Summary—This report summarizes the main features of the round table discussion on Fuzzy Automata and Decision Processes which took place during the 6th triennial IFAC World Congress at MIT in Boston in the late Summer of 1975. The meeting was organised by Professor M. M. Gupta following the resounding success of an earlier round table which he had organised on the same topic and reported in Automatica [22].

It was one of the more popular round table sessions held during the Congress and was attended by nearly 50 people. Contributions were offered by 14 authors. Those of whom were able to attend the session brought with them copies of full papers which were distributed to all participants. Because of the large number of contributions each author was asked to give only a brief account of his work leaving Professor Zadeh to make a larger introductory presentation. Some very interesting points were made during the discussion that followed the formal presentation.

This report gives a shortened account of what was said at the session. The list including addresses of all the authors present and most of the other participants is given in Section 5. The list of references at the end of this report has been formed by putting together all the key references in the full length papers submitted by the authors.

1. Introduction

The Chairman, Professor Gupta, in opening the session pointed out that this round table was characterised by the fact that the vast majority of the contributions were on practical applications of fuzzy-set-theory. These applications and many others are already pointing at the important future role of the theory. At the same time the theory itself is continuing to advance and in the process is sweeping aside several misconceptions in an understanding of humanistic processes.

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He began by saying that this was the appropriate time to take stock of what has happened in the field, in which direction fuzzy set theory is going and what impact it has on other theories. During the early days when he first introduced the theory of fuzzy sets, the question that was often put to him concerned the difference between randomness and fuzziness. This issue bothered many people and a lack of understanding of a basic difference between the two led many on wrong paths. The issue can be clarified to some extent by giving its compatibility to the concept such as 'flower' and another its probability (i.e. frequency of occurrence).

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about the degree of linearity and so on. Unfortunately because of the failure of system theory to take account of such fuzziness we do not have procedures for computing say the degree of linearity of a system or say the degree to which a random variable is stationary.

It was Professor Zadeh's contention that there is a cult of exactness which has put us in a straight-jacket. It ought to be realized that if one tries to say anything precise then the chance that it will be realistic is very small.

2. Formal contributions

The contributions consisted of several full length papers on various aspects of fuzzy logic theory and applications. Copies of these papers were available to all who attended the round table. The authors who attended the session formed the panel and gave a very brief presentation of their work. The time given to each had to be limited because of the large number of contributions received. Here we shall give a short summary of the contributions based on the oral presentation of the panelists and submitted extracts. The full length papers may be obtained from the author direct at addresses given below. The title of the work pertaining to each presentation is given in parenthesis against each panelist:

Professor S. S. L. Chang (Algorithms and Mapping by S. S. L. Chang), said that his paper concerned the question as to when one could use fuzzy algorithms to advantage. He defined algorithms as mappings on the decision space. If the decision space is a fuzzy set then we have a non-fuzzy decision if it were to be a fuzzy set then a fuzzy decision. Several examples of fuzzy and non-fuzzy decisions operating on both fuzzy and non-fuzzy systems are given in the paper.

"An algorithm is fuzzy if any of the instructions are fuzzy and the point is, when can you use such an algorithm to advantage?" Professor Chang had a fuzzy answer to this, namely: if there is distributed intelligence say when the executor of the instruction is going to use his own knowledge of the problem then one can use a fuzzy algorithm. An alternative condition for a fuzzy algorithm is irrelevance. It is difficult to think of a fuzzy algorithm when neither of these conditions are present.

The paper attempts to give a rigorous proof of this by first defining the concept of "optimum" or "non-inferior" with reference to fuzzy systems and then leading on to theorem 1 of the paper. This states that if a non-fuzzy decision is optimum (i.e. optimum for any membership function m) among all non-fuzzy decisions then it is m-optimum among all fuzzy lattice decisions. Here the idea of a fuzzy lattice decision is introduced to simplify the proof and because it is a good approximation of a fuzzy decision as can be shown rigorously.

"An important implication of this is the cutting out of certain areas of research namely how to execute a fuzzy algorithm? Or the question that since a fuzzy algorithm is defined as a fuzzy set on the decision space then how to interpret this? There are publications where an interpretation is made probabilistically and others where the maximum value is used". However according to Professor Chang these are not fuzzy algorithms. The conclusion is that if you do not have distributed intelligence then do not use a fuzzy algorithm. If the executor does have intelligence then leave it to him to decide which way to carry it out. If the result is immaterial then it does not matter which way it is carried out.

Professor J. C. Dunn (Indices of Partition Fuzziness and the Detection of Clusters in large Data Sets) by J. C. Dunn) said that his problem was the following: given a data set of finitely many vectors how does one determine the clusters in the data set and what element belongs to which cluster? The problem is particularly of interest when the data set is composed of vectors with a large number of dimensions.

In Professor Terrano's contribution he used algorithms which assigns a number for every partition on the data set in such a way that the value tends to become large the farther apart the partitions are separating real clusters in the data set. A recently developed algorithm is used to generate fuzzy k-portions of the data set, details of which can be supplied by him[8, 9, 17, 18]. The limiting partitions that are obtained extremize a fuzzy exten-

sion of the mean square error criterion function. In the experiments, three separate indices of partition fuzziness (entropy, H; normalised entropy, h; and a fuzzy intersection coefficient, F) are computed and plotted against k for limiting partitions for a variety of data sets.

Professor Dunn discussed the results of one of the experiments in which the data set was constructed by corrupting five different signals with additive Gaussian white noise to produce a data set of twenty five functions. The results containing plots of scalar indices of partition fuzziness indicated relatively conspicuous clustering structure at a value which can be anticipated from the data set.

Dr. Michio Sugeno (Analytical Representation of Fuzzy Systems by M. Sugeno and T. Terano) said that in his previous research with Professor Terrano they have proposed the concepts of fuzzy measures and fuzzy integrals[45, 46, 59].

A fuzzy measure is similar to a probability measure and is interpreted as a subjective state of the so called grade of fuzziness. However, the mathematical meaning of a fuzzy measure is quite different from a probability measure because a fuzzy measure does not always have additivity but only has monotonicity. Fuzzy integrals are defined using fuzzy measures and they also have monotonicity. These are similar to Lebesgue integrals and can be viewed as a fuzzy expectation. To these two concepts they have recently added the concept of conditional fuzzy measures described in their second contribution at the round table (see Terrano—next page).

Most of the fuzzy systems studied so far are those derived by replacing a deterministic input-output relation or a tension of states by a fuzzy relation. Such systems are therefore algebraic. The point of their research is that using fuzzy measures, fuzzy integrals and conditional fuzzy measures it is possible to express and study a system analytically. The paper describes how this can be applied and it is to be hoped that other researchers will adopt this approach of studying fuzziness in an analytical way.

Professor F. Donati (A Fuzzy cluster of the Demand within a Regional Service System by D. Carlucci and F. Donati) began by saying that the construction of a mathematical model is one of the most important aspects in the solution of many problems. The model describes the state of our knowledge about the problem. When this is completely known we can construct a deterministic model. More often however there is a great deal of uncertainty in our knowledge. Identification methods can improve this knowledge iteratively by taking more and more measurements. But such measurements are expensive and the procedure can thus be uneconomical. Often stochastic models are used instead, but their usefulness may be in doubt if randomness is not actually present in the problem.

The problem considered by Professor Donati and his co-author is one of defining influence areas of bank services and can be stated as follows. Given a set of service centres in a region and a distribution of population within the region, classify the users into subsets whose elements are customers of the same service centres. In the past such problems have been tackled by deterministic as well as stochastic models, but in their case the results obtained were not very satisfactory. Given the subjectivity in the choice exercised by human beings it seems the concept of fuzziness is better suited to this problem, by stating it in terms of a fuzzy model.

The paper describes how one would go about generating a fuzzy model. The model developed by them has two main aspects of novelty. The first is that the human behaviour has been modelled as a fuzzy process. The other is that in the model the notion of 'crowding' of the service centres has been successfully considered.

Dr. Peter King (On Fuzzy Control Systems to Industrial Processes by P. J. King and E. H. Mamdani) said that his interests were in human beings controlling processes. Many processes for which it is difficult to design an automatic controller are those where human beings have been operating the plant and whose control is mainly of a heuristic nature. They have worked on the idea that human beings and their work has shown that his strategy can be effectively modelled by a set of fuzzy decision rules. To illustrate this, he began by showing some material taken from a standard work on cement kilns[36]. This was a portion
of the set of rules which the operator of a kiln uses as his control protocol. These rules are characteristically fuzzy in nature. In their work similar rules were set up for controlling other processes and these heuristics amounted to a fuzzy algorithm. The paper describes how these can be implemented on a digital computer using fuzzy set theory.

The paper gives details of how the authors have considered the fuzzy functions giving the fuzziness of each dimension of the event. This method then applies the least squares method to dimensionality reduction methods which introduce a fuzzy function as a heuristic search is to reduce the number of trials quickly.

These grades of fuzziness are conveniently expressed as fuzzy integrals. Conditional fuzzy measures are then applied to simulate the learning process.

Professor George Saridis (Fuzzy Decision Making in Prosthetic Devices and other Applications) made a few further remarks about his work. He said that the reaction of control engineers concerning his original work of a similar nature[31, 32] indicated that the motivation behind the approach was not properly appreciated. For example it was stated that the controller should be analysed as a non-linearity in the feedback loop. However, the controller represented a fuzzy system which was similar to an algebraic discrete system, and it is meaningless to apply the notion of non-linearity to such systems.

The question of the stability of the controller was more important. But before this is investigated further it was important to realize that usually the stability is carried out by first taking the system away from the time domain (into frequency domain, s-plane etc.) However systems such as algebraic discrete system are best expressed only in time domain and it is worth bearing this in mind when investigating the stability of fuzzy controllers.

In the first method the fuzzy function giving the fuzziness of an event is approximated by a linear model aggregating the fuzzy functions giving the fuzziness of each dimension of the event. This method then applies the least squares method to obtain an approximation with reduced dimensions.

The second is the typical dimension method which selects the typical dimension only as a representative dimension for decision making. The paper also studies the applicability of these methods by applying them: (a) in a ranking problem constructed using playing cards (b) in the evaluation of a person as dependent on his evaluation of his different qualities (fortune, figure and heart). In each case the fuzzy measures are obtained from human test subject participating in the experiment.

Professor T. Terrano (Macroscopic optimization Using conditional Fuzzy measures by T. Terrano and M. Sugeno) said that the optimization of multi-modal functions was an important problem in system theory. In many publications heuristic method were proposed which contained random searches and logical decisions. This can be considered as a problem of decision making under uncertainty.

Two forms of uncertainties arise in such problems. One arises from the lack of knowledge within a region. The best strategy in such a case is to choose the next search point in the area where this vagueness is vast because then the possibility of finding the extremum is large. The other uncertainty comes from the fact that one cannot say if the given maximum of searched points is really extremum. This is related to the economics of search as number of trials required to get a satisfactory maximum value. The purpose of a heuristic search is to reduce the number of trials quickly.

The paper gives details of how the authors have considered a decision model similar to human learning. The idea is to obtain a map of the space and to use this as a guide for the search. This method then applies the least squares method to dimensionality reduction methods which introduce a fuzzy function as a heuristic search is to reduce the number of trials quickly.

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A very interesting discussion ensued on Professor Chang's definition of a Fuzzy decision. A number of the audience felt that such an idea seemed meaningless; either a decision was made or it was not made. Professor Chang reiterated some of the major points of his presentation and gave an example, say of a company Chairman deciding to promote a long cigarette. This was a fuzzy decision because the manner in which it was to be done was fuzzy. There was a disagreement whether as a decision it could be classified as fuzzy. During the discussion it was suggested that perhaps one should distinguish between a fuzzy instruction and a fuzzy decision. In the example of Professor Chang the decision was non-fuzzy but from the point of view of the subordinate implementation of it, it was a fuzzy instruction.

Among other comments Professor Dunn felt that there was an important aspect of fuzzy set theory that should not be overlooked. This was that Fuzzy set theory can illuminate certain aspects of ordinary set theory in a way that is analogous to what complex variable theory does for real analysis. Some very interesting observations were made by Professor Dunn during the general discussion. He said that there was no doubt that the problem of translating humanistic systems into precise language had been done very well by fuzzy set theory. However he disagreed with those who seemed to suggest that this was the only theory that accounted for subjectivity. Utility theory is also based in subjective probability theory. He had a question and a challenge for researchers in the field; most of the applications of the theory seemed to be descriptive in nature, the theory had not quite addressed the prescriptive heart of the problem. This must surely be the proper direction forward for the future.

4. Conclusion

Most of what follows is based on the observations made by Professor Zadeh on the points raised during the general discussion. Professor Ho had made an important point regarding prescriptive against normative aspects of fuzzy set theory. However it is important to realise that the theory is only 10 years old and is still in its infancy. So far the theory has been primarily descriptive and serves as a language for describing vagueness which in itself is an important achievement and one should not be too demanding. At some point it will become prescriptive, so that the description of a strategy at one level can be used to infer the strategies at another level.

It is the application of the theory as described by Mamdani, King, Dunn, Saridis, Terrano, Donati, Kokawa and others at this round table that is a significant landmark in the development of the theory.

Such actual applications of the theory will highlight the advantages of the theory as well as point out the current limitations and thus show the direction in which the future work should progress. These are grounds for optimism that the theory will be used by more and more people.

Fuzzy logic will serve as a language in many fields where classical constructions have been found wanting. For example, at one time there was a great deal of discussion in the literature as to what is an adaptive system? Many different definitions were put forward. But unlike controllability etc., adaptivity is a fuzzy concept. Similarly symmetric matrix is a non fuzzy idea but sparseness of a matrix is a fuzzy concept. Such concepts are best described as a fuzzy algorithm which returns the degree of membership of the concept. There are currently many misconceptions about apparently exact concepts that are in fact fuzzy.

The round table ended with a vote of thanks for Professor Gupta for organising it.

5. List of participants

The following is a list of names and addresses of most of the participants at the round table including all the panelists:

1. M. M. GUPTA (Chairman). Systems and Adaptive Control Research Laboratory, University of Saskatchewan, Canada.

2. L. A. ZADEH, Department of Electrical Engineering, Computer Science and Electronics, University of California, Berkeley, California 94720, U.S.A.

3. E. H. MAMDANI (Secretary), Department of Electrical and Electronic Engineering, Queen Mary College (University of London), London E1 4NS, England.


5. J. C. DUNN, Cornell University, College of Engineering, Thurston Hall ithaca, N.Y. 14850, U.S.A.

6. M. SUGENO, Department of Control Engineering, Tokyo Institute of Technology, Ohokayama, Meguro-ku, Tokyo 152, Japan.

7. F. DONATI, Gruppo Informatica e Automatica, Istituto Elettronecnicazione Nazionale galileo Ferraris. Politecnico di Torino, Torino, Italy.


9. M. KOKAWA, Systems Development Laboratory, Hitachi Ltd., Tsukuba, Yokoahama, Japan.

10. T. TERRANO, Tokyo Institute of Technology, Ohokayama, Tokyo 152, Japan.

11. G. N. SARIDES, Purdue University, School of Electrical Engineering, West Lafayette, Indiana 47907, U.S.A.

12. D. CARLUCCI, IEN G. Ferraris, c.so M. D’Aegli, 40, 10126 Torino, Italy.

13. N. K. SINGHA, Research Group in Simulation, Optimization and Control, Faculty of Engineering, McMaster University, Hamilton, U.S.A.


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16. P. SARACHIK, Polytechnic Institute of New York, 333 Jay Street, Brooklyn, New York 11201, U.S.A.

17. B. HRUZ, State Forest Products Research Institute, Department of Automation, Bratislava, Czechoslovakia.

18. R. V. POTTER, Engineering Department, Oxford University, Oxford, U.K.

19. A. H. PORTER, Engineering Department, Cambridge University, Cambridge, U.K.

20. G. MEYER, NASA/Ames Research Centre, Moffett Field, California, U.S.A.


23. R. L. FRANKS, Bell Laboratories, Room IC-429, Holmdel, N.J. 07733, U.S.A.


25. H. STEPANOV, Purdue University, School of Electrical Engineering, West Lafayette, IN 47907, U.S.A.

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29. V. K. ARYA, Shell Development Co., P.O. Box 481, Houston, Texas 77001, U.S.A.

30. A. BAGCHI, Twente University of Technology, Enschede, The Netherlands.

31. L. F. KAZIMA, University of Michigan, Ann Arbor, Michigan 48105, U.S.A.

32. B. R. BARMISH, Yale University, Becton Hall, Room 313, New Haven, CT 06520, U.S.A.

33. A. Z. MEIRII, P.O. Box 7963, Tel Aviv, Israel.

34. J. C. CHOW, M.I.T. Liming Laboratory, Lexington, MA 02173, U.S.A.

35. J. SPEYER, C.S. Draper Laboratory, Cambridge, Massachusetts, U.S.A.

36. D. H. JACOBSON, National Research Institute for Mathematical Sciences, P.O. Box 395, Pretoria, South Africa.
37. O. ALBERTSEN, Technical University of Denmark, Building 423, 2800 Lyngby, Denmark.
38. P. M. LARSEN, Department of Electric Power Engineering, Building 325, Technical University of Denmark.

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