2.163

Let A be the event that current flows in design A and let B be defined similarly. Design A will function if (1 or 2) & (3 or 4) operate. Design B will function if (1 & 3) or (2 & 4) operate. Denote the event R_i = {relay i operates properly}, i = 1, 2, 3, 4. So, using independence and the addition rule,

$P(A) = (R_1 \cup R_2) \cap (R_3 \cup R_4) = (.9 + .9 - .9^2)(.9 + .9 - .9^2) = 0.9801$.

$P(B) = (R_1 \cap R_3) \cup (R_2 \cap R_4) = .9^2 + .9^2 - (.9^2)^2 = .9639$.

So, design A has the higher probability.

2.164

Using the notation from Ex. 2.163, $P(R_1 \cap R_4 \mid A) = P(R_1 \cap R_4 \cap A) / P(A)$.

Note that $R_1 \cap R_4 \cap A = R_1 \cap R_4$, since the event $R_1 \cap R_4$ represents a path for the current to flow. The probability of this above event is $.9^2 = .81$, and the conditional probability is in question is .81/.9801 = 0.8264.

2.165

Using the notation from Ex. 2.163, $P(R_1 \cap R_4 \mid B) = P(R_1 \cap R_4 \cap B) / P(B)$.

$R_1 \cap R_4 \cap B = (R_1 \cap R_4) \cap (R_1 \cap R_3) \cup (R_2 \cap R_4) = (R_1 \cap R_4 \cap R_3) \cup (R_2 \cap R_4)$. The probability of the above event is $.9^3 + .9^2 - .9^4 = 0.8829$. So, the conditional probability in question is .8829/.9639 = 0.916.

3.10

Denote R as the event a rental occurs on a given day and N denotes no rental. Thus, the sequence of interest is RR, RNR, $RNNR$, $RNNNR$, … . Consider the position immediately following the first R: it is filled by an R with probability .2 and by an N with probability .8. Thus, $P(Y = 0)$ = .2, $P(Y = 1) = .8(.2) = .16$, $P(Y = 2) = .128$, … . In general,

$$P(Y = y) = .2(.8)^y,\ y = 0, 1, 2, \ldots .$$