Design of fuzzy logic controllers based on generalized T-operators

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Abstract: Since Zadeh first proposed the basic principle of fuzzy logic controllers in 1968, the MIN and MAX operators have been popular in the design of fuzzy logic controllers. In this paper, the general concept of T-operators is introduced into the conventional design methods for fuzzy logic controllers so that a general and flexible methodology for the design of these fuzzy logic controllers is available. Then, by computer simulations, studies are made so as to determine the relations between the various T-operators and the performance of a fuzzy logic controller. It is concluded that the performance of the fuzzy logic controller for a given class of plants very much depends upon the choice of the T-operators.

Keywords: Fuzzy logic controller; T-norms; T-conorms; T-operators; control systems.

1. Introduction

Since Zadeh first introduced the rationale for fuzzy logic control in 1968 [1] and in 1973 [2], and Mamdani first implemented a fuzzy logic controller in 1974 [3], the basic principle of fuzzy logic controllers has not only found tremendous applications in a variety of industrial uses, but has also gone through substantial theoretical developments [4]. In these fuzzy logic control algorithms, however, the MIN and MAX operators have been extensively used in the membership functions for the intersection and the union of fuzzy sets, and for the implication functions and the composition rules of inference; these are the three most important concepts associated with a fuzzy logic controller. However, theoretical and experimental studies have indicated that some operators may work better than others in some situations. For example, the product operator \( T_2 \) in this paper may be better than the MIN operator \( T_1 \) in this paper [5, 6]. This implies that some operators may be more suitable in the context of a given decision-making process. On the other hand, sometimes one also has to consider what properties are expected of the operators to be chosen, whether the accuracy of the model using the operators chosen is good enough, and whether the control algorithms using the operators chosen are sufficiently simple for computers and hardware implementations, etc. In fact, the choice of an operator is always a matter of context, and it mostly depends on the real-world problem which is to be
modelled. It is meaningful, therefore, to use the general notion of T-operators in the design of fuzzy logic controllers so that more options and flexibility are available for the selection of T-operators that may be better suited for a given control problem.

In this paper, a general concept of a fuzzy control algorithm based on generalized T-operators is proposed. In this algorithm, the conventional min, max and negation function are replaced by the general concept of T-operators. Some typical t-operators are then used in a simple fuzzy logic controller which is based on the proposed fuzzy control algorithm and the effects of these T-operators on the controller's performance are studied in the hope of determining the relations which may exist between the various T-operators and the controller's performance.

2. Fuzzy control algorithm based on T-operators

As defined in [7] and summarized in Table 1, a set of T-operators consists of a T-norm, a T-conorm and a negation function which are related to each other in

<table>
<thead>
<tr>
<th>N</th>
<th>$T_p(x, y)$</th>
<th>$T_q(x, y)$</th>
<th>$N_p(x)$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\min(x, y)$</td>
<td>$\max(x, y)$</td>
<td>$1-x$</td>
<td>Zadeh</td>
</tr>
<tr>
<td>2</td>
<td>$x \cdot y$</td>
<td>$x + y - xy$</td>
<td>$1-x$</td>
<td>Goguen, Bandler, etc.</td>
</tr>
<tr>
<td>3</td>
<td>$\max(x + y - 1, 0)$</td>
<td>$\min(x + y, 1)$</td>
<td>$1-x$</td>
<td>Giles, etc.</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{xy}{x + y - xy}$</td>
<td>$\frac{x + y - 2xy}{1 - xy}$</td>
<td>$1-x$</td>
<td>Weber</td>
</tr>
<tr>
<td>5</td>
<td>$\lambda xy$</td>
<td>$\frac{\lambda(x + y) + xy(1 - 2\lambda)}{\lambda + xy(1 - 2\lambda)}$</td>
<td>$1-x$</td>
<td>Hamacher</td>
</tr>
<tr>
<td>6</td>
<td>$\max(1 - ((1 - x)^p + (1 - y)^q), 0)$</td>
<td>$\min((x^p + y^q)^{1/p}, 1)$</td>
<td>$1-x$</td>
<td>Yager</td>
</tr>
<tr>
<td>7</td>
<td>$\frac{1}{1 + \left(\frac{1}{\lambda} - 1 + \frac{1}{\lambda} - 1\right)^{1/\lambda}}$</td>
<td>$\frac{1}{1 + \left(\frac{1}{\lambda} - 1 + \frac{1}{\lambda} - 1\right)^{1/\lambda}}$</td>
<td>$1-x$</td>
<td>Dombi</td>
</tr>
<tr>
<td>8</td>
<td>$\frac{xy}{\max(x, y, \lambda)}$</td>
<td>$1 - \frac{(1 - x)(1 - y)}{\max(1 - x, 1 - y, \lambda)}$</td>
<td>$1-x$</td>
<td>Dubois et al.</td>
</tr>
<tr>
<td>9</td>
<td>$\frac{\lambda xy}{1 + \lambda xy}$</td>
<td>$\min(x + y + \lambda xy, 1)$</td>
<td>$1-x$</td>
<td>Weber</td>
</tr>
<tr>
<td>10</td>
<td>$\max((1 + \lambda)(x + y - 1) - \lambda xy, 0)$</td>
<td>$\min(x + y + \lambda xy, 1)$</td>
<td>$1-x$</td>
<td>Yu Yandong, etc.</td>
</tr>
</tbody>
</table>

| T-operators |

$\lambda \rightarrow 0$, $\rightarrow T_5$ and $T_2^*$

$\lambda = 1$, $\rightarrow T_1$ and $T_2^*$

$\lambda \rightarrow \infty$, $\rightarrow T_3$ and $T_2^*$

$\lambda \rightarrow 0$, $\rightarrow T_3$ and $T_3^*$

$\lambda = 1$, $\rightarrow T_1$ and $T_3^*$

$\lambda \rightarrow \infty$, $\rightarrow T_3$ and $T_3^*$

$\lambda \rightarrow 0$, $\rightarrow T_5$ and $T_2^*$

$\lambda = 1$, $\rightarrow T_1$ and $T_2^*$

$\lambda \rightarrow \infty$, $\rightarrow T_3$ and $T_2^*$

$\lambda \rightarrow 0$, $\rightarrow T_5$ and $T_2^*$

$\lambda = 1$, $\rightarrow T_1$ and $T_2^*$

$\lambda \rightarrow \infty$, $\rightarrow T_3$ and $T_2^*$
some way. The general concept of T-operators was extensively used in the study of probabilistic metric spaces and, later, they were introduced into fuzzy set theory. However, in the designs of almost all existing fuzzy controllers, only one of many sets of T-operators has been used, and these are $\min(x, y)$, $\max(x, y)$ and $(1 - x)$. A general fuzzy control algorithm will be described first.

A fuzzy logic controller consists of a set of linguistic conditional statements or rules which are taken from operators who have expert knowledge about the process to be controlled. These rules usually relate the process error ($e$) and the change in error ($\Delta e$) to the change in the process input ($\Delta u$) that has to be applied. For example, a section of control rules could be described as follows:

- If the error ($e$) is large and the change in error ($\Delta e$) is small, then the change in the process input ($\Delta u$) is medium;
- If the error ($e$) is small and the change in error ($\Delta e$) is zero, then the change in the process input ($\Delta u$) is small; and
- If . . .

Here, $e$, $\Delta e$ and $\Delta u$ are linguistic variables, and their values are fuzzy sets, such as large, small, zero, etc. which are denoted by $E_i$, $AE_i$ and $AU_i$ over the universes of $E$, $AE$ and $AU$, respectively.

In general a fuzzy controller has $N$ rules; these rules can be expressed in an ensemble form:

\[
\bigcup_{i=1}^{N} \text{if } e \text{ is } E_i \text{ and } \Delta e \text{ is } AE_i, \text{ then } \Delta u \text{ is } AU_i.
\]

The $i$-th rule can be described by a fuzzy relation $R_i$ on a universe of discourse $E \times AE \times AU$ and its membership function is given by

\[
\mu_{R_i}(e, \Delta e, \Delta u) = f_\wedge (\mu_{E_i}(e), \mu_{AE_i}(\Delta e), \mu_{AU_i}(\Delta u))
\]

where $f_\wedge (\cdot, \cdot, \cdot)$ is a general representation of the implication functions and $e \in E$, $\Delta e \in AE$ and $\Delta u \in AU$.

What is important here is to represent the implication function $f_\wedge (\cdot, \cdot, \cdot)$ by T-operators. By representing T-norms by $T[\cdot]$ and T-conorm by $T^*[\cdot]$, the following three implication functions are defined as:

\begin{align*}
(i) \quad & \mu_{R_i}(e, \Delta e, \Delta u) = T[\mu_{E_i}(e), \mu_{AE_i}(\Delta e), \mu_{AU_i}(\Delta u)], \\
(ii) \quad & \mu_{R_i}(e, \Delta e, \Delta u) = T^*[T[\mu_{E_i}(e), \mu_{AE_i}(\Delta e), \mu_{AU_i}(\Delta u)], \\
& \qquad N[T[\mu_{E_i}(e), \mu_{AE_i}(\Delta e)]]], \\
(iii) \quad & \mu_{R_i}(e, \Delta e, \Delta u) = T^*[\mu_{AU_i}(\Delta u), N[T[\mu_{E_i}(e), \mu_{AE_i}(\Delta e)]]].
\end{align*}

This first equation is an extension of Mamdani’s implication functions, and the other two are extensions of Zadeh’s implication functions. Similar extensions can be made to other implication functions which may be found in [8]. The choice of an implication function is a matter of the context in a given problem.

The overall fuzzy relation $R$ is then given by

\[
\mu_{R_i}(e, \Delta e, \Delta u) = \bigwedge_{i=1}^{N} [\mu_{R_i}(e, \Delta e, \Delta u)].
\]
Thus, all the control rules are put together and described by a single fuzzy relation $R$. Now if the actual process error, the actual change in error and the actual change in the process input take on fuzzy values $E'$, $\Delta E'$ and $\Delta U'$ respectively at a particular moment, the actual change in the process input $\Delta U'$ can then be obtained as follows:

$$\Delta U' = E' \circ \Delta E' \circ R,$$

$$\mu_{\Delta U'}(\Delta u) = \sup_{(e, \Delta e)} T[\mu_{E'}(e), \mu_{\Delta E'}(\Delta e), \mu_{R}(e, \Delta e, \Delta u)]. \quad (6)$$

Equation (6), in fact, represents a generalized form of the compositional rule of inference.

Equations (2) to (6) constitute the general fuzzy control algorithm based on the T-operators which indeed provide more options and flexibility for fuzzy control systems designers.

3. A simple controller structure based on T-operators

In this section, a fuzzy controller with the simplest structure is used in which the proposed fuzzy control algorithm is utilized. This controller is used to control a given class of processes for given reference inputs, and the system response is a function of the T-operators. Given various typical T-operators, the purpose of the following simulation studies is to see what the system responses look like when other parameters are constant.

A block diagram of the T-operator based simple fuzzy logic controller is shown in Figure 1. For simplicity, only a single-input and single-output structure is considered.

Also, the following set of control rules are proposed for the controller based on the knowledge of an expert operator.

- If $e$ is negative and $\Delta e$ is negative, then $\Delta u$ is negative,
- If $e$ is negative and $\Delta e$ is positive, then $\Delta u$ is zero,
- If $e$ is positive and $\Delta e$ is negative, then $\Delta u$ is zero,
- If $e$ is positive and $\Delta e$ is positive, then $\Delta u$ is positive.

![Fig. 1. Block diagram of the fuzzy controller.](image-url)
The following fuzzy sets are used to represent the fuzzy concepts involved in the above control rules:

- **NE**: negative error,
- **PE**: positive error,
- **NΔE**: negative change in error,
- **PΔE**: positive change in error,
- **NΔU**: negative change in process input,
- **ZΔU**: zero change in process input,
- **PΔU**: positive change in process input.

The membership functions for these fuzzy sets are defined in Figure 2.

This is the simplest fuzzy controller which takes the minimum number of control rules and uses simple piecewise fuzzy quantities. For the purpose of simulation studies, the controller is designed to be the one that uses on-line fuzzy logic.

The actual error and the actual change in error at any sampling instant are calculated as follows:

\[
e(n) = [x(n) - y(n)] \times GE, \tag{7}
\]
\[
\Delta e(n) = [y(n) - y(n - 1)] \times GΔE, \tag{8}
\]

where \(y(n)\) is the process output at the \(n\)-th sampling, \(GE\) and \(GΔE\) are scaling factors, and \(x(n)\) is the reference input.

![Fig. 2. Membership functions of fuzzy sets.](image-url)
\( e(n) \) and \( \Delta e(n) \) are always non-fuzzy values and, therefore, Eq. (6) is further simplified to give a fuzzy change in the process input \( \Delta U' \) as follows:

\[
\mu_{\Delta U'}(\Delta u) = \mu_R[e(n), \Delta e(n), \Delta u]
\]

\[
= \prod_{i=1}^{4} [\mu_R(e(n), \Delta e(n), \Delta u)].
\]  

(9)

If the implication function described by Eq. (2) is utilized, then we have

\[
\mu_{\Delta U'}(\Delta u) = \prod_{i=1}^{4} [T[\mu_{E}(e(n)), \mu_{AE}(\Delta e(n)), \mu_{AU}(\Delta u)]]
\]

\[
T[\mu_{NE}(e(n)), \mu_{NAE}(\Delta e(n)), \mu_{NAU}(\Delta u)],
\]

\[
T[\mu_{PXE}(e(n)), \mu_{PAE}(\Delta e(n)), \mu_{PAU}(\Delta u)],
\]

\[
T[\mu_{PE}(e(n)), \mu_{PE}(\Delta e(n)), \mu_{PAU}(\Delta u)].
\]  

(10)

Thus, given two non-fuzzy inputs \( e(n) \) and \( \Delta e(n) \), the fuzzy controller infers a fuzzy change in the process input \( \Delta U' \) from the control rules. However, the process needs a crisp input and, therefore, a defuzzification algorithm is required. Here, the method of 'Center of Gravity' is chosen. Thus, the non-fuzzy change in process input \( \Delta u(n) \) is calculated as

\[
\Delta u(n) = \frac{\sum_{i=1}^{4} \mu_{\Delta U'}(\Delta u) \times \Delta u}{\sum_{i=1}^{4} \mu_{\Delta U'}(\Delta u)}.
\]  

(11)

4. Simulations studies and discussions

Computer simulations were used to examine the impact of each of the following five typical T-operators on the proposed fuzzy controller’s performance and to reveal the differences among these T-operators in terms of control problems. Under a given controller model, given plants and given T-operators, a large number of simulations were conducted which are shown in Figures 3 to 8.

Table 1 gives the various T-operator developed in [7]. First, consider the following mathematical facts for five typical T-norms (\( T_1 \) to \( T_5 \)) and five typical T-conorms (\( T_1^* \) to \( T_5^* \)):

(i) \( T_5 \leq T \leq T_1 \), \( T_5^* \leq T^* \leq T_1^* \);
(ii) \( T_5 < T_3 < T_2 < T_1 \), \( T_5^* < T_3^* < T_2^* < T_1^* \);
(iii) \( T_i \) (or \( T_i^* \)) \((i = 6, 7, \ldots, 11)\) takes two of \( T_j \) (or \( T_j^* \)) \((j = 1, 2, \ldots, 5)\) as its two limits when the parameter \( \lambda \) or \( p \) goes to its two extremes as shown in Table 1.

Based on the above knowledge, it was decided to use only five T-conorms (\( T_1 \) to \( T_5 \)) and five T-norms (\( T_1^* \) to \( T_5^* \)) in the proposed general fuzzy control algorithm to investigate the effects of T-operators on the controller’s performance.
Fig. 3a. Output of first order plant, $y' + y = u$ (column 1).

Fig. 3b. Outputs of second order plant, $y'' + 2y' + 1.25y = u$ (column 1).
Fig. 3c. Output of first order plant with time delay, $e^{-0.25/(s+1)}$ (column 1).

Fig. 3d. Outputs of nonlinear plant, $y'' + ([y'] - 1.5)y' + y = u$ (column 1).
Design of fuzzy logic controllers

Fig. 4a. Output of first order plant, \( y' + y = u \) (column 2).

Fig. 4b. Outputs of second order plant, \( y'' + 2y' + 1.25y = u \) (column 2).
First Order Plant with Time Delay

$$e^{-0.25/(s+1)}$$

Fig. 4c. Output of first order plant with time delay, $e^{-0.25/(s+1)}$ (column 2).

Nonlinear Plant $y'' + (|y'| - 1.5)y' + y = u$

Fig. 4d. Outputs of nonlinear plant, $y'' + (|y'| - 1.5)y' + y = u$ (column 2).
**Design of fuzzy logic controllers**

First Order Plant

\[ y' + y = u \]

\[ GE = 1.0 \]

\[ GΔE = 0.5 \]

\[ GΔU = 0.5 \]

Fig. 5a. Output of first order plant (column 3).

Second Order Plant

\[ y'' + 2y' + 1.25y = u \]

\[ GE = 0.5 \]

\[ GΔE = 0.5 \]

\[ GΔU = 0.5 \]

Fig. 5b. Outputs of second order plant (column 3).
First Order Plant \[ y' + y = u \]

GE = 0.5
GAE = 0.5
GAU = 0.5

Second Order Plant \[ y'' + 2y' + 1.25y = u \]

GE = 0.5
GAE = 0.5
GAU = 0.2

Fig. 6a. Output of first order plant (column 4).

Fig. 6b. Outputs of second order plant (column 4).
First Order Plant  \( y' + y = u \)

\[
\begin{align*}
T_4 & \quad T_1 \\
T_4 & \quad T_2 \\
T_4 & \quad T_3 \\
T_4 & \quad T_4 \\
T_4 & \quad T_5
\end{align*}
\]

Fig. 7a. Output of first order plant (column 5).

Second Order Plant  \( y'' + 2y' + 1.25y = u \)

\[
\begin{align*}
T_4 & \quad T_1 \\
T_4 & \quad T_2 \\
T_4 & \quad T_3 \\
T_4 & \quad T_4 \\
T_4 & \quad T_5
\end{align*}
\]

Fig. 7b. Outputs of second order plant (column 5).
First Order Plant  \[ y' + y = u \]

GE = 0.5  
G\AE = 0.5  
G\AU = 0.5

Second Order Plant  \[ y'' + 2y' + 1.25y = u \]

GE = 0.5  
G\AE = 0.5  
G\AU = 0.5

Fig. 8a. Output of first order plant (column 6).

Fig. 8b. Outputs of second order plant (column 6).
For the above T-norms and T-conorms, given the actual process error $e(n)$ and the actual change in error $\Delta e(n)$ at any sampling instant, the fuzzy change in the process input $\Delta U'$ is given by

$$
\mu_{\Delta U'}(\Delta u) = \prod_{k=1}^{4} \left[ T_i[\mu_{E_k}(e(n)), \mu_{\Delta E_k}(\Delta e(n)), \mu_{\Delta U_k}(\Delta u)] \right], \quad i, j = 1, \ldots, 5. \quad (12)
$$

If $i = j$ in (12), each $T_i$ works with its own $T_i^*$, while when $i \neq j$, mixed logic is used. For example, $T_1$ may work with any of the five T-conorms. This makes the twenty-five couples of T-norms and T-conorms as shown in Table 2. The purpose here is to investigate how each couple affects the performance of the proposed simple fuzzy controller.

In order to obtain more general conclusions, several different plants are used which are given as follows:

1. First-order plant: $y' + ay = bu$.
2. Second-order plant: $y'' + ay' + by = cu$.
3. Plant with time delay: $y' + ay = u(t - t_0)U(t - t_0)$, where

$$
U(t) = \begin{cases} 
1 & t > 0, \\
0 & t \leq 0.
\end{cases}
$$

4. Nonlinear plant: $y'' + (|y| - a)y' + by = cu$.

Here $a$, $b$, $c$ and $t_0$ are constants.

The simulation results for twenty-five couples of T-norms and T-conorms are shown in Figures 3 to 8. The results of the first column in Table 2 for the above four plants with the same step reference input are given in Figure 3. For example, Figure 3(a) shows the responses of the first-order plant for a step input. Each curve in a figure represents the output of the plant controlled by the fuzzy controller which uses the indicated forms of T-norm and T-conorm. Similarly, Figures 4 to 8 show the results of column 2 to 6 in Table 2. Figure 4 shows the output of all four plants for column 2. For simplicity, however, only two plants are used in columns 3 to 6.

It must be stated, however, that because of the complexity involved in fuzzy inference processes and defuzzification, obtaining an analytical expression of the non-fuzzy process input in terms of a T-norm, a T-conorm, the process error $e(n)$

<p>| Table 2. Six columns of $T$ and $T^*$ and their simulation results |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|</p>
<table>
<thead>
<tr>
<th>Response</th>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
<th>Column 5</th>
<th>Column 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasing speed of system response</td>
<td>$T_1$, $T_3^*$</td>
<td>$T_1$, $T_1^*$</td>
<td>$T_2$, $T_1^*$</td>
<td>$T_3$, $T_1^*$</td>
<td>$T_4$, $T_1^*$</td>
<td>$T_5$, $T_1^*$</td>
</tr>
<tr>
<td>Increase speed of system response</td>
<td>$T_2$, $T_2$</td>
<td>$T_1$, $T_2^*$</td>
<td>$T_2$, $T_2^*$</td>
<td>$T_3$, $T_2^*$</td>
<td>$T_4$, $T_2^*$</td>
<td>$T_5$, $T_2^*$</td>
</tr>
<tr>
<td>Increase speed of system response</td>
<td>$T_2$, $T_2$</td>
<td>$T_1$, $T_2^*$</td>
<td>$T_2$, $T_2^*$</td>
<td>$T_3$, $T_2^*$</td>
<td>$T_4$, $T_2^*$</td>
<td>$T_5$, $T_2^*$</td>
</tr>
<tr>
<td>Increase speed of system response</td>
<td>$T_1$, $T_1^*$</td>
<td>$T_1$, $T_3^*$</td>
<td>$T_2$, $T_3^*$</td>
<td>$T_3$, $T_3^*$</td>
<td>$T_4$, $T_3^*$</td>
<td>$T_5$, $T_3^*$</td>
</tr>
</tbody>
</table>
and the change in the process error $\Delta e(n)$ is extremely difficult. Therefore, it seems to be impossible to obtain analytical solutions and conclusions on the studies given in this paper.

From the process outputs shown in Figures 3 to 8, a general conclusion that can be drawn is that there are differences between using one couple of T-norms and T-conorms or another, under the conditions proposed here for most of the twenty five couples of the T-norms and T-conorms.

Consider first the results of column 1 in Table 2 in which every T-norm was used with its dual. In Figure 3, which represent the responses of a first-order plant, a second-order plant, a first-order plant with time delay, and a nonlinear plant to a step input, it is obvious that the response is faster for the couple with a smaller T-norm and a larger T-conorm. However, there is an exception to this conclusion and it is the couple $T_3$ and $T_3^*$. The responses of all the four plants controlled by the fuzzy controller which uses $T_3$ and $T_3^*$ tend to be oscillatory when they approach the steady states, although they are much faster than others. However, the steady-state error may be reduced if appropriate scaling factors, $GE$, $GAE$ and $GAU$, are chosen or if the poles of the plant (first- or second-order) are farther from the $j\omega$ axis. In general, whatever the plant, the fuzzy controller with $T_5$ and $T_5^*$ is the best, that with $T_1$ and $T_1^*$ is the worst, and the others are in between except that with $T_3$ and $T_3^*$. It appears that this order does not change, although the curves of the process outputs may come closer to each other, or become more separated when the scaling factors are changed. From the results of column 2 to column 6 in Table 2, it seems that the process outputs in each column come even closer than those given in column 1. However, an interesting result is that when a T-norm, $T_i$, works with a smaller T-conorm $T_j$ ($i, j = 1, 2, 3, 4, 5$), the process responses are faster as shown in Figures 3 to 8. For $T_3$ and $T_3^*$, this conclusion is not very clear as shown in Figure 6. All of these results are summarized in Table 2.

It must be noted that the conclusions given above are obtained under the conditions proposed in this study. However, they may or may not be universal. More extensive studies of this problem will have to be carried out in future work.

5. Conclusions

The fuzzy control algorithm proposed in this paper is an extension of the conventional one. This is done by replacing the $\text{MIN}$ and $\text{MAX}$ operators in the conventional control algorithm by the generalized notion of T-operators. This new fuzzy control algorithm provides a general and flexible methodology for the design of fuzzy logic controllers.

By computer simulations, the effects of some typical T-operators on the performance of a simple fuzzy logic controller which has utilized the proposed general fuzzy control algorithm are studied and the difference among these typical T-operators in terms of control problems are investigated. Generally speaking, in the proposed fuzzy controller, using one couple of T-norms and T-conorms or another does make a difference in terms of the system responses. Some
interesting results are obtained. It is hoped that these results will help controller
designers in the selection of T-operators for their particular control problems.
Further studies are underway using the notion of adaptivity which optimizes the
location and width of the membership function during the operation of the plant.

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