LETTER

Multiple Access Systems with QPSK Modulation

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SUMMARY This letter considers multiple access systems without bandwidth expansion. To improve the spectral efficiency, each user employs a QPSK modulation. The orientation of QPSK constellations is designed to maximize the minimum distance of the superimposed symbol constellation. The upper and lower bounds for the error performance of the proposed design demonstrate its advantage.

key words: CDMA, interference cancellation, QPSK modulation

1. Introduction

Conventional multiple access techniques, such as time division multiple access (TDMA), frequency division multiple access (FDMA) and code division multiple access (CDMA), always expand the transmission bandwidth. Recently, trellis coded multiple access (TCMA) is proposed in [1] and [2] to improve the bandwidth efficiency. In the proposed TCMA system, \( K \) users use QPSK modulation to access the same channel bandwidth typically used by a single user. Although it is shown in [2] that a reliable transmission can be achieved with the use of channel coding and branch metric update multuser detector, the performance of uncoded systems is not considered in [2].

Motivated by the work in [1], [2], this letter considers the design of signal constellations for uncoded multiple access systems using QPSK modulation. It may first appear that the problem under consideration is similar to that of finding two-dimensional signal constellations for a single-user system, a classical problem dated back to the 70’s [3], [4]. However, in the move from a single-user system to a multi-user system, there is a constraint due to the multiple access nature, that limit the choices of the superimposed constellations. The design criterion here is to maximize the minimum distance of the superimposed constellation.

2. The Multiple Access System with QPSK Modulation

The system under consideration consists of \( K \) users. Each user transmits independent and identically distributed data using QPSK modulation. The two data bits of user \( k \) at time index \( l \) is mapped onto one QPSK user symbol \( s_{k,l} \). The user symbols are then passed through an AWGN channel. The received signal can be represented by the following discrete-time baseband model:

\[
 r_l = \sum_{k=1}^{K} w_{k,l} e^{j\theta_{k,l}} s_{k,l} + n_l = u_l + n_l
\]

Here, the amplitude and phase offset of user \( k \) at time index \( l \) are denoted by \( w_{k,l} \) and \( \theta_{k,l} \), respectively. The noise \( n_l \) is a complex noise with two-sided power spectral density of \( N_0/2 \). The signal \( u_l \) in (1) denotes the resulting symbol in the superimposed constellation. This superimposed constellation is the joint constellation of all user-specific constellations and it has at most \( 4^K \) symbols.

Assuming equal received amplitudes \( w_{i,l} = w_{j,l} = \sqrt{E_s} = \sqrt{2E_b} \), where \( E_s \) and \( E_b \) are the energy per QPSK symbol and energy per bit, respectively) and phase synchronization \( (\phi_{i,l} - \phi_{j,l} = \theta_{i,l} \) is the same for all \( l \) for all users, the minimum distance \( d_{\text{min}} \) in the superimposed constellation is calculated when the angles among QPSK constellations \( \theta_{i,l} \) rotate. In the case of two users, the distance profile in Fig. 1 shows that \( d_{\text{min}} \) achieves the maximum value of \( \sqrt{4 - 2\sqrt{3}} \sqrt{E_s} \approx 0.73 \sqrt{E_s} \approx 1.03 \sqrt{E_b} \) when the phase offset between the two QPSK constellations is \( \pi/6 \). At this peak value of the \( d_{\text{min}} \), the superimposed constellation gives the best performance because of the maximum possible separation of superimposed symbol points. Note that when the phase offset is \( \pi/4 \) then \( d_{\text{min}} \) is \( \sqrt{6 - 4\sqrt{2}} \sqrt{E_s} \approx \sqrt{2.87} \sqrt{E_s} \approx \sqrt{2.87} \sqrt{E_b} \).

Fig. 1 The distance profile.
Fig. 2 The constellations of two users with QPSK for different cases of phase offsets: (a),(d),(g): viewed as four BPSK users; (b),(e),(h): two QPSK users; (c),(f),(i): superimposed constellation.

0.58 $\sqrt{E_s} \approx 0.82 \sqrt{E_s}$.

Figure 2 plots the superimposed constellations for the two-user case, corresponding to $\pi/6$, $0$ and $\pi/4$ phase offsets between the two QPSK constellations. The number of distinct signal points of the case of zero phase offset in Fig. 2(f) is 9 instead of 16 in Figs. 2(c) and 2(i). This is because some combinations of users’ symbols are indistinguishable. In this case, the superimposed symbols are nonequivalent. In [2], the superimposed constellation corresponding to $\pi/4$ phase offset in Fig. 2(i) was presented as optimum in terms of BER. However the proposed superimposed constellation corresponding to $\pi/6$ phase offset in Fig. 2(c) is better in terms of BER performance due to increased separation of the inner signal points.

Observe that the QPSK constellation with Gray mapping can be considered as a superimposed constellation of a 2-user multiple access system employing BPSK modulation. The amplitude of both BPSK constellations is $\sqrt{2}/2 \sqrt{E_s}$ and they are offset by $\pi/2$. From this perspective, the superimposed constellation of $K$ users with QPSK modulations can be viewed as the superimposed constellation of a system with $2K$ users, each having BPSK modulation. This is illustrated in Figs. 2(a), (d), (g), where each signal point represents the signature waveform of one BPSK user. The $k$th BPSK user is identified by its own phase $\varphi_k$, $k = 1, \ldots, 2K$.

With the above model, the BER performance of the uncoded multiple access systems can be upper and lower bounded using the same technique for optimal multiuser detection as in [5]. In essence, the bounds can be derived from the $2K \times 2K$ correlation matrix $R = [\rho_{ij,k}]$ of the waveforms of BPSK users. The crosscorrelations between the $i$th and the $k$th “users” is $\rho_{ik,k} = \Re \{e^{j(\varphi_i - \varphi_k)}\} = \Re \{e^{j\theta}\} = \cos(\theta)$. The derivation of bounds from the correlation matrix $R$ is summarized below [5].

Let $E_k = \{e \in \{-1,0,+1\}^{2K} | e_k \neq 0\}$ be the set of error vectors that affects the $k$th user of $2K$ users system. Note that $E = \bigcup_{k=1}^{2K} E_k$. Let the number of nonzero components of an error vector and the energy of an hypothetically multiuser signal modulated by $e$ are denoted, respectively, by $w(e) = \sum_{k=1}^{2K} |e_k|$ and $\|Z(e)\|^2 = \int_0^T (\sum_{k=1}^{2K} e_k s_k(t))^2 dt = e^T R e$. An error vector $e \in E$ is decomposable into $e = e' \in E$ and $e'' \in E$ if

1. $e = e' + e''$
2. if $e_k = 0$, then $e'_k = e''_k = 0$; and
3. $e'^T R e'' \geq 0$.

The subset of indecomposable vectors in $E_k$ is denoted by $F_k$. It can be shown if $e \in F_k$ then $-e \notin F_k$. Thus it is convenient to denote $F_k^+ = F_k \cup F_k^-$ the sets of indecomposable vectors in $F_k$ that have $e_k = 1$ and $e_k = -1$, respectively. Note that $F_k = F_k^+ \cup F_k^-$. Let $d_{k,min} = \min_{e \in F_k^+} \|Z(e)\|$ and $u_{k,min} = \min_{e \in F_k^-} \|Z(e)\| = d_{k,min} w(e)$. Here, $d_{k,min}$ is interpreted as one half of minimum distance between two multiuser signals that differ in the $k$th bit or minimum distance of the $k$th user. With the above notations, the error probability of the user $k$ is bounded as follows [5]:

$$2^{1-w_{k,min}} Q(yd_{k,min}) \leq P_k[\text{error}]
\leq \sum_{e \in F_k^+} 2^{-w(e)+1} Q(yZ(e))$$

(2)

where $\gamma = \sqrt{P_s/\sigma} = \sqrt{2E_s/2\sigma}$ is the signal to noise ratio.

As an example, the set of indecomposable vectors $F_1$ for the case of two users using QPSK modulations with $\pi/6$ phase offset is listed in Table 1. Due to the symmetry of the constellation, $F_2$ and $F_3$ are the same as $F_1$ and $F_3$, respectively. In fact, the bounds of error performance corresponding to $F_1$ and $F_3$ are the same. For QPSK constellations with phase offset $\pi/4$, all $F_1$, $F_2$, $F_3$ and $F_4$ are identical. From the set of indecomposable vectors and Eqn. (2), the probability of error for each user is bounded as:

$$P_{\pi/6}^L = \frac{1}{2} Q\left(\gamma \sqrt{2 - \sqrt{3}}\right) \leq P_{\pi/6}[\text{error}]
\leq \frac{5}{4} Q\left(\gamma \sqrt{2 - \sqrt{3}}\right) + \frac{1}{8} Q\left(\gamma \sqrt{4 - 2 \sqrt{3}}\right)
+ \frac{3}{4} Q\left(\gamma\right) + \frac{1}{2} Q\left(\gamma \sqrt{3}\right) = P_{\pi/6}^U$$

(3)

and

$$P_{\pi/4}^L = \frac{1}{4} Q\left(\gamma \sqrt{3 - 2 \sqrt{2}}\right) \leq P_{\pi/4}[\text{error}]$$

Table 1

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<thead>
<tr>
<th>$F_k(\epsilon_1 = -1)$</th>
<th>$F_k(\epsilon_1 = 1)$</th>
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<tbody>
<tr>
<td>$-1$ $-1$ $1$ $1$</td>
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Fig. 3 Comparison of error performance of multiple access systems using different QPSK constellations.

\[
\begin{align*}
&\leq \frac{1}{2} Q\left(\gamma \sqrt{3 - 2 \sqrt{2}}\right) + Q\left(\gamma \sqrt{2 - \sqrt{2}}\right) \\
&+ \frac{1}{4} Q\left(\gamma \sqrt{3 - 3 \sqrt{2}/2}\right) + Q(\gamma) \\
&+ \frac{1}{8} Q\left(\gamma \sqrt{4 - 2 \sqrt{2}}\right) + \frac{1}{2} Q\left(\gamma \sqrt{2}\right) = P_{\pi/4}^{U} \quad (4)
\end{align*}
\]

Figure 3 plots the bounds of error performance corresponding to \(\pi/4\) and \(\pi/6\) phase offset. It can be clearly observed that using \(\pi/6\) phase offset between the two QPSK constellations yields better error performance than using \(\pi/4\) phase offset. More specifically, the lower bound when using \(\pi/6\) phase offset is superior to the upper bound when using \(\pi/4\) phase offset at high signal-to-noise ratio region.

The optimal QPSK constellations for systems with more users can be found numerically (for example by using the optimization toolbox in MATLAB). An example, the optimum angle of QPSK constellations for three-user system are \(\theta_{1,2} = 17^\circ 17'\) and \(\theta_{1,3} = 35^\circ 03'\) and the corresponding minimum distance is \(d_{\text{min}} = 0.6095 \sqrt{E_b}\).

Finally, it should be noted that, for the systems under consideration, since all users are signalling over a narrow-band channel (using modulations on the same two-dimensional plane), the number of users is severely limited compared to the traditional spread spectrum systems. If more users are desired, then more bandwidth needs to be spent to create more orthogonal two-dimensional planes.

3. Conclusions

Maximization of the minimum Euclidean distance of the superimposed symbol constellation was considered to improve the performance of an uncoded multiple access system in a two-dimensional signalling space. In particular, QPSK constellations were found. The bounds of the error performance for a two-user system was also presented to justify the proposed design.

References


