Estimating the Costs of a Horizontal Merger: Methodology and the Case of the 1887-1914 Sugar Monopoly

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Abstract: Monopoly merging costs refer to the costs incurred during monopoly formation or the costs of maintaining an existing monopoly. This paper estimates that enforcing the Sherman act increased the sugar monopoly's merging costs from at most 35% of industry profits in 1887 to at least 252% of industry profits in 1914. This estimation is based on closed-form estimations for bounds of monopoly merging costs with linear demand and costs. The paper also provides analogous estimations with quadratic demand and linear costs. Finally, the paper provides dollar estimations using cost and demand forms estimated in Genesove and Mullin (1998).

Key Words: Core, Estimation of transaction costs, Merger precondition, Sugar monopoly

JEL Classification Number: C62, C71, L14, L41

1. Introduction

Since the seminal work of Coase (1937), transaction cost analysis has maintained a steady interest over the past 70 years in business, economics, and other social science disciplines. It not only generated large theoretical literatures such as transaction cost economics (TCE) and property right theory (PRT), but also generated large empirical literatures on vertical integration and long-term contracting,[1] and on estimating the size of transaction costs in a variety of sectors from agriculture to financial markets.[2]

This paper makes two theoretical contributions and one empirical contribution to merger study and transaction cost analysis. The first theoretical contribution is closed-form estimations for the lower (upper) bound of merging costs for observed (dissolved) monopoly with linear costs and linear or quadratic demands. These estimations are obtained by following the estimation procedure outlined in Zhao (2009), which is based on two merger preconditions: the profitability precondition and the non-empty core precondition. Such results are proof that there exist new insights and benefits of cooperative game theory that can not be obtained from non-cooperative game theory. Note that it is not possible to obtain closed-form estimations with loglinear and exponential demands, because the calculation involves solving equations whose unknown can not be obtained analytically. The second theoretical contribution is that the core is non-empty and has a non-empty (relative) interior

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[1] See Williamson (1975, 1985) and Klein et al (1978) for TCE, and Grossman and Hart (1986) and Hart and Moor (1990) for PRT. Shelanski and Klein (1995) found that the empirical literature is remarkably consistent with the predictions of TCE, although David and Han (2004) found only 47% support for TCE (among 63 articles), they found strong support for the predictions of asset specificity and weak support for the predictions of uncertainty; Whinston (2003) suggested that PRT offers a richer set of predictions that need more empirical tests.

[2] See Wang (2003) for a survey of such estimations. See also Boerner and Macher (2003) for surveys of transaction cost studies in social sciences, Jones (2002) in financial markets, Rindfleisch and Heide (1997) in marketing, and Joskow (1988) in vertical integration.

in Cournot oligopolies with quadratic demand and linear costs, whereas it is only previously known that the core is non-empty in Cournot oligopolies with linear demand and costs.

Empirically, the paper provides numerical estimations for lower and upper bounds of the sugar monopoly's merging costs, based on the demand forms constant marginal cost estimated by Genesove and Mullin (1998). For example, under the assumption of linear demand and cost, the paper shows that monopoly merging costs were at most 35% of pre-merger total profits at its formation in 1887, and at least 252% of post-dissolution total profits at its dissolution in 1914. To put it differently, the paper shows that the enforcement of Sherman act increased the sugar monopoly merging costs from at most 35% of pre-merger total profit to at least 252% of post-dissolution total profits. The paper also provides numerical estimations based on the quadratic, loglinear and exponential demands reported in Genesove and Mullin (1998).

The rest of the paper is organized as follows. Section 2 describes the historical background of the sugar monopoly, Section 3 describes the theoretical model. Sections 4 and 5 provide theoretical and empirical estimations for monopoly merging cost, with linear and quadratic demands, respectively. Section 6 concludes the paper, and the Appendix provides all proofs.

2. Summary of the 1887-1914 Sugar Monopoly

There were 27 sugar refineries before the sugar monopoly or American Sugar Refining Company (ASRC) was formed in 1887, which consolidated 18 of the refineries and controlled 80% of the industry's capacity. In the monopoly's first year of operation, it reduced its 20 plants into ten, its production to 68% of its capacity, and its market share to 72.7%. Consequently, prices quickly rose 16%, entry followed, and price war soon occurred.

The monopoly's production further decreased to 59% of its capacity, and its market share fell to 66% in 1889. Buying out the competition was the monopoly's most logical choice. After acquiring seven competing refineries, its market share increased to 95% in 1892.

Two more price wars occurred during 1892-1901, which forced the monopoly to form and reform by buying out the competition or reaching cartel agreements. Cartel agreements were the monopoly's main control method after 1892. Such change in conduct seemed to be a natural response to the enforcement of antitrust policy, because a cartel is not as easy as a merger to be targeted by antitrust officials. No price war occurred after 1901, perhaps because the refiners "buried the hatchet," a phrase attributed to John Arbuckle who entered the sugar business in 1898 and became the monopoly's main competitor.

The monopoly's market share declined from 95% in 1892 to 85% in 1894, 77% in 1902, and 70% in 1905. There were 12 cane refining companies in 1905 who supplied 96% of the market.[3] The monopoly effectively ceased to exist in 1914 when the federal government dropped its lawsuit under Sherman Act because of the monopoly's declining market share and because of its involuntary partial dissolution. See Eichner (1969), Zerbe (1969), and Genesove and Mullin (1997, 1998) for more detailed descriptions.

Genesove and Mullin (1998) reported that the marginal cost of sugar production remained roughly a constant during 1887-1914, and they estimated it as equal to

$$c = 0.26 + 1.075\, P_{raw}\,,$$

where P_{raw} is the price of raw sugar in 1898 dollar. Our estimation of the sugar monopoly's merging cost is based on the above value of marginal cost and the four types of demand

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[3] The remaining 4% of the market were beet sugar supplied by about 57 beet factories owned by perhaps 30 companies. As reported in Zerbe (1969), the monopoly's control of beet factories declined from 70% in 1905 to 60% in 1907 and 54% in 1911, and in the same time, beet factories' share of sugar market increased from 4% in 1905 to 12% in 1907 and 14% in 1911, respectively.

estimated by the same authors. It is important to keep in mind that the estimation in this paper and in Genesove and Mullin (1998) exclude sugars supplied by fringe refineries, so the demand function used in both studies is actually the difference between the entire market demand for sugar and fringe firms' supply of sugar.

3. Description of the Model

There is a representative consumer with the following quasi-linear utility function:

$$u(Q, I) = aQ - \frac{b\gamma Q^{(1+\gamma)/\gamma}}{1+\gamma} + I,$$

where Q is output consumption, I is a composite measure of all other consumptions, $a > 0$ and $b\gamma > 0$. Maximizing $u(Q, I)$ subject to the budget constraint $pQ+I \leq M$ leads to the following inverse demand function $P(Q) = a - bQ^{1/\gamma}$, of which the demand form is

$$Q(P) = \beta(a - P)^{\gamma}, \tag{1}$$

where $\beta = b^{-\gamma} > 0$ represents demand size, γ is an index of convexity, and a is the maximum willingness to pay when $\gamma > 0$. The above form (1) includes four types of empirically-relevant demand functions: linear ($\gamma = 1$), quadratic ($\gamma = 2$), log-linear ($a = 0$ and $\gamma < 0$), and exponential (a, $\gamma \to \infty$ and a/γ constant). Our estimation of monopoly merging costs for the 1890-1914 American sugar monopoly are based on the estimated demand forms in Genesove and Mullin (1998), which are reproduced in Table 1.

Assume that each firm i ($i = 1,\ldots,n$) has a linear cost function $C_i(q_i) = c_i q_i$, $0 \leq q_i \leq z_i$, where $c_i > 0$ is its constant marginal (and unit) cost, and $z_i > 0$ is its capacity. Without loss of generality, assume $c_1 \leq c_2 \leq \ldots \leq c_n$, so firm 1 is the most efficient firm, and firm n is the least efficient firm. For each output vector $q = \{q_1, q_2, \ldots, q_n\}$, firm i's profit is given by

$$\pi_i(q) = [a - c_i - b(\Sigma q_j)^{1/\gamma}]q_i, \text{ all } i. \tag{2}$$

The above description of Cournot oligopoly can be alternatively defined as a normal form game given below:

$$\Gamma = \{N, Z_i, \pi_i\}, \tag{3}$$

where for each firm $i \in N$, its feasible set is $Z_i = [0, z_i]$, with $z_i > 0$ as its production capacity, and its payoff function is $\pi_i(q)$ given in (2).

We now define the pre- and postmerger equilibrium for the monopoly merger and its core. The following assumption *A0* is assumed throughout the paper:

A0: *For each merger $S \subseteq N$, its premerger and postmerger equilibria are unique and are interior solutions, and its capacity and cost function are given by*

$$z_S = \Sigma_{j \in S} z_j, \text{ and} \tag{4}$$

$$C_S(q) = Min\ \{\Sigma_{j \in S} C_j(x_j) \mid q = \Sigma_{j \in S} x_j \leq z_S, x_j \geq 0, j \in S\}. \tag{5}$$

A0 assumes that each merger involves both the exit of all inefficient members and an increase in the efficient member's capacity (up to $z_S = \Sigma_{j \in S} z_j$).

Solving the first order conditions yields the Cournot equilibrium given below:

$$q_i = \frac{\beta\gamma\,[n\gamma(a-\bar{c})]^{(\gamma-1)}}{(1+n\gamma)^{(\gamma-1)}}\,\frac{a-(1+n\gamma)c_i+n\gamma\bar{c}}{1+n\gamma}, \text{ all } i, \tag{6}$$

where $\bar{c} = (\Sigma c_j)/\text{n}$ is the average marginal costs, and $\beta = b^{-\gamma}$. It is straightforward to check that the total supply, price level, individual markup, and individual profits at the Cournot equilibrium are equal to the values given below:

$$Q = \Sigma q_j = \beta\left\{\frac{n\gamma(a-\bar{c})}{1+n\gamma}\right\}^{\gamma},\ P = P(Q) = \frac{a+n\gamma\bar{c}}{1+n\gamma},$$

$$P - c_i = \frac{a-(1+n\gamma)c_i+n\gamma\bar{c}}{1+n\gamma}, \text{ and} \tag{7}$$

$$\pi_i = \frac{(1+n\gamma)^{(\gamma-1)}q_i^{\,2}}{\beta\gamma\,[n\gamma(a-\bar{c})]^{(\gamma-1)}} = \frac{\beta\gamma\,[n\gamma(a-\bar{c})]^{(\gamma-1)}}{(1+n\gamma)^{(\gamma-1)}}\left\{\frac{a-(1+n\gamma)c_i+n\gamma\bar{c}}{1+n\gamma}\right\}^2.$$

Note that the markup-supply ratio for each firm i is identical and equal to the absolute value of the slope of inverse demand: $(P - c_i)/q_i = -P(Q)' = (1+n\gamma)^{(\gamma-1)}/\{\beta\gamma\,[n\gamma(a-\bar{c})]^{(\gamma-1)}\}$, all i .

It follows from A0 and from (7) that the monopoly supply, price and profits are:

$$Q_M = \beta\{\frac{\gamma(a-c_1)}{1+\gamma}\}^{\gamma},\ P_M = \frac{a+\gamma c_1}{1+\gamma},$$
$$\pi_M = \frac{(1+\gamma)^{(\gamma-1)}Q_M^2}{\beta\gamma\,[\gamma(a-c_1)]^{(\gamma-1)}} = \frac{\beta\gamma^{\gamma}(a-c_1)^{(\gamma+1)}}{(1+\gamma)^{(\gamma+1)}}, \tag{8}$$

where $c_1 = Min\{c_i \mid i \in N\}$ is the minimum of all marginal costs.

Denote a monopoly merger contract as a triplet (N, Q_M, θ), where N is the set of firms, Q_M is monopoly output in (8), and θ is a split of the monopoly profits (i.e., $\theta \geq 0,\ \Sigma\theta_i = \pi_M$). Now, the core of market (3) or the core of the monopoly merger is defined below. For each coalition or non-monopoly merger $S \neq N$, its guaranteed profit is given by

$$v(S) = \underset{x_S}{Max}\ \underset{y_T}{Min}\ \Sigma_{i\in S}\pi_i(x_S, y_T) = \underset{y_T}{Min}\ \underset{x_S}{Max}\Sigma_{i\in S}\pi_i(x_S, y_T), \tag{9}$$

where $T = \{i \mid i \notin S\}$ is the set of outsiders, $(x_S, y_T) = (x_S, y_{-S})$ is a vector $w \in \boldsymbol{R}^n$ with $w_i = x_i$ if $i \in S$, $= y_i$ if $i \notin S$; the feasible regions for x_S and y_T are $\{x_S \in \boldsymbol{R}_+^S \mid \Sigma_{j\in S} x_j \leq z_S\}$ and $Z_T = \Pi_{j\in T} Z_j$.

A profit vector $\theta \in \boldsymbol{R}_+^n$ is in the core[4] of market (3) if it is a split of the monopoly profits and if it gives each coalition S no less than its guaranteed profits $v(S)$ (i.e., if $\Sigma\theta_i = \pi_M$ and $\Sigma_{i\in S}\theta_i \geq v(S)$ for all $S \neq N$). Let $Core(\Gamma)$ denote the set of all core vectors for (3), q and $\pi_j(q)$ be the premerger equilibrium and profits given in (6)-(7). Our estimation of monopoly merging costs is based on the following two preconditions for a monopoly merger:

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[4] Note that in general situations, one would have

$$v_{\alpha}(S) = \underset{x_S}{Max}\ \underset{y_T}{Min}\ \Sigma_{i\in S}\pi_i(x_S, y_T) < v_{\beta}(S) = \underset{y_T}{Min}\ \underset{x_S}{Max}\Sigma_{i\in S}\pi_i(x_S, y_T),$$

which implies β-core $\subseteq$ α-core (Aumann, 1959), where the α- and β-core are defined by replacing the above $v(S)$ by $v_{\alpha}(S)$ or $v_{\beta}(S)$. As shown in Zhao (1999), α-core = β-core always holds in oligopoly markets, so there is no need to make the α- and β-distinction here, and one can simply use the term *core*.

Proposition 1 (**Necessary conditions for the monopoly merger**, Zhao [2009]): *Suppose that a monopoly merger* (N, Q_M, θ) *has been observed. Then, the following two preconditions hold: (i)* $\theta_j \geq \pi_j(q)$ *for all j; and (ii)* $\theta \in Core(\Gamma)$.

As commented in Zhao (2009), at least one firm is worse off with the merger if the profitability precondition in part (i) fails, and at least one blocking coalition is worse off with the merger if the non-empty core precondition in part (ii) fails. Both conditions hold because the failure of either would be absurd under profit-seeking behavior. The two preconditions are independent of each other, as shown in the following two examples from Zhao (2009):

Example 1: $n = 3$, $P = 6-\Sigma q_j$, $C_i(q_i) = 0.8q_i$, $0 \leq q_i \leq 1.5$, $i = 1, 2, 3$. The premerger and monopoly profits are $\pi_i \equiv 1.69$, $\pi_m = 6.76$. Assume that monopoly merging cost is $MCM = 1.65$. Then, one can show: there is an incentive to merge ($v(123) = (\pi_m - MCM) = 5.11 > \Sigma\pi_j = 5.07$), and the core is empty.

Example 2: $n = 3$, $P = 6-\Sigma q_j$, $C_i(q_i) = 0.5q_i$, $0 \leq q_i \leq 2$, $i = 1, 2, 3$. The premerger and monopoly profits are $\pi_i \equiv 1.89$, $\pi_m = 7.56$. Assume that monopoly merging cost is $MCM = 2$. Then, one can show: there is no incentive to merge ($v(123) = (\pi_m - MCM) = 5.56 < \Sigma\pi_j = 5.67$), and core is non-empty.

The non-empty core condition could be easily misunderstood as below. Under *A0*, $v(S)$ in (9) is equal to $Max\{\Sigma_{i\in S}\pi_i(x_S, z_{-S})|x_S\}$, so outsiders produce at full capacity z_{-S}. Because producing z_{-S} is not credible, the non-empty core condition is of no use. Such argument, however, derives from a misunderstanding of the core. The core *does not require any S to produce* z_{-S}. Instead, it requires that total profits be maximized and divided in such a way that each $S \neq N$ *receives no less than* its worst profits $v(S)$. Given z_{-S}, the above $v(S)$ is typically small, implying that the core is possibly large. However, the large size of the core strengthens, rather than weakens, the non-empty core precondition that $\theta \in Core(\Gamma)$ holds.

A short paragraph about new developments on the refinement of core will be added here before conference presentation.

4. Estimation of Monopoly Merging Costs with Linear Demand

Applying the non-empty core precondition requires calculation of the core. I use the MNBP (minimum no-blocking payoff) method of Zhao (2001) in such calculation. The payoffs in (8)-(9) define the following coalitional transferable utility (TU) game:

$$\Gamma = \{N, v\}, \tag{10}$$

where the payoff $v(N)$ for the grand coalition is given in (8), and the payoff $v(S)$ for each $S \neq N$ is given in (9). Given the game (10), its minimum no-blocking payoff is defined as

$$MNBP = Min\ \{\Sigma_{i\in N}\theta_i \mid \theta\in \mathbf{R}^n_+;\ \Sigma_{i\in S}\ \theta_i \geq v(S) \text{ for all } S \neq N\}. \tag{11}$$

Then, it is straightforward to show that the core (the core's interior) is non-empty if and only if $MNBP \leq v(N)$ ($MNBP < v(N)$) holds. The core is empty in Example 1, because $v(123) = 5.11 < MNBP = 5.13$; the core is non-empty and has a non-empty interior in Example 2, because $v(123) = 5.56 > MNBP = 4.59$.

Proposition 2: *Let* $\pi_j(q)$ *and* π_M *be the profits in (7)-(8), MNBP be given in (11), and let the terms* MCM^* *and* MCM_0 *be defined as below:*

$$MCM^* = \pi_M - Max\{\Sigma\pi_j(q), MNBP\}, \text{ and} \tag{12}$$

$$MCM_0 = \pi_M - Min\{\Sigma\pi_j(q), MNBP\}. \tag{13}$$

Then, the merging cost for a completed monopoly merger is at most MCM^**, and the merging cost for an unobserved monopoly merger (or a dissolved monopoly) is at least* MCM_0*.*

The above proposition refines earlier estimations in Zhao (2009), which reported the above upper bound (i.e., first part of Proposition 4 in Zhao [2009]) and a preliminary lower bound (i.e., second of Proposition 4 in Zhao [2009]) whose value is higher than that of (13).

As readers will see, the main task of computing the above bounds for monopoly merging costs is to compute MNBP in (11) based on the payoffs in (2) and the values in (9). The American sugar industry during 1890-1914 with linear demand (see Genesove and Mullin [1998]) can be modeled by Assumption 1 below:

A1: i) $Q(P) = \beta(a-P)$; and ii) $c_j = c > 0$, $z_j = z > 0$, all j.

Part ii) of **A1** assumes that firms have symmetric marginal cost and symmetric capacity. Under **A0**-**A1**, simplifying (7)-(8) and applying the known result with $b = 1$ in Zhao (2009) lead to the following results:

Proposition 3: *Under* ***A0*** *and* ***A1****, firm i's premerger profits in (7), the monopoly profit in (8), and the MNBP in (11) are equal to values given below:*

$$\pi_i = \beta(a-c)^2/(n+1)^2,\ \pi_M = \beta(a-c)^2/4, \text{ and } \\ mnbp(\Gamma) = n\beta(a-c-z/\beta)^2/[4(n-1)]. \tag{14}$$

Proposition 4 below provides estimations of the lower (upper) bounds of merging costs for observed (dissolved) monopoly mergers in all linear Cournot oligopolies with symmetric costs and symmetric capacity.[5]

Proposition 4: *Let* $\tau_1 = n-2\sqrt{n-1}$, $\tau_2 = n+2\sqrt{n-1}$, and $\tau \geq 0$ *be the rate of excessive capacity defined by* $z = (1+\tau)\beta(a-c)/(n+1)$. *Then, under the same assumptions of Proposition 3, the upper bound MCM* in (12) and lower bound* MCN_0 *in (13) are equal to*

$$MCM^* = \begin{cases} \dfrac{n\beta(a-c)^2}{(n+1)^2}\,\dfrac{(n-1)(n+1)^2-n(n-\tau)^2}{4n(n-1)} & \text{if } \tau < \tau_1 \text{ or } \tau > \tau_2 \\[2ex] \dfrac{n\beta(a-c)^2}{(n+1)^2}\,\dfrac{(n-1)^2}{4n} & \text{if } \tau_1 \leq \tau \leq \tau_2; \end{cases} \tag{15}$$

$$MCM_0 = \begin{cases} \dfrac{n\beta(a-c)^2}{(n+1)^2}\,\dfrac{(n-1)^2}{4n} & \text{if } \tau < \tau_1 \text{ or } \tau > \tau_2 \\[2ex] \dfrac{n\beta(a-c)^2}{(n+1)^2}\,\dfrac{(n-1)(n+1)^2-n(n-\tau)^2}{4n(n-1)} & \text{if } \tau_1 \leq \tau \leq \tau_2. \end{cases} \tag{16}$$

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[5] Note that MCM* in (15) becomes the earlier estimation in Zhao (2009) when $\beta = 1$ and $\tau < \tau_1$ (i.e., Proposition 5 in Zhao [2009]).

Table 1. Estimated Bounds of Monopoly Merging Costs with Linear Demand
(as a proportion of **premerger industry profits**, determined by (15) and (16))

	Upper Bounds for Successful Monopoly*						**Lower Bounds for Unobserved Monopoly****					
tau n	0.00	0.10	0.20	0.30	0.50	1.00	0.00	0.10	0.20	0.30	0.50	1.00
2	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.22	0.32	0.40	0.56	0.88
3	0.21	0.28	0.33	0.33	0.33	0.33	0.33	0.33	0.35	0.42	0.55	0.83
4	0.23	0.30	0.36	0.42	0.54	0.56	0.56	0.56	0.56	0.56	0.56	0.81
5	0.24	0.30	0.36	0.42	0.53	0.80	0.80	0.80	0.80	0.80	0.80	0.80
6	0.24	0.30	0.36	0.42	0.53	0.79	1.04	1.04	1.04	1.04	1.04	1.04
7	0.24	0.30	0.36	0.42	0.53	0.79	1.29	1.29	1.29	1.29	1.29	1.29
8	0.25	0.30	0.36	0.41	0.52	0.78	1.53	1.53	1.53	1.53	1.53	1.53
9	0.25	0.30	0.36	0.41	0.52	0.78	1.78	1.78	1.78	1.78	1.78	1.78
10	0.25	0.30	0.36	0.41	0.52	0.78	2.03	2.03	2.03	2.03	2.03	2.03
11	0.25	0.30	0.36	0.41	0.52	0.77	2.27	2.27	2.27	2.27	2.27	2.27
12	0.25	0.30	0.36	0.41	0.52	0.77	**2.52**	2.52	2.52	2.52	2.52	2.52
13	0.25	0.30	0.36	0.41	0.51	0.77	2.77	2.77	2.77	2.77	2.77	2.77
14	0.25	0.30	0.36	0.41	0.51	0.77	3.02	3.02	3.02	3.02	3.02	3.02
15	0.25	0.30	0.36	0.41	0.51	0.77	3.27	3.27	3.27	3.27	3.27	3.27
16	0.25	0.30	0.35	0.41	0.51	0.77	3.52	3.52	3.52	3.52	3.52	3.52
17	0.25	0.30	0.35	0.41	0.51	0.76	3.76	3.76	3.76	3.76	3.76	3.76
18	0.25	0.30	**0.35**	0.41	0.51	0.76	4.01	4.01	4.01	4.01	4.01	4.01
19	0.25	0.30	0.35	0.41	0.51	0.76	4.26	4.26	4.26	4.26	4.26	4.26
20	0.25	0.30	0.35	0.41	0.51	0.76	4.51	4.51	4.51	4.51	4.51	4.51
21	0.25	0.30	0.35	0.41	0.51	0.76	4.76	4.76	4.76	4.76	4.76	4.76
22	0.25	0.30	0.35	0.41	0.51	0.76	5.01	5.01	5.01	5.01	5.01	5.01
23	0.25	0.30	0.35	0.41	0.51	0.76	5.26	5.26	5.26	5.26	5.26	5.26
24	0.25	0.30	0.35	0.41	0.51	0.76	5.51	5.51	5.51	5.51	5.51	5.51
25	0.25	0.30	0.35	0.40	0.51	0.76	5.76	5.76	5.76	5.76	5.76	5.76
50	0.25	0.30	0.35	0.40	0.50	0.76	12.01	12.01	12.01	12.01	12.01	12.01
75	0.25	0.30	0.35	0.40	0.50	0.75	18.25	18.25	18.25	18.25	18.25	18.25
100	0.25	0.30	0.35	0.40	0.50	0.75	24.50	24.50	24.50	24.50	24.50	24.50

Table 1 shows the values of (15) and (16) as a proportion of premerger total profits (i.e., MCM^* and MCM_0 divided by $n\beta(a-c)^2/(n+1)^2$), which are invariant for all possible values of the parameter (a, β, c). Because β is canceled out, the left half of the table are the same as those in Table 1 of Zhao (2009). Note also that the table has excluded the case of $\tau > \tau_2$, which is out of range of our sugar monopoly.

To find the bounds of monopoly merging costs for our sugar industry, recall that the monopoly consolidated 18 refineries and the excess capacity rate was about 20%. Finding the row with n = 18 and the left column with τ = 20% gives the number 0.35 in bold face, indicating that the sugar monopoly's merging costs were at most 35% of pre-merger total profit at its formation in 1887. There were about 12 refineries in 1914 which operated at near capacity level. Finding the row with n = 12 and the right column with τ = 0% gives the number 2.52 in bold face, indicating that the sugar monopoly's merging costs, or the cost of avoiding dissolution and keeping the monopoly operation, were at least 252% of post-dissolution total profit at its dissolution in 1914.[6]

5. Estimation of Monopoly Merging Costs with Quadratic Demand

The American sugar industry during 1890-1914 with quadratic demand (see Genesove and Mullin [1998]) can be modeled by Assumption 2 below:

A2: i) $Q(P) = \beta(a-P)^2$; and ii) $c_j = c > 0,\ z_j = z > 0$, all j.

Under **A0** and **A2**, substituting $\gamma = 2$ into (7)-(8) and simplifying the expressions lead to pre- and post merger profits as given below. The expression for MNBP in (18) below is obtained by analyzing and solving the linear programming problem (11).

Proposition 5: *Under **A0** and **A2**, firm i's premerger profits in (7), the monopoly profit in (8), and the MNBP in (11) are equal to values given below:*

$$\pi_i = 4n\beta(a-c)^3/(2n+1)^3,\ \pi_M = 4\beta(a-c)^3/27,\ \text{and} \tag{17}$$

$$mnbp = \frac{2n\beta(a-c)^3}{27(n-1)}\left[2-\left(1+\frac{12n(1+\tau)}{(2n+1)^2}\right)^{0.5}\right]^2\left[1+\left(1+\frac{12n(1+\tau)}{(2n+1)^2}\right)^{0.5}\right], \tag{18}$$

where τ is the rate of excessive capacity defined by $z = 4n\beta(1+\tau)(a-c)^2/(2n+1)^2$.

Proposition 6 below shows that the core is non-empty and has a non-empty (relataive) interior in Cournot oligopolies with linear costs and quadratic demand, denoted as a $2(n+1)$-

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[6] A footnote with dollar estimations will be added in the final version.

vector $(a,\beta,c,z) \in \mathbf{R}^{2n+2}_{++}$ with $c=(c_1,..., c_n)$, $z=(z_1, ..., z_n)$, whose the demand and cost functions are: $Q(p)= \beta(a-p)^2$, $C_i(q_i) = c_i q_i$, $0\leq q_i \leq z_i$, all i.

Proposition 6: *Let the market (3) be given by* $(a,\beta,c,z)\in \mathbf{R}^{2n+2}_{++}$, *and assume* ***A0***. *Then, the core of (3) is non-empty and has a non-empty (relative) interior.*

Proposition 7 below provides estimations of the lower (upper) bounds of merging costs for observed (dissolved) monopoly mergers in Cournot oligopolies with quadratic demand, symmetric linear costs and symmetric capacity.

Proposition 7: *Under the same assumptions of Proposition 5, let r be defined as* $r = [1+12n(1+\tau)/(2n+1)^2]^{0.5}$, *and* $h(n,\tau)$ *be defined as*

$$h(n,\tau) = 2n - \frac{(2n+1)^3(2-r)^2(1+r)}{27(n-1)}. \tag{19}$$

Then, the upper bound MCM in (12) and lower bound* MCN_0 *in (13) are equal to*

$$MCM^* = \begin{cases} \dfrac{2\beta(a-c)^3[2(n-1)- n(2-r)^2(1+r)]}{27(n-1)} & \text{if } h(n,\tau) < 0 \\[2ex] \dfrac{4\beta(a-c)^3[(2n+1)^3- 27n^2]}{27(2n+1)^3} & \text{if } h(n,\tau) \geq 0\,; \end{cases} \tag{20}$$

$$MCM_0 = \begin{cases} \dfrac{4\beta(a-c)^3[(2n+1)^3- 27n^2]}{27(2n+1)^3} & \text{if } h(n,\tau) < 0 \\[2ex] \dfrac{2\beta(a-c)^3[2(n-1)- n(2-r)^2(1+r)]}{27(n-1)} & \text{if } h(n,\tau) \geq 0\,. \end{cases} \tag{21}$$

Table 2. Estimated Bounds of Monopoly Merging Costs with Quadratic Demand
(as a proportion of **premerger industry profits**, determined by (20) and (21))

	Upper Bounds for Successful Monopoly*						**Lower Bounds for Unobserved Monopoly****					
tau n	0.00	0.10	0.20	0.30	0.50	1.00	0.00	0.10	0.20	0.30	0.50	1.00
2	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.35	0.43	0.59	0.89
3	0.27	0.35	0.41	0.41	0.41	0.41	0.41	0.41	0.42	0.49	0.63	0.91
4	0.30	0.37	0.44	0.51	0.64	0.69	0.69	0.69	0.69	0.69	0.69	0.93
5	0.32	0.39	0.46	0.52	0.65	0.94	1.0	0.97	0.97	0.97	0.97	0.97
6	0.33	0.40	0.46	0.53	0.66	0.95	1.3	1.26	1.26	1.26	1.26	1.26
7	0.33	0.40	0.47	0.54	0.66	0.96	1.6	1.55	1.55	1.55	1.55	1.55
8	0.34	0.41	0.47	0.54	0.67	0.97	1.8	1.84	1.84	1.84	1.84	1.84
9	0.34	0.41	0.48	0.54	0.67	0.97	2.1	2.14	2.14	2.14	2.14	2.14
10	0.35	0.41	0.48	0.55	0.67	0.98	2.4	2.43	2.43	2.43	2.43	2.43
11	0.35	0.42	0.48	0.55	0.68	0.98	2.7	2.72	2.72	2.72	2.72	2.72
12	0.35	0.42	0.48	0.55	0.68	0.99	**3.0**	3.02	3.02	3.02	3.02	3.02
13	0.35	0.42	0.49	0.55	0.68	0.99	3.3	3.31	3.31	3.31	3.31	3.31
14	0.35	0.42	0.49	0.55	0.68	0.99	3.6	3.61	3.61	3.61	3.61	3.61
15	0.35	0.42	0.49	0.55	0.68	1.00	3.9	3.90	3.90	3.90	3.90	3.90
16	0.36	0.42	0.49	0.55	0.68	1.00	4.2	4.20	4.20	4.20	4.20	4.20
17	0.36	0.42	0.49	0.55	0.68	1.00	4.5	4.49	4.49	4.49	4.49	4.49
18	0.36	0.42	**0.49**	0.56	0.69	1.00	4.8	4.79	4.79	4.79	4.79	4.79
19	0.36	0.42	0.49	0.56	0.69	1.00	5.1	5.09	5.09	5.09	5.09	5.09
20	0.36	0.43	0.49	0.56	0.69	1.00	5.4	5.38	5.38	5.38	5.38	5.38
21	0.36	0.43	0.49	0.56	0.69	1.01	0.4	0.43	0.49	0.56	0.69	1.01
22	0.36	0.43	0.49	0.56	0.69	1.01	0.4	0.43	0.49	0.56	0.69	1.01
23	0.36	0.43	0.49	0.56	0.69	1.01	0.4	0.43	0.49	0.56	0.69	1.01
24	0.36	0.43	0.49	0.56	0.69	1.01	0.4	0.43	0.49	0.56	0.69	1.01
25	0.36	0.43	0.49	0.56	0.69	1.01	6.9	6.86	6.86	6.86	6.86	6.86
50	0.37	0.43	0.50	0.56	0.70	1.02	14.3	14.26	14.26	14.26	14.26	14.26
75	0.37	0.43	0.50	0.57	0.70	1.03	21.7	21.67	21.67	21.67	21.67	21.67
100	0.37	0.43	0.50	0.57	0.70	1.03	29.1	29.08	29.08	29.08	29.08	29.08

Table 2 shows the values of (20) and (21) as a proportion of premerger total profits (i.e., $MCM*$ and MCM_0 divided by $4n^2\beta(a-c)^3/(2n+1)^3$), which remain constant for all possible values of (a, β, c). To find the bounds of monopoly merging costs in percentage terms for each industry with quadratic demand and symmetric linear cost, one only need to know the number of firms and the excess capacity rate.

In our sugar industry, the monopoly consolidated 18 refineries (the dissolution resulted in 12 refineries) and the excess capacity rate was about 20% (0%) in 1887 (in 1914). Finding the row with n = 18 (n = 12) and the left (right) column with τ = 20% (τ = 0%) gives the number 0.49 (3.0) in bold face, so the sugar monopoly's merging costs increased from at most 49% of pre-merger total profit at its formation in 1887 to at least 300% of post-dissolution total profit at its dissolution in 1914.

Comparing the above estimations with the earlier ones under linear demand, one sees that the estimated bounds of monopoly merging costs in percentage terms under quadratic demand are higher than under linear demand. However, the comparison between the estimated bounds of monopoly merging costs in dollars under quadratic and linear demands is ambiguous, because it is not clear under which demand assumption the premerger (or monopoly) profits are higher.[7]

6. Conclusion

The above analysis has provided percentage estimations for the lower (upper) bound of merging costs of observed (dissolved) monopoly in all Cournot oligopolies with linear costs and linear or quadratic demands. Applying the result to the Sugar monopoly, it showed that the enforcement of Sherman act increased, for example with linear cost and demand, its merging costs from at most 35% of pre-merger total profit at its formation in 1887 to at least 252% of post-dissolution total profits at its dissolution in 1914. Although it is not possible to obtain such general estimations with loglinear and exponential demands, the method made it possible to obtain numerical estimations with such demand forms based on known cost and demand forms, such as those in Genesove and Mullin (1998).

Because the estimation method can be applied in a straightforward manner empirically to other monopolies and theoretically to differentiated multi-product nonlinear Cournot (or Bertrand) oligopolies, readers are left with a rich source of future empirical and

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[7] By comparing (17) with (14), one sees that monopoly profits are higher under quadratic demand than under linear demand if and only if $(a-c) > 27/16$, and individual premerger profits are higher under quadratic demand than under linear demand if and only if $(a-c) > (2n+1)^3/[4n(2n+1)^2]$.

theoretical studies on estimating merging costs. The author hopes readers will be encouraged to empirically estimate the impact of enforcing antitrust regulations on monopoly merging costs in all sectors in which the cost and demand forms have been estimated or can be estimated in future studies.

Appendix

Proof of Proposition 2: It is similar to that of Proposition 4 in Zhao (2009). **Q.E.D**

Proof of Proposition 3: Simplifying (7)-(8) leads to each firm *i*'s profit and monopoly profit given in (14). To derive the formula for MNBP, we use the known results for $b = 1$ in Zhao (2009). First, rearranging the payoff in (2) and using A1 yields:

$$\pi_i(q) = b\,[(a - c)/b - \Sigma q_j]q_i.$$

By the above expression, equilibrium output with $P(Q) = a - bQ$ can be obtained from earlier output with $P(Q) = A - Q$ by replacing $(A\text{-}c)$ in Zhao (2009) with $(a\text{-}c)/b$. Then, equilibrium profit with $P(Q) = a - bQ$ can be obtained by multiplying b to earlier profit with $P(Q) = A - Q$ and by replacing $(A\text{-}c)$ with $(a\text{-}c)/b$.

Hence, earlier formula $v(S) = (A - c - (n-k)z)^2/4$ (equation (17) in Zhao [2009], where $k = |S|$ is number of firms in S) becomes

$$v(S) = b((a - c)/b - (n-k)z)^2/4 = [a - c - (n-k)zb]^2/[4b]$$
$$= \beta[a - c - (n-k)z/\beta]^2/4.$$

Keep in mind that $\beta = 1/b$. Similarly, earlier formula $mnbp(\Gamma) = n(A - c - z)^2/[4(n-1)]$ (i.e., (19) in Zhao [2009]) becomes

$$mnbp(\Gamma) = nb((a - c)/b - z)^2/[4(n-1)] = n(a - c - zb)^2/[4(n-1)b]$$
$$= n\beta(a - c - z/\beta)^2/[4(n-1)],$$

which is the value (14). **Q.E.D**

Proof of Proposition 4: Substituting (14) and $z = (1+\tau)\beta(a\text{-}c)/(n+1)$ into $[MNBP\text{-}\Sigma\pi_j(x)]$ and simplifying the expression, one sees that the sign of $[MNBP\text{-}\ \Sigma\pi_j(x)]$ is given by the sign of $[\tau^2\text{-}2n\tau+(n\text{-}2)^2]$, whose two roots are $\tau_1 = n\text{-}2\sqrt{n\text{-}1}$ and $\tau_2 = n+2\sqrt{n\text{-}1}$. Therefore,

$$Max\{\Sigma\pi_j(q),\ MNBP\} = \begin{cases} MNBP & \text{if } \tau<\tau_1 \text{ or } \tau>\tau_2 \\ \Sigma\pi_j(q) & \text{if } \tau_1\le\tau\le\tau_2; \end{cases}$$

$$Min\{\Sigma\pi_j(q),\ MNBP\} = \begin{cases} \Sigma\pi_j(q) & \text{if } \tau<\tau_1 \text{ or } \tau>\tau_2 \\ MNBP & \text{if } \tau_1\le\tau\le\tau_2. \end{cases}$$

Substituting (14) and the above expressions into (12)-(13) and simplifying, one obtains the results of (15) and (16). **Q.E.D**

Proof of Proposition 5: It is straightforward to obtain the premerger and monopoly profits. The expression for mnbp is obtained by establishing the following three lemmas. **Q.E.D**

For any $1\le k\le n$, let $S(k) = \{T\subseteq N \mid |T| = k\}$ denote the set of coalitions who have precisely k members. Lemma 1 below obtains the expression of $v(S)$ for all $S\neq N$.

Lemma 1: *Under A0 and A2, for each k = 1, ..., n-1, the guaranteed profit for all* $S\in S(k)$ *is equal to*

$$v(k) = \frac{2\beta(a-c)^3}{27}\left[2-\left(1+\frac{12n(1+\tau)(n-k)}{(2n+1)^2}\right)^{0.5}\right]^2\left[1+\left(1+\frac{12n(1+\tau)(n-k)}{(2n+1)^2}\right)^{0.5}\right], \tag{22}$$

where τ *is the rate of excessive capacity defined by* $z = 4n\beta(1+\tau)(a-c)^2/(2n+1)^2$.

Proof of Lemma 1: Under A2, the profit in (2) becomes $\pi_i(q) = [a-c-b(\Sigma q_j)^{1/2}]q_i$, all i. So the maximand of the maximization problem (9) for each S becomes

$$f(y)= (a-c-\beta^{0.5}(q+(n-k)z)^{0.5})q = (a-c-\beta^{0.5}y)(y^2-(n-k)z),$$

where $y^2=q+(n-k)z$. Solving the first order condition for $Max\{f(y)\,|\,y\ge 0\}$ yields a unique maximum equal to

$$y_2 = \frac{\beta^{0.5}(a-c)}{3}\left[1+\left(1+\frac{12n(1+\tau)(n-k)}{(2n+1)^2}\right)^{0.5}\right].$$

Substituting y_2 into f(y), simplifying and factoring yield the expression in (22). **Q.E.D**

Now, for each $k = 1, ..., n-1$, consider the minimum value defined by

$$MV(k) = \{Min\ \Sigma_{i\in N}x_i \mid x\in \mathbf{R}^n_+;\ \Sigma_{i\in S}\, x_i \ge v(k),\ \text{for all } S \in S(k)\}. \tag{23}$$

The next lemma provides a formula for each $MV(k)$ and their relations.

Lemma 2: Under *A0 and A2*, the following two claims hold. (i) $MV(k) = nv(k)/k$; and (ii) $MV(k+1) \ge MV(k)$, for $k = 1, ..., n-1$.

Proof of Lemma 2: Part (i). (23) has $\binom{n}{k}= n\,(n-1)...(n-k+1)/k!$ constraints, so $v(k)$ on the right-hand side appears $n(n-1)\dots(n-k+1)/k!$ times. Each x_i on the left-hand side appears $\binom{n-1}{k-1} = (n-1)\dots(n-k+1)/(k-1)!$ times. By summing over all constraints, one has

$$[(n-1)\dots(n-k+1)/(k-1)!]\ \Sigma x_i \geq [n\dots(n-k+1)/k!]\ v(k), \text{ or } \Sigma x_i \geq n\ v(k)/k.$$

Hence, the minimum value of (23) is equal to $MV(k) = n\ v(k)/k$.

Part (ii). By the above proof of Lemma 1, the maximands for $v(k)$ and $v(k+1)$ can be rewritten respectively as

$$f(q;k)= f(y;k)= (a-c-\beta^{0.5}y)(y^2-(n-k)z), \text{ and}$$

$$f(q;k+1)= f(y;k+1)= (a-c-\beta^{0.5}y)(y^2-(n-k-1)z).$$

We first evaluate the sign of $g(k)=(y^2-(n-k)z)/k-(y^2-(n-k-1)z)/(k+1)$. Substituting $y^2=q+(n-k)z$ into

$$g(k) = (y^2-nz)/[k(k+1)] = (y^2-nz)/[k(k+1)],$$

one has $g(k) = (q-kz)/[k(k+1)]$. Since the constraint for the maximization problem (9) is $q \leq kz$,

$$g(k) \leq 0 \text{ or } f(y;k)/k \leq f(y;k+1)/(k+1) \tag{24}$$

holds. It follows from (24) that

$$Max\{f(y;k)/k \mid y \geq 0\} = Max\{f(y;k) \mid y \geq 0\}/k = v(k)/k$$
$$\leq v(k+1)/(k+1) = Max\{f(y;k+1)/(k+1) \mid y \geq 0\}.$$

By part (i), part (ii) of the lemma holds. **Q.E.D**

Lemma 3: Under *A0 and A2*, the *MNBP* defined in (11) is given by

$$mnbp = \frac{2n\beta(a-c)^3}{27(n-1)}\left[2-\left(1+\frac{12n(1+\tau)}{(2n+1)^2}\right)^{0.5}\right]^2\left[1+\left(1+\frac{12n(1+\tau)}{(2n+1)^2}\right)^{0.5}\right]. \tag{25}$$

Proof of Lemma 3: Let *FR* and *FR(k)* below denote the "feasible region" in (11) and (23):

$$FR = \{x \in \boldsymbol{R}^n_+ \mid \Sigma_{i \in S} x_i \geq v(k), \text{ for all } S \in S(k),\ k = 1, \dots, n-1\},$$

$$FR(k) = \{x \in \boldsymbol{R}^n_+ \mid \Sigma_{i \in S} x_i \geq v(k), \text{ for all } S \in S(k)\},\ k = 1, \dots, n-1.$$

The above two expressions lead to

$$FR = \cap^{n-1}_{i=1} FR(k). \tag{26}$$

Now, for each $k = 1, \dots, n-1$, define

$$FR(k)^* = \{x \in \boldsymbol{R}^n_+ \mid \Sigma_{i \in N} x_i \geq n\ v(k)/k\}.$$

Using arguments similar to those in the proof of part (i) in Lemma 2, one sees that $FR(k) \subseteq FR(k)^*$ for each $k = 1, \dots, n-1$. By (26) and part (ii) in Lemma 2, one has

$$FR \subseteq FR^* = \cap_{i=1}^{n-1} FR(k)^* = \{x \in \mathbf{R}_+^n \mid \Sigma_{i \in N} x_i \geq n\, v(n-1)/(n-1)\}. \qquad (27)$$

Observe that the minimum value of $\{Min\ \Sigma_{i \in N} x_i \mid x \in FR^*\}$ is equal to

$$\{Min\ \Sigma_{i \in N} x_i \mid x \in FR^*\} = nv(n-1)/(n-1) = MV(n-1). \qquad (28)$$

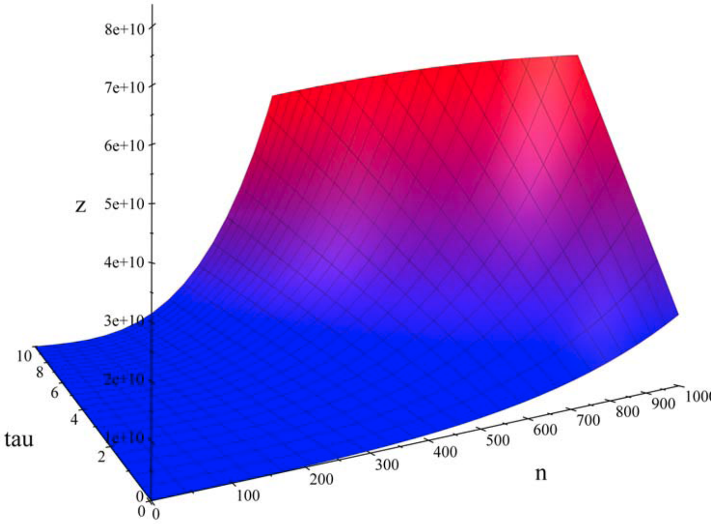
Since the symmetric Min solution (i.e., $x_i = v(n-1)/(n-1)$, all i) is included in FR, it follows from (27) and (28) that $\{Min \Sigma_{i \in N} x_i | x \in FR\} = \{Min \Sigma_{i \in N} x_i | x \in FR^*\}$. Substituting $v(n-1)$ in (22) into (28) leads to the expression for mnbp in (25). **Q.E.D**

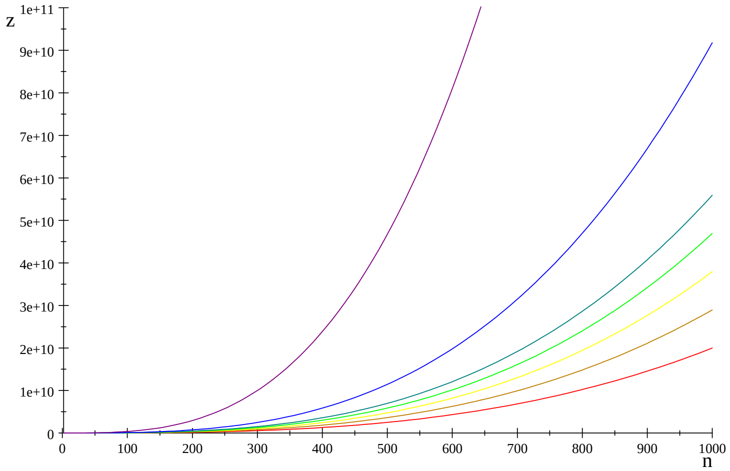
Proof of Proposition 6: We first prove the special case of symmetric markets, and then extend it to asymmetric markets. The proof consists of three part.

Part 1: In symmetric case, A2 holds. Rearranging (17)-(18) leads to

$$\pi_M - mnbp = \leq mnbp(a, c', z') \leq n(a - z_{min} - c_1)^2/[4(n-1)]. \qquad (29)$$

completed by establishing the following four lemmas. Given a game $\Gamma = \{N, v\}$, recall that its minimum no-blocking payoff (MNBP) is defined in (11), and that its core is denoted as $Core(\Gamma)$.





We first establish core existence using the following four assumptions:

With *A1*, the unconstrained optimal supply of *S* in finding its $v(S)$ is an interior solution (i.e., less than or equal to its capacity). With *A2*, the price facing any *S* is always greater than its marginal cost c_S, this guarantees that each coalition produces a positive amount of output. With *A3* and *A4*, firms are symmetric in marginal cost and in capacity.

Lemma 1: Given a game $\Gamma = \{N, v\}$, let $\Gamma' = \{N, v'\}$ be a new game such that $v'(S) \leq v(S)$ for all $S \neq N$. Then $mnbp(\Gamma') \leq mnbp(\Gamma)$.

Proof of Lemma 1: Since $v(S)$ is reduced to $v'(S)$, the feasible region for Γ given in (11) is enlarged to that for Γ'. Thus, the minimum value of the same objective function in (11), $\Sigma_{i \in N} x_i$, is reduced from $mnbp(\Gamma)$ to $mnbp(\Gamma')$. **Q.E.D**

Lemma 4: Given (a, c, z), let $v(S) = v_\alpha(S) = v_\beta(S)$ be given by (9), and $mnbp(\Gamma) = mnbp(a, c, z)$ be given in (11). Define a market (a, c', z') by $c' = (c_1, \ldots, c_1) \in \mathbf{R}^n_+$, and $z' = (z_{min}, \ldots, z_{min}) \in \mathbf{R}^n_+$, where $c_1 = Min\ \{c_i \mid i \in N\}$, $z_{min} = Min\ \{z_i \mid i \in N\}$, let the *MNBP* in this new market be denoted as $mnbp(a, c', z')$. Then

$$mnbp(a, c, z) \leq mnbp(a, c', z') \leq n(a - z_{min} - c_1)^2/[4(n-1)]. \qquad (30)$$

Proof of Lemma 4: Consider the second inequality. If *A1* and *A2* are both satisfied in *(a, c′, z′)*, then Lemma 3 and (22) yield $mnbp(a, c', z') = n(a - z_{min} - c_1)^2/[4(n-1)]$. Note that *v(S)* will be reduced to a smaller value if either *A1* or *A2* or both are violated. If *A1* is violated, *v(S)* will fall below $(a-(n-s)z_{min} - c_1)^2/4$ (where *s* denote the number of firms in *S*), because now *v(S)* is achieved at constrained solution. If *A2* is violated, *v(S)* becomes zero. Thus, by Lemma 1, the *MNBP* will be reduced, so

$$mnbp(a, c', z') \leq n(a - z_{min} - c_1)^2/[4(n-1)].$$

Now consider the first inequality of (30). Assume for a moment that *A1* and *A2* are both satisfied in *(a, c, z)*. By *A1* and *A2*, the value for *S* in *(a, c, z)* satisfies

$$v(S) = (\overline{x}_S)^2 = (a - c_S - \Sigma_{j \notin S} z_j)^2/4 \leq (a - c_1 - (n-s)z_{min})^2/4. \qquad (31)$$

By earlier arguments, the above *v(S)* in (*a, c, z*) will fall below $(a - c_S - \Sigma_{j \notin S} z_j)^2/4$, if either *A1* or *A2* or both be violated. Since $(a - c_1 - (n-s)z_{min})^2/4$ is the value of *S* in *(a, c′, z′)*, (31) and Lemma 1 lead to $mnbp(a, c, z) \leq mnbp(a, c', z')$. **Q.E.D**

By parts (ii-iii) of *A0*, the monopoly profit is $\overline{\pi} = (a - c_1)^2/4$. By part (i) of Lemma 2, this is equal to *MV(n)* defined in (23) for the new market (*a, c′, z′*). It follows from Lemmas 3-4 and part (ii) of Lemma 2 that

$$\begin{aligned} v(N) = \overline{\pi} = MV(n) > MV(n-1) &= n(a - z_{min} - c_1)^2/[4(n-1)] \\ &\geq mnbp(a, c', z') \geq mnbp(a, c, z) = mnbp(\Gamma). \end{aligned}$$

Since *v(N)* is greater than *mnbp(Γ)*, the core has a non-empty interior. **Q.E.D**

Proof of Proposition 7: Substituting (17)-(18) into

$$\Sigma\pi_i - mnbp = \frac{4n^2\beta(a-c)^3}{(2n+1)^3} - \frac{2n\beta(a-c)^3}{27(n-1)}\left[2-\left(1+\frac{12n(1+\tau)}{(2n+1)^2}\right)^{0.5}\right]^2\left[1+\left(1+\frac{12n(1+\tau)}{(2n+1)^2}\right)^{0.5}\right]$$

$$= \frac{2n\beta(a-c)^3}{(2n+1)^3}\left\{2n - \frac{(2n+1)^3}{27(n-1)}\left[2-\left(1+\frac{12n(1+\tau)}{(2n+1)^2}\right)^{0.5}\right]^2\left[1+\left(1+\frac{12n(1+\tau)}{(2n+1)^2}\right)^{0.5}\right]\right\}$$

$$= \frac{2n\beta(a-c)^3}{(2n+1)^3} h(n,\tau),$$

one sees that $Max\{\Sigma\pi_i, mnbp\} = mnbp$ if $h(n,\tau) < 0$, and $Max\{\Sigma\pi_i, mnbp\} = \Sigma\pi_i$ if $h(n,\tau) \geq 0$. Hence, for $h(n,\tau) < 0$,

$$MCM^* = \pi_M - mnbp$$

$$= \frac{4\beta(a-c)^3}{27} - \frac{2n\beta(a-c)^3}{27(n-1)} [2-(1+\frac{12n(1+\tau)}{(2n+1)^2})^{0.5}]^2 [1+(1+\frac{12n(1+\tau)}{(2n+1)^2})^{0.5}]$$

$$= \frac{2\beta(a-c)^3[2(n-1)- n(2-r)^2(1+r)]}{27(n-1)};$$

and for $h(n,\tau) \geq 0$,

$$MCM^* = \pi_M - \Sigma\pi_i = \frac{4\beta(a-c)^3}{27} - \frac{4n^2\beta(a-c)^3}{(2n+1)^3} = \frac{4\beta(a-c)^3[(2n+1)^3 - 27n^2]}{27(2n+1)^3},$$

which are the expressions of (20).

Similarly, one can obtain (21). **Q.E.D**

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