Monotonicity and Complexity of Digraph Search Problems

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Abstract

Given a digraph, suppose that there is an intruder hiding on vertices or along edges. A digraph search problem is to find the minimum number of searchers required to capture the intruder under some conditions. In this talk, we consider the monotonicity and complexity of five digraph search problems: directed searching, mixed directed searching, internal directed searching, internal strong searching, and internal weak searching. In the first three search problems, both searchers and intruder must follow the edge directions when they move along edges. In the internal strong search problem, the intruders must move in the edge directions but searchers need not. In the internal weak search problem, searchers must move in the edge directions but intruders need not. There are three actions for searchers in the first two search problems: placing, removing and sliding, and there are only two actions for searchers in the last three internal search problems: placing and sliding. Note that the internal strong searching is a "strong" version of the internal directed searching, the internal weak searching is a "weak" version of the internal directed searching, and the internal edge searching is an analogy of the internal directed searching on undirected graphs. We prove that the first three problems are monotonic and the last two problems are non-monotonic, respectively. It is interesting that the internal directed searching is monotonic while the internal strong searching, the internal weak searching and the the internal edge searching are all non-monotonic. We also show that the first four problems are $\mathcal{NP}$-complete and the last problem is $\mathcal{NP}$-hard. We solve the open problem on whether a non-monotonic searching problem can be $\mathcal{NP}$-complete.

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