This volume is dedicated to
THE FUTURE GENERATION

for the preservation of essentials
of humanistic processes
in this advancing technological world.

Aśno bhadrah ātvo yāntu viśvataḥ
Let noble thoughts universally flow to us
Rg Veda
Much progress has been made in the development of fuzzy set theory and its application since the first volume in this field was published by North-Holland in 1977. Considerable work has been done, for example, on the development of a well-defined body of knowledge on the properties and characteristics of fuzzy sets, fuzzy switching mechanisms and decision making in fuzzy situations. As well, a wide variety of applications has taken place in many areas of engineering and in medicine and the social sciences. A number of the more significant of these developments are reported in this Volume. They show a maturing of the field - the "fuzziness" of the fuzzy sets is being replaced by a much more rigorous mathematical basis and a much better appreciation as to how it can be applied. They also show that a fundamental relationship exists between many problems in these various fields - they have, in other words, much in common.

This Volume contains 35 papers written by 49 authors from 11 countries; thus, it is ample proof of the very rapid expansion of this field. It includes papers dealing with the basic theory and papers on application. As well, an up-to-date bibliography, which contains 1,800 references, is included. Hopefully, you will enjoy reading the works of the authors as much as they enjoyed doing the work and reporting it for the benefits of all of us.

I also wish to congratulate the editors of this book for a job well done. In particular, I wish to acknowledge Dr. M.M. Gupta who was largely the driving force behind it.

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This is a second volume on Fuzzy Systems published by North-Holland.

In many applied fields such as engineering, the social sciences and medical diagnostics, the sources of vague or fuzzy data are numerous and diverse both in origin and in magnitude. Conventionally, in the construction of mathematical models for most of these fuzzy systems, this imprecision is standardly portrayed as random processes. The introduction of Fuzzy Set Theory has changed this conventional approach. Fuzzy Set Theory has initiated a growing interest not only on the part of mathematicians, but as well among engineers and other applications-oriented workers. A major portion of this current interest and research is oriented toward the applications.

The mathematical sophistication required to understand the properties of fuzzy processes and the role they play in the corresponding applications is considerable; this tends to create a gap between mathematically oriented academics, with limited interest in design and development, and applications-oriented engineers, who tend to dismiss some of the available mathematical tools as being too complicated or of questionable usefulness.

This volume on "Advances in Fuzzy Set Theory and Applications" has been edited to present a coherent view of this growing field. One purpose of the present collection of papers which has been drawn from mathematics, engineering, the social sciences and medical diagnostics is to contribute to the narrowing of the gap between theory and practice.

The literature on the subject of the present volume is rich in contributions both in theory and applications of the fuzzy field. For this reason the selection of papers has been a difficult task. The principal criteria that have been used are the significance and quality of the contributions, their relevance to the central thesis of this volume, and in the context of the historical evolution of the theory and its applications. Many outstanding contributions could not be included here because of limitations of space. An effort was made throughout to present a coherent sequence of the papers that enhances the aspect of interaction between engineering considerations and the underlying mathematical ideas.

The first part of the volume gives a broad perspective view of fuzzy set theory. The second part is concerned for the greater part, into recent theoretical developments, while in Part III the papers deal with applications of fuzzy sets. A synthesis of the volume is presented in Chapter I. To further aid the reader in his/her studies, an extensive bibliography containing 1,799 references
is included in this Volume.

The editors believe that this volume of outstanding contributions should provide reference material which will be useful both to the theoretician and to the applications-oriented researchers. It is hoped that it will also be equally valuable to students, and teachers.

Madan M. Gupta
Hammonin A. Bagade
Ronald R. Yager
ACKNOWLEDGEMENTS

The task of organizing and editing a collection of papers on a subject as new and as diverse as the theory of fuzzy sets has not been an easy one to accomplish. I am deeply appreciative of the spirit of cooperation and understanding manifested by all of the contributors to this Volume. The editorial assistance of Dr. Rammohan Ragade and Dr. Ronald Yager is gratefully acknowledged. The Editor records his appreciations to Mrs. Janice Friesen who was very helpful during all the editorial phases of this Volume. I am indebted to my wife, Suman, who not only helped me since the inception of this Volume but also provided moral support by not demanding much of my time.

Madan M. Gupta
Editor
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