Efficient Generation of Hamilton Cycles in Restricted and Generalized Rotator Graphs

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Abstract. The rotator graph has vertices labeled by the permutations of n in one line notation, and there is an arc from u to v if a prefix of u's label can be rotated to obtain v's label. In other words, it is the directed Cayley graph whose generators are $\sigma_k := (1\ 2\ \cdots\ k)$ for $2 \leq k \leq n$ and these rotations are applied to the indices of a permutation.

In a restricted rotator graph the allowable rotations are restricted from $k \in \{2, 3, \ldots, n\}$ to $k \in G$ for some smaller (finite) set $G \subseteq \{2, 3, \ldots, n\}$. We construct Hamilton cycles for restricted rotator graphs with $G = \{n-1, n\}$ and $G = \{2, 3, n\}$, and provide efficient iterative algorithms for generating them. The Hamilton cycles are grown recursively using simple sequence operations that we call 'reusing' and 'recycling'.

In a generalized rotator graph the vertices are instead labeled by the permutations of a given multiset M. Generalized rotator graphs can be restricted in the same way as rotator graphs. We construct a Hamilton cycle for every restricted generalized rotator graph with $G = \{n-1, n\}$. The Hamilton cycles are obtained by applying the necklace-prime algorithm to the cool-lex order of the permutations of the multiset M.

This talk involves collaborative work with Alexander Holroyd, Frank Ruskey, Joe Sawada, Brett Stevens, and Dennis Wong.

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