<u>Midterm Exam (Solutions)</u>

1(b):
Define F as "failure to learn". Then, $P(F|A) = 0.2$, $P(F|B) = 0.1$, $P(A) = 0.7$, $P(B) = 0.3$.
By Bayes' rule, $P(A|F) = \frac{P(F|A)\times P(A)}{P(F|A)\times P(A)+P(F|B)\times P(B)} = \frac{0.2\times 0.7}{0.2\times 0.7+0.1\times 0.3} = \frac{0.14}{0.17} \cong 82.35\%$.

2(b):
Since A and B are mutually exclusive, $A \cap B = \emptyset$, and consequently $P(A \cap B) = 0$.
For A and B to be independent, we need to have $P(A \cap B) = P(A)P(B)$. This is possible if and only if $P(B) = 0$.

3(b):
There are $\binom{50}{3}$ = 19,600 ways to choose the 3 winners. Each of these is equally likely.

a. There are $\binom{4}{3}$ = 4 ways for the organizers to win all of the prizes. The probability is 4/19600.

b. There are $\binom{4}{2}\binom{46}{1}$ = 276 ways the organizers can win two prizes and one of the other 46 people to win the third prize. So, the probability is 276/19600.

c. $\binom{4}{1}\binom{46}{2}$ = 4140. The probability is 4140/19600.

d. $\binom{46}{3}$ = 15,180. The probability is 15180/19600.

4(b):
Let Y = # of homeowners preferring brand A. Thus, Y is binomial with $n = 15$ and $p = 0.5$.
Using the Appendix, $P(Y \geq 10) = 1 - P(Y \leq 9) = 1 - 0.849 = 0.151$.

5(b):
Let Y = # of tosses until the first head. Thus, Y has geometric distribution with $p=q=1/2$.
$P(Y \geq 12 \mid Y > 10) = P(Y > 11 \mid Y > 10) = P(Y > 1) = 1/2$.