_{k,i})$ can be written in terms of the Fourier transforms of $s_k(t)$ and $s_i(t)$. As will be seen later, the new expression for $\text{var}(I_{k,i})$ is very helpful when formulating and solving the optimization problem considered in this paper.

Since $s_k(t)$ and $s_i(t)$ are time limited to $[0, T]$, it follows that $R_{k,i}(\tau) = R_{k,i}(T - \tau)$ and $\int_0^T R_{k,i}^2(\tau) d\tau = \int_{-T}^0 R_{k,i}^2(\tau) d\tau$. Therefore the integral in (6) becomes

$$\int_0^T \left[ R_{k,i}^2(\tau) + R_{k,i}^*(\tau) \right] d\tau = \int_{-T}^0 \left[ R_{k,i}^2(\tau) + R_{k,i}^*(\tau) \right] d\tau$$

(7)

Define $v_{k,i}(\tau) = \int_{-T}^0 s_k(t)s_i(t + T - \tau) d\tau$ and let $V_{k,i}(f) = \mathcal{F}\{v_{k,i}(\tau)\}$, where $\mathcal{F}\{\cdot\}$ denotes the Fourier transform. Since $v_{k,i}(\tau) = s_k(\tau) \circ s_i(-\tau)$, $\circ$ denotes the convolution operation, where $s_i(\tau) = s_i(\tau + T)$, then $V_{k,i}(f) = S_k(f)S_i^*(f) e^{-j2\pi fT}$ and $|V_{k,i}(f)|^2 = |S_k(f)|^2 |S_i(f)|^2$. Let $f(\tau) = v_{k,i}(\tau) + v_{k,i}^*(\tau)$, then $f(\tau)$ is time-limited to $[0, 2T]$. Furthermore, it can be shown that $f(\tau) = f(2T - \tau)$, i.e., $f(\tau)$ is an even function about $T$. Since $f(\tau) = R_{k,i}^2(\tau) + R_{k,i}^*(\tau)$ for $0 \leq \tau \leq T$, the right hand side of (7) can be written as

$$\int_{-T}^0 \left[ R_{k,i}^2(\tau) + R_{k,i}^*(\tau) \right] d\tau$$

$$= \frac{1}{2} \int_{-T}^T \left[ v_{k,i}^2(\tau) + v_{k,i}^2(\tau) \right] d\tau$$

(8)

$$= \frac{1}{2} \int_{-\infty}^\infty \left( |V_{k,i}(f)|^2 + |V_{k,i}(f)|^2 \right) df$$

Now, combining (6), (7) and (8) gives

$$\text{var}(I_{k,i}) = \frac{1}{2T} \int_{-\infty}^\infty |S_k(f)|^2 |S_i(f)|^2 df$$

(9)

and the SIR in (5) becomes

$$\text{SIR}_k = \left[ \left( \frac{2E_k}{N_0} \right)^{-1} + \frac{1}{2T^2} \sum_{i=1,i \neq k}^{K} \frac{P_i}{P_k} \int_{-\infty}^\infty |S_k(f)|^2 |S_i(f)|^2 df \right]^{-1}$$

(10)

2.2 Asynchronous DS-CDMA Systems with Random Signature Sequences

In this section, the model of asynchronous DS-CDMA systems using random signature sequences and multiple chip waveforms is introduced. The SIR at the outputs of the correlation receivers is also obtained as a function of the number of users, the processing gain and the chip waveforms.

Let $g_1(t), g_2(t), \ldots, g_D(t)$ be the $D$ distinct chip waveforms time-limited to $[0, T_c]$ whose energies are normalized so that

$$\int_0^{T_c} g_m^2(t) dt = T_c, \quad m = 1, 2, \ldots, D$$

(11)

Then the signature waveform of user $k$ is constructed as follows

$$s_k(t) = \sum_{j=0}^{M-1} \sum_{m=1}^{D} s_k(Dj + d - 1)g_m(t - (Dj + d - 1)T_c)$$

(12)

where $s_k = [s_k(0), s_k(1), \ldots, s_k(N - 1)]$ is modeled as a vector of i.i.d. random variables taking values in $\{-1, 1\}$ with equal probability. To simplify our analysis, it has been assumed in (12) that the processing gain $N = T/T_c$ is an integer multiple of $D$, i.e., $N = DM$.

To evaluate the SIR in this case, the variance of $I_{k,i}$ in (5) needs to be re-evaluated, taking into account the randomness of the signature sequences. That is,

$$\text{var}(I_{k,i}) = E_{s_k,s_i} \left[ E_{\tau_i} \left[ E_{\varphi_i} \left( E_{b_i} (I_{k,i}^2 | s_k, s_i, b_i, \varphi_i, \tau_i) \right) \right] \right]$$

$$= E_{s_k,s_i} \left\{ \frac{1}{2T} \int_{-T}^T \left[ R_{k,i}^2(\tau) + R_{k,i}^*(\tau) \right] d\tau \right\}$$

(13)

$$= \int_{-\infty}^\infty s_k(t)s_i(t - \tau) d\tau$$

Let $v_{k,i}(\tau) = \int_{-\infty}^\infty s_k(t)s_i(t - \tau) d\tau$ and $V_{k,i}(f) = \mathcal{F}\{v_{k,i}(\tau)\}$ Then $V_{k,i}(f) = |S_k(f)|^2$. Since $v_{k,i}(\tau)$ is an even function, time-limited to $[-T, T]$ and $v_{k,i}(\tau) = v_{k,i}(\tau)$ for $0 \leq \tau \leq T$, one has

Note that with this definition, $v_{k,i}(\tau)$ is different from $v_{k,i}(\tau)$ defined in Sect. 2.1.
Comparing (8) and (14) leads to the following identity,
\[
\int_0^T \left[ R_{k,l}^2(\tau) + R_{i,k}^2(\tau) \right] d\tau = 2 \int_0^T \tilde{R}_k(\tau) \tilde{R}_l(\tau) d\tau
\]  
(15)

Thus (13) can be written as
\[
\text{var}(I_{k,i}) = E_{\text{ini}} \left\{ \frac{1}{T} \int_0^T \tilde{R}_k(\tau) \tilde{R}_l(\tau) d\tau \right\}
\]  
(16)

To further evaluate (16), let \( I = \lfloor \tau/T_c \rfloor \) be the integer part of \( \tau/T_c \) and \( r = \tau - IT_c \). Then \( I \) and \( r \) are random variables uniformly distributed over \([0, 1, \ldots, N-1]\) and \([0, T_c]\) respectively. Since the components of vector \( \tilde{s}_k \) are i.i.d. random variables, it is not hard to see that \( E_{\text{ini}} \{ \tilde{R}_k(\tau) \} \) is nonzero only when \( 0 \leq \tau \leq T_c \) (i.e., when \( I = 0 \)). More specifically,
\[
E_{\text{ini}} \{ \tilde{R}_k(\tau) \} = \begin{cases} \frac{N}{D} \sum_{m=1}^D \tilde{h}_m(r), & \text{if } I = 0 \\ 0, & \text{otherwise} \end{cases}
\]  
(17)

In (17), \( \tilde{h}_m(r) = \int_0^{T_c} g_m(t) g_m(t-r) dt \), \( 0 \leq r \leq T_c \), is the partial correlation function of the chip waveform \( g_m(t) \). Let
\[
I = \frac{1}{D^2} \sum_{m=1}^D \left( \sum_{d=1}^D \tilde{h}_d(r) \right)^2 dr
\]  
be the normalized interference parameter. Then the SIR in (5) can be written in terms of \( I \) as
\[
\text{SIR}_k = \left( \frac{2E_k}{N_0} \right)^{-1} + \frac{I}{N \sum_{i=1}^N \sum_{j=1}^N P_i P_j}^{-1}
\]  
(18)

Note that when \( g_1(t) = g_2(t) = \ldots = g_D(t) \), \( I \) is just the normalized mean-squared partial chip correlation defined in [3], [5] and the SIR in (18) agrees with the result given in [5] for the single chip pulse. Similarly as in Sect. 2.1, to facilitate the design of optimal chip waveforms considered in this paper, it is convenient to express the parameter \( I \) in terms of the Fourier transforms of the chip pulses. This can be done as follows. For \( m = 1, 2, \ldots, D \), let
\[
u_m(\tau) = \int_{-\infty}^\infty g_m(t) g_m(t-\tau) dt
\]  
be the autocorrelation function of the chip pulse \( g_m(t) \). Then \( u_m(\tau) \) is an even function confined to \([-T_c, T_c]\) with \( u_m(f) = |G_m(f)|^2 \). Now using the fact that \( \tilde{h}_m(\tau) = u_m(\tau) \) for \( 0 \leq \tau \leq T_c \) and applying Parseval’s theorem one has
\[
I = \frac{1}{D^2 T_c^3} \int_0^T \left( \sum_{m=1}^D \tilde{h}_m(\tau) \right)^2 d\tau
\]  
(19)

Again, when \( g_1(t) = g_2(t) = \ldots = g_D(t) \), (19) reduces to the normalized integration of the fourth power of the magnitude spectrum of the single chip waveform as shown in [3] and [6].

3. Design Problems with Bandwidth Consideration

3.1 Design of Signature Waveforms

It is desired to obtain the signature waveforms to maximize the SIR in (5) for every user. However this is a very difficult, if not an impossible task. Thus an alternative objective, namely to minimize the average MAI variance at the outputs of all correlation receivers is considered. The (normalized) average MAI variance is defined by
\[
J = \frac{1}{K T^2} \sum_{k=1}^K \sum_{i \neq k}^K \frac{P_i}{P_k} \text{var}(I_{k,i})
\]  
(20)

where \( J \) has been normalized to be independent of \( T \). Minimizing \( J \) ensures an average performance level over all users. It is conceivable that when the number of users \( K \) is fixed, the minimum value of \( J \) decreases as the transmission bandwidth of the system increases. Thus, for a given bandwidth \( W \), the optimal set of signature waveforms is the one that minimizes \( J \).

The bandwidth of a communication system is usually judged based on the power spectral density (PSD) of the transmitted signal. Let \( \mathcal{P}(f) \) be the PSD of the transmitted signal \( y(t) \). It can be shown [14] that \( \mathcal{P}(f) \) is proportional to \( \sum_{k=1}^K P_k |\mathcal{S}_k(f)|^2 \). Two commonly used bandwidth measurements are the root-mean-square (RMS) and fractional out-of-band power (FOBP) bandwidth [13]. The RMS bandwidth \( W \) of the system is defined as [15]
\[
W^2 = \int_{-\infty}^{\infty} f^2 \mathcal{P}(f) df = \frac{\sum_{k=1}^K \int_{-\infty}^{\infty} f^2 |\mathcal{S}_k(f)|^2 df}{T \sum_{k=1}^K P_k}
\]  
(21)

where \( b^2(s_k(t)) = \frac{1}{T} \int_{-\infty}^{\infty} f^2 |\mathcal{S}_k(f)|^2 df \) is the square of the...
RMS bandwidth of $s_k(t)$. The system is said to have FOBP bandwidth $W$ at level $\eta$, $0 < \eta < 1$, if

$$
\frac{\int_{|f|>W} |P(f)|^2 df}{\int_{-\infty}^{\infty} |P(f)|^2 df} = \frac{\sum_{k=1}^{K} \int_{|f|>W} |P_k S_k(f)|^2 df}{T \sum_{k=1}^{K} P_k} = \frac{\sum_{k=1}^{K} P_k \epsilon(s_k(t))}{\sum_{k=1}^{K} P_k} = \eta
$$

where $\epsilon(s_k(t)) = \frac{1}{T} \int_{|f|>W} |S_k(f)|^2 df$ is the fraction of the energy of $s_k(t)$ lying outside the frequency band $[-W, W]$.

It is clear from (21) and (22) that the signature waveforms directly determine the transmission bandwidth. Thus the design problem for signature waveforms with bandwidth proportional to $\sum_{k=1}^{K} |\Phi_k|^2$ can be written as

$$
\text{minimize } \int_{-\infty}^{\infty} f^2 |G_m(f)|^2 df = W^2
$$

Likewise, for $0 < \eta < 1$, the FOBP bandwidth constraint can be written as

$$
\frac{1}{DT} \sum_{m=1}^{D} \int_{-\infty}^{\infty} |G_m(f)|^2 df = \eta
$$

Thus the design problem for multiple chip waveforms can be stated as follows.

**Problem 1:** Consider an asynchronous CDMA system equipped with correlation receivers. Given a signaling interval $T$, a transmission bandwidth $W$ and a set of the received powers $\{P_k\}_{k=1}^{K}$, find a set of $K$ signature waveforms $\{s_1(t), s_2(t), \ldots, s_K(t)\}$ that minimize $J$ in (20) subject to the energy constraint of (2) and the RMS bandwidth constraint of (21) (or the FOBP bandwidth constraint of (22)).

### 3.2 Design of Multiple Chip Waveforms

Equation (18) shows that in asynchronous DS-CDMA systems using random signature sequences, the SIR for every user increases as $I$ decreases. It follows that to maximize SIR for all users, one needs to find multiple chip waveforms to minimize $I$ in (19). This also implies that, different from the design of multiple signature waveforms, the design of chip waveforms is independent of the received power levels of all users.

The PSD of the signal $y(t)$ can be shown to be proportional to $\sum_{m=1}^{D} |G_m(f)|^2$. Thus the RMS bandwidth constraint is as follows

$$
\frac{1}{DT} \sum_{m=1}^{D} \int_{-\infty}^{\infty} f^2 |G_m(f)|^2 df = W^2
$$

Likewise, for $0 < \eta < 1$, the FOBP bandwidth constraint can be written as

$$
\frac{1}{DT} \sum_{m=1}^{D} \int_{|f|>W} |G_m(f)|^2 df = \eta
$$

Thus the design problem for multiple chip waveforms can be stated as follows.

**Problem 2:** Consider an asynchronous DS-CDMA system using random signature sequences and multiple chip waveforms. Given a signaling interval $T$ and a transmission bandwidth $W$, find a set of $D$ chip waveforms $\{g_1(t), g_2(t), \ldots, g_D(t)\}$ that minimize $I$ in (19) subject to the energy constraint of (11) and the RMS bandwidth constraint of (23) (or the FOBP bandwidth constraint of (24)).

The two optimization problems stated in this section concern finite sets of time-limited waveforms and are very similar. The only difference lies in the objective functions. These problems are very difficult to solve explicitly due to the complexity of the objectives and the constraints. Nevertheless, the expansion technique employed in [3] can be applied here to simplify the design problems.

In [3] the authors obtain the optimal single chip waveforms for offset quadrature DS-CDMA systems under bandwidth, phase and envelope constraints. The method is to approximate the solution by a finite series expansion over a complete set of basis functions. As pointed out in [3], the choice of a proper set of basis functions is very important to reduce the dimensionality of the equivalent discrete optimization problem. Which basis set is chosen is governed by the bandwidth criterion under consideration. If the FOBP bandwidth is used, it is suggested in [3] that the prolate spheroidal functions should be used to expand the Fourier transforms of the signature waveforms (and chip waveforms respectively). For the RMS bandwidth constraint, the set of time-truncated sinuisoids $\{\sin(n \pi t/T), 0 \leq t \leq T\}_{n=1}^{\infty}$ is selected for the expansion of the signature waveforms (and chip waveforms respectively). This selection is natural since the functions $\{\sin(n \pi t/T), 0 \leq t \leq T\}_{n=1}^{\infty}$ form a complete set for all continuous functions time limited to $[0, T]$ and more importantly, they achieve the minimum RMS bandwidth [16]. For brevity of presentation, only the RMS bandwidth constraint is pursued in this paper.

### 4. Optimal Signature Waveforms

#### 4.1 Problem Simplification

Let $\psi_n(t) = \sqrt{2/T} \sin(n \pi t/T) p_T(t)$, where $p_T(t) = 1$ for $0 \leq t \leq T$ and $p_T(t) = 0$ otherwise. Then the RMS bandwidth of $\psi_n(t)$ is $b(\psi_n(t)) = n/2T$. To simplify the calculation of the objective function in (20) introduce the shifted (and possibly negated) versions $\phi_n(t)$ of $\psi_n(t)$, defined by

$$
\phi_n(t, T) = \begin{cases} 
\sqrt{T} \cos \left( \frac{n \pi t}{T} \right), & -\frac{T}{2} \leq t \leq \frac{T}{2}, \text{if } n \text{ is odd} \\
\sqrt{T} \sin \left( \frac{n \pi t}{T} \right), & -\frac{T}{2} \leq t \leq \frac{T}{2}, \text{if } n \text{ is even} 
\end{cases}
$$

Let $\Phi_n(f, T) = \mathcal{F}\{\phi_n(t, T)\}$. Note that when $n$ is odd, the function $\phi_n(t, T)$ is even, hence $\Phi_n(f, T)$ is a real function. On the other hand, when $n$ is even, $\phi_n(t, T)$ is an odd function and $\Phi_n(f, T)$ is purely imaginary. Write $\Phi_n(f, T) = X_n(f, T)$ when $n \neq 2l - 1$ and $\Phi_n(f, T) = jY_n(f, T)$ when $n = 2l, l$.
where $\text{sinc}(x) = \sin(\pi x)/\pi x$.

Let $\widetilde{s}_k(t) = s_k(t + T/2)$ be the shifted version of the signature waveform $s_k(t)$. Since $\{\phi_k(t)\}_{k=1}^K$ form a complete set for all continuous functions time limited to $[-T/2, T/2]$, $\widetilde{s}_k(t)$ can be expanded as follows

$$\text{(27)}$$

$$\text{(27)}$$

where the coefficients $x_{kl}$ and $y_{kl}$ are given by $x_{kl} = \int_{-T/2}^{T/2} \widetilde{s}_k(t)\phi_{2l-1}(t, T)\,dt$ and $y_{kl} = \int_{-T/2}^{T/2} \widetilde{s}_k(t)\phi_{2l}(t, T)\,dt$. The RMS bandwidths of $s_k(t)$ and $\widetilde{s}_k(t)$ are the same and can be computed as follows [16],

$$\text{(28)}$$

$$\text{(28)}$$

Due to the constraint on the system bandwidth in (21), it follows from (28) that the coefficients $x_{kl}$ and $y_{kl}$ should be very small when $l$ is large. Therefore, for all practical purposes, it is sufficient to truncate each sum in (27) to a finite length of $L$ terms, that is

$$\text{(29)}$$

Using the truncated expansion in (29), the constraints in (2) and (21) can be written in terms of $x_{kl}$ and $y_{kl}$, $k = 1, 2, \ldots, K$ and $l = 1, 2, \ldots, L$ as follows.

$$\text{(30)}$$

and

$$\text{(31)}$$

The above optimization problem can be solved numerically, for example, by means of sequential quadratic programming routines. Since the number of users $K$ is not controllable, to reduce the dimensionality of the optimization problem, it is desired to use as small value for $L$ as possible. In general, such value of $L$ depends on the bandwidth-time product $WT$ (due to the bandwidth constraint) and can be numerically determined from the plot of $J(L)$, the minimum value of $J(L)$ over the signature waveforms for a given $L$.

### 4.2 A Numerical Example

A numerical result is given in this section to demonstrate the optimal signature waveforms obtained from solving Problem 3. In general, the optimal waveforms need to be found for each set of received powers. For simplicity and in the interest of space, a CDMA system with four ($K = 4$) equal-power users ($P_k = P$, $k = 1, 2, 3, 4$) is examined as an illustrative example. The RMS bandwidth value ranges from $0.6T$ to $2.0T$, which is the range of interest to us.

Given the number of users and the RMS bandwidth, different values of $L$ were used in Problem 3 to obtain the corresponding signature waveforms and the minimum value of the objective function, namely $J^*(L)$. Plotting $J^*(L)$ versus $L$ reveals that, for all RMS bandwidth values considered, $J^*(L)$ decreases very slowly when $L \geq 4$. For example, Fig. 2 plots $J^*(L)$ for $WT = 1.4$ and $WT = 2.0$. Thus $L = 4$ is used in the example of this section to obtain the signature waveforms. It should be noted, however, that for systems with a larger number of users and wider bandwidth the value of $L$ can be larger.

To evaluate the performance of the designed signature waveforms, the average MAI variance ($J$) achieved by the optimal signature waveforms is plotted in Fig. 3 as a function of the time-bandwidth product $WT$. As expected, the MAI reduces as the bandwidth increases. In Fig. 3, the performance of the designed signature waveforms is also compared with that of the signature waveforms optimally designed for synchronous CDMA systems. Such a design of

![Fig 2 Influence of L on the minimum value of the objective function, K = 4.](image)
signature waveforms was considered in [15] with the same optimality criterion and bandwidth constraint. For convenience, we shall refer to the waveforms designed specifically for asynchronous CDMA systems in this paper as the asynchronous signature waveforms and the waveforms designed for synchronous CDMA systems in [15] as the synchronous signature waveforms. In Fig. 3 two curves are plotted for the performance of the synchronous signature waveforms: one over synchronous systems and the other over asynchronous systems (system-mismatch situation).

Orthogonal signature waveforms are available in synchronous CDMA systems when $(2WT)^2 \geq (K+1)(2K+1)/6$ [15]. Therefore the MAI produced by the synchronous signature waveforms in synchronous systems is zero when $WT \geq 1.369$ as shown in Fig. 3. Assuming that the same orthogonal synchronous signature waveforms are used for larger RMS bandwidths, the MAI produced by the synchronous signature waveforms in asynchronous systems is the same for $WT \geq 1.369$, as can be seen in Fig. 3. The superiority of the asynchronous signature waveforms over synchronous ones in asynchronous systems is clearly observed from Fig. 3 for all values of RMS bandwidth.

It is also of interest to notice from Fig. 3 that for very small values of RMS bandwidth, the MAI produced by synchronous waveforms in synchronous systems is larger than that produced by the asynchronous waveforms in asynchronous systems. This is counter-intuitive since the performance of synchronous systems is usually taken as the lower bound for the performance of the asynchronous ones. This observation can be explained as follows. When the RMS bandwidth is very small, all the signature waveforms possess very similar shapes (in order to satisfy the bandwidth constraint). This means that the synchronous correlations among signature waveforms are very high, causing a huge MAI in synchronous systems. On the other hand, the MAI in the asynchronous systems depends on the particular delays among users and can be very small for certain delays. Therefore, after averaging over the entire range of the delays (the symbol duration $T$), the average MAI in asynchronous systems can be significantly smaller than that in synchronous ones. Nevertheless, when RMS bandwidth increases, the MAI in synchronous systems approaches zero much faster than that in asynchronous systems. In other words, it requires much larger bandwidth for the asynchronous system to perform at satisfactory level (when MAI is small) compared to that of synchronous systems, even though optimal signature waveforms are used in both scenarios.

Finally, optimal asynchronous signature waveforms are demonstrated in Figs. 4 and 5 for selected values of RMS bandwidth occupancies. These sets of optimal waveforms are unique and can be assigned to users in any order. Note that the signature waveforms in Figs. 4 and 5 possess (even or odd) symmetry about the midpoint of the symbol duration.

### 5. Optimal Chip Waveforms

#### 5.1 Problem Simplification

Similar to signature waveform design, the problem of de-
signing multiple chip waveforms (Problem 2) can be reduced to a finite-dimensional optimization problem. To this end, expand the delayed version of each chip waveform as follows:

$$\tilde{g}_m(t) = g_m(t + T_c/2)$$

$$\approx \sum_{l=1}^{L} x_{ml} \phi_{2l-1}(t, T_c) + \sum_{l=1}^{L} y_{ml} \phi_2(t, T_c)$$  \hspace{1cm} (32)

where $x_{ml} = \int_{-T_c/2}^{T_c/2} \tilde{g}_m(t) \phi_{2l-1}(t, T_c) dt$ and $y_{ml} = \int_{-T_c/2}^{T_c/2} \tilde{g}_m(t) \phi_2(t, T_c) dt$.

Using this expansion, the constraints in (11) and (23) can be written in terms of $x_{ml}$ and $y_{ml}$, $m = 1, 2, \ldots, D$ and $l = 1, 2, \ldots, L$, as

$$\sum_{l=1}^{L} (x_{ml}^2 + y_{ml}^2) = T_c, \hspace{1cm} m = 1, 2, \ldots, D$$  \hspace{1cm} (33)

and

$$\sum_{m=1}^{D} \sum_{l=1}^{L} (2l-1)^2 x_{ml}^2 + 4l^2 y_{ml}^2 = 4D(WT_c)^2 T_c$$  \hspace{1cm} (34)

Appendix B expresses the objective in (19) in terms of $x_{ml}$ and $y_{ml}$. Therefore Problem 2 is now equivalent to the following finite-dimensional optimization problem.

**Problem 4:** Find 2DL coefficients $x_{ml}$ and $y_{ml}$, $m = 1, 2, \ldots, D$ and $l = 1, 2, \ldots, L$, that minimize $I(D, L)$ in (A-19) subject to the constraints in (33) and (34).

As for Problem 3, Problem 4 can be solved numerically. It should be noted, however, that the dimensionality of Problem 4 does not depend on the number of users $K$ and is usually much smaller than that of Problem 3. This is due to the following two reasons. First, the number of unknowns in Problem 4 depends only on $D$ and $L$, which can be selected to achieve a compromise between performance and complexity. Secondly, since the value of $WT_c$ is usually less than 3.0 for DS-CDMA systems, the practical value of $L$ in Problem 4 is much smaller than that in Problem 3.

Finally, the following proposition justifies the advantage of using multiple chip waveforms in DS-CDMA systems with random signature sequences.

**Proposition 1:** Consider a DS-CDMA system using random signature sequences and D chip waveforms. Let the $D$ chip waveforms be the solutions of Problem 4 for some fixed value of $L$. Let $I^*(D, L)$ be the corresponding interference parameter and $K$ be an integer number. Then

$$I^*(KD, L) \leq I^*(D, L)$$  \hspace{1cm} (35)

Proof: The proof is trivial, by noting that the equality in (35) is achieved when using $K$ copies of the set of $D$ optimal chip waveforms for the set of $kD$ chip waveforms.

5.2 Numerical Results

Multiple chip waveforms obtained from solving Problem 4 are presented in this section. Up to $D = 3$ is considered. The values of $W$ are from $0.5/T_c$ to $3/T_c$, which is the range of interest for us. For this range of RMS bandwidth, using $L = 6$ yields sufficient accuracy for optimal chip waveforms.

Numerical results indicate that the improvement from using multiple chip waveforms over single chip waveforms is quite significant. This is illustrated in Fig. 6 where the ratios between the interference parameters achieved by the optimum double and triple chip waveforms (denoted by $I_{2c}$ and $I_{3c}$, respectively) and that of the optimum single chip waveform (denoted by $I_{1c}$) are plotted versus $WT_c$. Observe that the largest gain is achieved by moving from single chip waveform to double chip waveforms and there is not much improvement with triple chip waveforms (note the values shown in vertical axis of Fig. 6 in order to verify this observation). When $W = 0.5/T_c$ there exists only one chip pulse of duration $T_c$, namely the half-sine waveform $\sqrt{2/T_c} \sin (\pi/T_c) r(t)$, therefore there is no advantage to use multiple chip waveforms for interference suppression.

However, as the bandwidth increases, the interference reduction capability of multiple chip waveforms increases and saturates at about $W = 2.4/T_c$. At $W = 2.4/T_c$, the interference can be reduced by about 10% by using multiple chip waveforms instead of single chip waveform. This is equivalent to a 10% saving in transmission bandwidth for a given SIR requirement. Thus in general, there exists an minimum value of $WT_c$ for multiple chip waveforms to minimize the multiple access interference.

It is also of interest to compare the performance of optimal chip waveforms (single, double or triple) among themselves when varying the chip duration $T_c$. For a fair comparison, the bandwidth $W$ and the symbol duration $T$ are fixed. Since $N = T/T_c$, it follows from (18) that to maximize SIR, one needs to minimize $IT_c$, or equivalently to minimize $IWT_c$. This parameter is plotted against $WT_c$ in Fig. 7. It can be seen that the performance improves with in-
increasing chip duration $T_c$ and saturates at about $T_c = 1.4/W$ for the single chip waveform and $2.4/W$ for both double and trip chip waveforms. Thus in general, there exists a minimum value of $WT_c$ for multiple chip waveforms that minimizes the multiple access interference. Note that the RMS bandwidths of some common chip waveforms are relatively small, which make them not efficient in terms of minimizing MAI. This is illustrated in Fig. 8 where $IWT_c$ achieved by the half-sine, raised cosine, Blackman [9] and four-term odd cosine series [10] chip waveforms are shown (in the range of small $WT_c$). The equations for these chip waveforms are as follows.

1) Half-sine: $p_2(t) = \sqrt{2} \sin \left( \frac{\pi t}{T_c} \right) p_{Tc}(t)$.

2) Raised cosine: $p_3(t) = \frac{\sqrt{2}}{3} \left[ 1 - \cos \left( \frac{2\pi t}{T_c} \right) \right] p_{Tc}(t)$.

3) Blackman [9]:

$$p_4(t) = \begin{bmatrix} c \left[ k_1 - k_2 \cos \left( \frac{2\pi t}{T_c} \right) + k_3 \cos \left( \frac{4\pi t}{T_c} \right) \right] p_{Tc}(t), \end{bmatrix}$$

where $c^2 = (k_1^2 + k_2^2 + k_3^2 - 2k_1k_3)^{-1}$ and $k_1 = 0.42$, $k_2 = 0.5$ and $k_3 = 0.08$.

4) Four-term odd cosine series [10]:

$$p_5(t) = \left[ 0.868 - 0.686 \cos \left( \frac{2\pi t}{T_c} \right) - 0.149 \cos \left( \frac{4\pi t}{T_c} \right) - 0.033 \cos \left( \frac{6\pi t}{T_c} \right) \right] p_{Tc}(t).$$

It can be seen from Fig. 8 that the performance of the raised cosine chip is closest to that of the optimal single chip, followed by the Blackman and cosine series chips.

Examples of optimal single and double chip waveforms are plotted in Fig. 9 for $WT_c = 2.4$. This value of $WT_c$ is chosen since it gives the optimal chip duration for double and triple chip waveforms as discussed above. The advantage of using those optimal single and double chip waveforms in terms of bit error rate is also shown in Fig. 10 for a CDMA system having $K = 32$ users and a RMS bandwidth value such that $N = 32$ if the optimal double chip waveforms are used (i.e., $WT = 32 \times 2.4$). Figure 10 is obtained by using the improved Gaussian approximation developed in [17] for double chip waveforms. The improved GA takes into account the actually shapes of the chip waveforms (through their correlation functions) to approximate the error proba-
bility and it has been shown in [1], [11], [17] that this approximation is very accurate. The relative performances of different chip waveform(s) in Fig. 10 agree very well with the values of parameter $I_{WT}$, plotted in Fig. 8. More specifically, both Figs. 8 and 10 show that the optimal double chip waveforms perform the best, followed by the optimal single chip waveform, the raised cosine, Blackman and the half-sine waveforms. Finally, it can be seen from Fig. 10 that at the BER level of $10^{-2}$ a gain of about 2 dB in $E_b/N_0$ can be attained by using the optimal double chip waveforms compared to the raised cosine waveform (which performs the best among the common chip waveforms).

6. Conclusions

Two problems of designing signature waveforms and multiple chip waveform for asynchronous CDMA systems have been considered in this paper. The bandwidth constraint is explicitly taken into account in the design process so that the available bandwidth of the system is optimally utilized. Appropriate performance parameters have been derived for both design problems when correlation receiver is used. These performance parameters are expressed in terms of the Fourier transforms of the signature and chip waveforms respectively, which facilitates the use of the series expansion method to simplify design problems. Various design examples have also been given to demonstrate the superiority of the proposed signature and multiple chip waveforms. In particular, it has been shown that in DS-CDMA systems with random signature sequences, 10% of MAI can be reduced (or equivalently, 10% of transmission bandwidth can be saved) by using two chip waveforms instead of a conventional single chip waveform.

References


The improved GA to error probability was originally proposed by Holtzman in [1] for a single rectangular chip waveform. It has been extended to an arbitrary single chip waveform in [11], [17] and arbitrary double chip waveforms in [17].
Expression, each term in (A-13) can be expressed in Appendix A. In particular, let \( J(L, D, T_c) \) be the expression in (A-3) when replacing \( K \) by \( D \) and \( T \) by \( T_c \), then
\[
\frac{1}{D^2 T_c^3} \sum_{m=1}^{D} \sum_{n=1}^{D} \int_{-\infty}^{\infty} \left| G_m(f) \right|^2 \left| G_n(f) \right|^2 df = J(L, D, T_c) \quad (A-11)
\]

Let \( \hat{G}_m(f) = F\{ g_m(t) \} \). Then from (32) one has
\[
\hat{G}_m(f) = \sum_{l=1}^{L} x_{ml} X_l(f, T_c) + j \sum_{l=1}^{L} y_{ml} Y_l(f, T_c) \quad (A-12)
\]

Thus the integral in the first term of (A-10) can be calculated as follows:
\[
\int_{-\infty}^{\infty} \left| G_m(f) \right|^4 df = \int_{-\infty}^{\infty} \left( \sum_{l=1}^{L} x_{ml} X_l(f, T_c) \right)^4 df
\]
\[
+ \int_{-\infty}^{\infty} \left( \sum_{l=1}^{L} y_{ml} Y_l(f, T_c) \right)^4 df + 2 \int_{-\infty}^{\infty} \left( \sum_{l=1}^{L} x_{ml} X_l(f, T_c) \right) \left( \sum_{l=1}^{L} y_{ml} Y_l(f, T_c) \right)^2 df \quad (A-13)
\]
The first two terms of (A-13) can be written as [3]
\[
\int_{-\infty}^{\infty} \left( \sum_{l=1}^{L} x_{ml} X_l(f, T_c) \right)^4 df = \sum_{k_0, \ldots, k_L} \gamma(k_0, \ldots, k_L) \prod_{l=0}^{L} x_{ml}^{k_l} \quad (A-14)
\]
and
\[
\int_{-\infty}^{\infty} \left( \sum_{l=1}^{L} y_{ml} Y_l(f, T_c) \right)^4 df = \sum_{k_0, \ldots, k_L} \chi(k_0, \ldots, k_L) \prod_{l=0}^{L} y_{ml}^{k_l} \quad (A-15)
\]
where
\[
\gamma(k_0, \ldots, k_L) = \frac{4!}{k_0! k_1! \cdots k_L!} \left( \int_{-\infty}^{\infty} \prod_{l=0}^{L} X_{l}^{k_l}(f, T) df \right) \quad (A-16)
\]
and
\[
\chi(k_0, \ldots, k_L) = \frac{4!}{k_0! k_1! \cdots k_L!} \left( \int_{-\infty}^{\infty} \prod_{l=0}^{L} Y_{l}^{k_l}(f, T) df \right) \quad (A-17)
\]
The last term of (A-13) can be expressed as
\[
2 \int_{-\infty}^{\infty} \left( \sum_{l=1}^{L} x_{ml}(f, T_c) \right)^2 \left( \sum_{l=1}^{L} y_{ml}(f, T_c) \right)^2 \, df
= 2 \sum_{l=1}^{L} \sum_{n=1}^{L} \sum_{\beta=1}^{L} \sum_{\gamma=1}^{L} (x_{ml}x_{nl}y_{ml}y_{nl}) \beta_{l,n,p,q}(T_c)
\]

(A·18)

Finally, combining (A·10), (A·11), (A·13), (A·14), (A·15) and (A·18) gives

\[
I(D, L) = \frac{1}{2D^2T_c^3} \sum_{m=1}^{D} \sum_{k_0, \ldots, k_L} \left( \gamma(k_0, \ldots, k_L) \prod_{l=0}^{L} x_{ml}^k + \chi(k_0, \ldots, k_L) \prod_{l=0}^{L} y_{ml}^k \right) + \frac{J(L, D, T_c)}{D}
\]

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