Assignment 6 (Solutions):

4.39 The distance Y is uniformly distributed on the interval A to B, If she is closer to A, she has landed in the interval (A, $\frac{A+B}{2}$). This is one half the total interval length, so the probability is .5. If her distance to A is more than three times her distance to B, she has landed in the interval ($\frac{3B+A}{4}$, B). This is one quarter the total interval length, so the probability is .25.

4.42 The distribution function is $F(y) = \frac{y - \theta_1}{\theta_2 - \theta_1}$, for $\theta_1 \le y \le \theta_2$. For $F(\phi_{.5}) = .5$, then $\phi_{.5} = \theta_1 + .5(\theta_2 - \theta_1) = .5(\theta_2 + \theta_1)$. This is also the mean if the distribution.

4.48 Let Y = location of the selected point. Then, Y has a uniform distribution on the interval (0, 500).
a. $P(475 \le Y \le 500) = 1/20$
b. $P(0 \le Y \le 25) = 1/20$
c. $P(0 < Y < 250) = 1/2$.

4.50 Let Y = time when the phone call comes in. Then, Y has a uniform distribution on the interval (0, 5). The probability is $P(0 < Y < 1) + P(3 < Y < 4) = .4$.

4.59 **a.** $z_0 = 0$.
b. $z_0 = 1.10$
c. $z_0 = 1.645$
d. $z_0 = 2.576$

4.64 **a.** Note that the value 450 is (450 – 400)/20 = 2.5 standard deviations above the mean. So, $P(Z > 2.5) = .0062$.
b. The probability is .00618.
c. The top scale is for the standard normal and the bottom scale is for a normal distribution with mean 400 and standard deviation 20.

4.75 Let Y = volume filled, so that Y is normal with mean μ and $\sigma = .3$ oz. They require that $P(Y > 8) = .01$. For the standard normal, $P(Z > z_0) = .01$ when $z_0 = 2.33$. Therefore, it must hold that $2.33 = (8 - \mu)/.3$, so $\mu = 7.301$.

4.77 **a.** Let Y = SAT math score. Then, $P(Y < 550) = P(Z < .7) = 0.758$.

4.88 Let Y have an exponential distribution with $\beta = 2.4$.

a. $P(Y > 3) = \int_3^\infty \frac{1}{2.4} e^{-y/2.4} dy = e^{-3/2.4} = .2865$.

b. $P(2 \le Y \le 3) = \int_2^3 \frac{1}{2.4} e^{-y/2.4} dy = .1481$.

4.127 For α = β = 1, $f(y) = \frac{\Gamma(2)}{\Gamma(1)\Gamma(1)} y^{1-1}(1-y)^{1-1} = 1,\ 0 \le y \le 1$, which is the uniform distribution.