Force Balancing of Robotic Mechanisms Based on Adjustment of Kinematic Parameters

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Force balancing is a very important issue in mechanism design and has only recently been introduced to the design of robotic mechanisms. In this paper, a force balancing method called adjusting kinematic parameters (AKP) for robotic mechanisms or real-time controllable (RTC) mechanisms is proposed, as opposed to force balancing methods, e.g., the counterweights (CW) method. Both the working principle of the AKP method and the design equation with which to construct a force balanced mechanism are described in detail. A particular implementation of the AKP method for the RTC mechanisms where two pivots on a link are adjustable is presented. A comparison of the two methods, namely the AKP method and the CW method, is made for two RTC mechanisms with different mass distribution. The joint forces and torques are calculated for the trajectory tracking of the RTC mechanisms. The result shows that the AKP method is consistently better than the CW method in terms of the reduction of the joint forces and the torques in the servomotors, and the smoothing of the fluctuation of the joint force. [DOI: 10.1115/1.1864116]

1 Introduction

Mechanisms driven by real-time controllable (RTC) motors (or servomotors) are called RTC mechanisms. The RTC mechanisms are building blocks in many machine tools and advanced robots due to their flexibility in terms of adapting to different applications without the need of changing the dimension and shape of their components. The main generic task of a RTC mechanism can be defined as trajectory tracking. There is a controller in the RTC mechanism for the trajectory tracking task. Therefore, designing a RTC mechanism involves both the mechanical and the controller designs. This may give a unique opportunity for integration of these two design aspects into an optimal one.

In this paper, the mechanical design of a RTC mechanism concerns about force balancing. Force balancing is a very important issue in mechanism design. It is defined as a set of conditions under which the weight of the links of a mechanism does not require any torque or force at the actuators under static conditions for any configuration of the mechanism. The force balancing is important for the dynamic balancing of RTC mechanisms. There are many benefits for a RTC mechanism that is force balanced, in particular in the application areas such as machine tools and flight or driving simulators.

Force balancing is typically achieved by introducing additional mechanical elements to the system. There are two different principles for force balancing: (i) making the total mass center of a mechanism stationary [1–5] using the counterweights, and (ii) making the total potential energy of a mechanism constant [7–10] using the elastic elements (add-springs). Many studies have been conducted on force balancing of planar and spatial mechanisms based on principle (i). According to this principle, in order to balance the linked mechanisms, it is necessary to make the total mass center of the mechanisms invariant. For this, the linear independent vector approach was widely used [1]. Lowen et al. [3] developed this theory and solved the problem of full force balancing of general planar linkages by means of mass redistribution [3]. The mass redistribution method is also called the counterweights (CW) method. Some other investigations using principle (i) were reported by Kochev [4] and by Yu [5]. Agrawal et al. [6] discussed the force balancing problem for a planar mechanism using auxiliary parallelograms. The CW method has the disadvantages in the increase of joint forces and the driving torques. Furthermore, the introduction of additional masses in the mechanism tends to increase the inertia. When the mechanism runs at a varying high speed, the increase of the inertia leads to the increase of energy consumption, and this is undesired.

Studies based on principle (ii), specifically by adding springs, are reported. Streit and Gilmore [7] studied the force balancing of a rigid body, Gosselin [8] discussed the force balancing of a parallel mechanism, and Ebert-Uphoff et al. [9] discussed the balancing of a spatial parallel platform. The spring balancer and gravity balancer was compared for a simple mechanism in Ref. [10]. It should be noted that the add-springs method for force balancing relies on the use of springs, which is difficult to implement in some applications. Also, a mechanism using this method can only be balanced in one direction, i.e., the gravity direction. Furthermore, the effects of the masses of the added springs are very difficult to be taken into account in design.

An observation is made that all these studies in essence apply those methods applicable to non-RTC mechanisms and RTC mechanisms. As implied earlier, the controller for the RTC mechanism may affect the mechanical design—mass distribution for force balancing in this case. This is a basic idea behind the proposed adjusting kinematic parameters (AKP) approach reported in this paper. This paper is organized as follows. Section 2 presents the working principle of the AKP method. Section 3 derives the design equation of the AKP method. Section 4 gives two examples to demonstrate the effectiveness of the AKP method, as opposed to the CW method, in terms of the joint forces and the driving torques. Section 5 presents conclusions with discussion.

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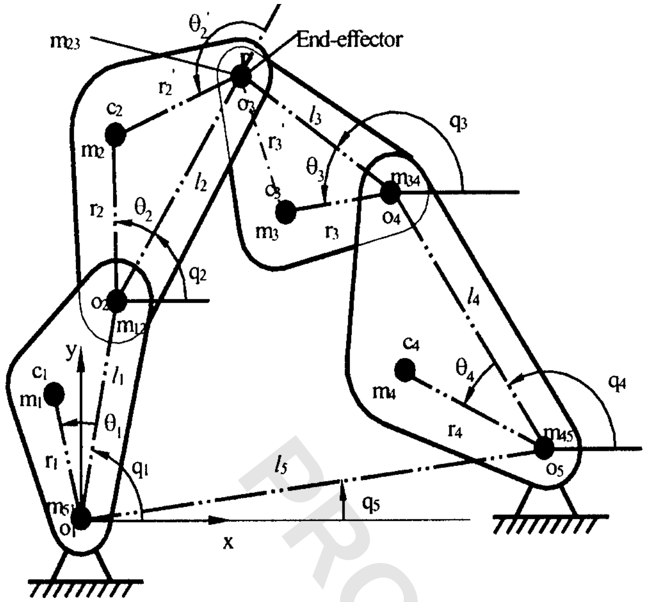

Fig. 1 Scheme of a RTC mechanism

2 The Working Principle of the AKP Method

The AKP method starts with principle (i). According to Ref. [1], by taking the whole mechanism as a separate force diagram, we have the following equation:

$$f_0 = -\frac{d}{dt}\{M\dot{r}_c\} + Mg \tag{1}$$

where M is the total mass of the mechanism, f_0 is the total force applied on the mechanism, g is the gravitational acceleration, and $\dot{r}_c$ is the velocity of the *mass center* (MC) of the mechanism. It can be seen from Eq. (1) that the undesired shaking force results from the change in the system's linear momentum. This item becomes zero if the MC does not change in any configuration (i.e., r_c=constant). In order for a dynamic system to achieve the configuration invariant MC, the dimension and the mass distribution scheme must be subject to some constraints. These constraints are represented in a form of equations, called the condition equation. In the following, the LVP [1] is applied to an exemplar system shown in Fig. 1. In Fig. 1, two rotary servomotors are mounted at joints O_1 and O_5 and described by joint variables q_1 and q_4, respectively. Also, O_1 and O_5 are the parts of the ground. Therefore, q_5 is constant. Point P (overlapping with O_3) represents the end effector of the system.

Now with slight departure from the standard procedure of the LVP approach, we separately define the mass of the link component, denoted by m_i (i is the identifier of a link), and the mass of the joint between any two links, denoted by $m_{i,j}$ (the link i and link j are directly connected). After that, we can write the position vector of the MC of the system as

$$\mathbf{r}_c = \frac{1}{M}\left(\sum_{i=1}^{4} m_i\mathbf{r}_i + \sum_{i=1}^{4} m_{i,i+1}\mathbf{l}_i\right) \tag{2}$$

where $\mathbf{r}_i$ is the position vector of the mass center of link I, and $\mathbf{l}_i$ is the link vector (see Fig. 1). The vectors $\mathbf{r}_i$ and $\mathbf{l}_i$ can be expressed as

$$\begin{cases} \mathbf{r}_1 = r_1 e^{\mathbf{i}(q_1+\theta_1)} \\ \mathbf{r}_2 = l_1 e^{\mathbf{i}q_1} + r_2 e^{\mathbf{i}(q_2+\theta_2)} \\ \mathbf{r}_3 = l_5 e^{\mathbf{i}q_5} + l_4 e^{\mathbf{i}q_4} + r_3 e^{\mathbf{i}(q_3+\theta_3)} \\ \mathbf{r}_4 = l_5 e^{\mathbf{i}q_5} + r_4 e^{\mathbf{i}(q_4+\theta_4)} \end{cases} \tag{3}$$

$$\begin{cases} \mathbf{l}_1 = l_1 e^{\mathbf{i}q_1} \\ \mathbf{l}_2 = l_1 e^{\mathbf{i}q_1} + l_2 e^{\mathbf{i}q_2} \\ \mathbf{l}_3 = l_5 e^{\mathbf{i}q_5} + l_4 e^{\mathbf{i}q_4} \\ \mathbf{l}_4 = l_5 e^{\mathbf{i}q_5} \end{cases} \tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (2) leads to

$$\begin{aligned} M\mathbf{r}_c &= (m_3 + m_4 + m_{34} + m_{45})l_5 e^{\mathbf{i}q_5} + (m_1 r_1 e^{\mathbf{i}\theta_1} + m_2 l_1 + m_{12} l_1 \\ &\quad + m_{23} l_1)e^{\mathbf{i}q_1} + (m_2 r_2 e^{i\theta_2} + m_{23} l_2)e^{\mathbf{i}q_2} + m_3 r_3 e^{\mathbf{i}\theta_3} e^{\mathbf{i}q_3} \\ &\quad + (m_4 r_4 e^{\mathbf{i}\theta_4} + m_{34} l_4 + m_3 l_4)e^{\mathbf{i}q_4} \end{aligned} \tag{5}$$

The unit vectors $e^{\mathbf{i}q_1}$, $e^{\mathbf{i}q_2}$, $e^{\mathbf{i}q_3}$, and $e^{\mathbf{i}q_4}$ are constrained by the kinematic closed-loop equation $l_1 e^{\mathbf{i}q_1} + l_2 e^{\mathbf{i}q_2} - l_3 e^{\mathbf{i}q_3} - l_4 e^{\mathbf{i}q_4} - l_5 e^{\mathbf{i}q_5} = 0$. Substituting this constraint equation into Eq. (5) leads to

$$\begin{aligned} M\mathbf{r}_c &= (m_3 + m_4 + m_{34} + m_{45} + (m_2 r_2 e^{i\theta_2} + m_{23} l_2)/l_2)l_5 e^{\mathbf{i}q_5} \\ &\quad + (m_1 r_1 e^{i\theta_1} + (m_2 + m_{12})l_1 - l_1 m_2 r_2 e^{i\theta_2}/l_2)e^{\mathbf{i}q_1} + (m_3 r_3 e^{i\theta_3} \\ &\quad + m_{23} l_3 + l_3 m_2 r_2 e^{i\theta_2}/l_2)e^{\mathbf{i}q_3} + (m_4 r_4 e^{i\theta_4} + (m_3 + m_{23} + m_{34})l_4 \\ &\quad + l_4 m_2 r_2 e^{i\theta_2}/l_2)e^{\mathbf{i}q_4} \end{aligned} \tag{6}$$

It should be noted that q_5 is constant as mentioned before. Therefore, there are three configuration-dependent variables in Eq. (6), namely, q_1, q_3, and q_4. In order to make the MC stationary, all the coefficients of those three variables must be set to zero. This results in the following equations:

$$m_1 r_1 e^{i\theta_1} + (m_2 + m_{12})l_1 - l_1 m_2 r_2 e^{i\theta_2}/l_2 = 0 \tag{7}$$

$$m_3 r_3 e^{i\theta_3} + m_{23} l_3 + l_3 m_2 r_2 e^{i\theta_2}/l_2 = 0 \tag{8}$$

$$m_4 r_4 e^{i\theta_4} + (m_3 + m_{23} + m_{34})l_4 + l_4 m_2 r_2 e^{i\theta_2}/l_2 = 0 \tag{9}$$

Using the relationship $r_2 e^{\mathbf{i}\theta_2} = l_2 + r_2' e^{\mathbf{i}\theta_2'}$, Eq. (7) can be rewritten as

$$l_2 m_1 r_1 e^{i\theta_1} = l_1 m_2 \tilde{r}_2' e^{i\tilde{\theta}_2'} \tag{10}$$

where $\tilde{r}_2' = \sqrt{(r_2' \cos\theta_2' - m_{12} l_2/m_2)^2 + (r_2' \sin\theta_2')^2}$, $\tilde{\theta}_2' = \tan^{-1}[r_2' \sin\theta_2' / (r_2' \cos\theta_2' - m_{12} l_2/m_2)]$

Similarly, Eq. (8) can be rewritten as

$$l_2 m_3 r_3 e^{i\theta_3} + l_3 m_2 \tilde{r}_2 e^{i\tilde{\theta}_2} = 0 \tag{11}$$

where $\tilde{r}_2 = \sqrt{(r_2 \cos\theta_2 + m_{23} l_2/m_2)^2 + (r_2 \sin\theta_2)^2}$, $\tilde{\theta}_2 = \tan^{-1}[r_2 \sin\theta_2 / (r_2 \cos\theta_2 + m_{23} l_2/m_2)]$ Likewise, Eq. (9) can be rewritten as

$$l_2 m_4 r_4 e^{i\theta_4} + l_4 m_2 \hat{r}_2 e^{i\hat{\theta}_2} = 0 \tag{12}$$

where $\hat{r}_2 = \sqrt{(r_2 \cos\theta_2 + (m_3 + m_{23} + m_{34})l_2/m_2)^2 + (r_2 \sin\theta_2)^2}$

$$\hat{\theta}_2 = \tan^{-1}\left(\frac{r_2 \sin\theta_2}{r_2 \cos\theta_2 + (m_3 + m_{23} + m_{34})l_2/m_2}\right)$$

Equations (10)–(12) are called the condition equations for the mechanism shown in Fig. 1. They can be further written into the following equations:

$$l_2 m_1 r_1 = l_1 m_2 \tilde{r}_2' \text{ and } \theta_1 = \tilde{\theta}_2' \tag{13}$$

$$l_2 m_3 r_3 = l_3 m_2 \tilde{r}_2 \text{ and } \theta_3 = \tilde{\theta}_2 + \pi \tag{14}$$

$$l_2 m_4 r_4 = l_4 m_2 \hat{r}_2 \text{ and } \theta_4 = \hat{\theta}_2 + \pi \tag{15}$$

The condition equations can be satisfied through two ways: (1) redistributing the mass center while maintaining the kinematic parameters unchanged, and (2) changing the kinematic parameters, l_i $(i=1,2,3,4)$ while maintaining the total mass of the mechanism unchanged. It is noted that when the kinematic parameter changes, the parameters r_i and θ_i will change, accordingly. The first method is the CW method, and the second method is the

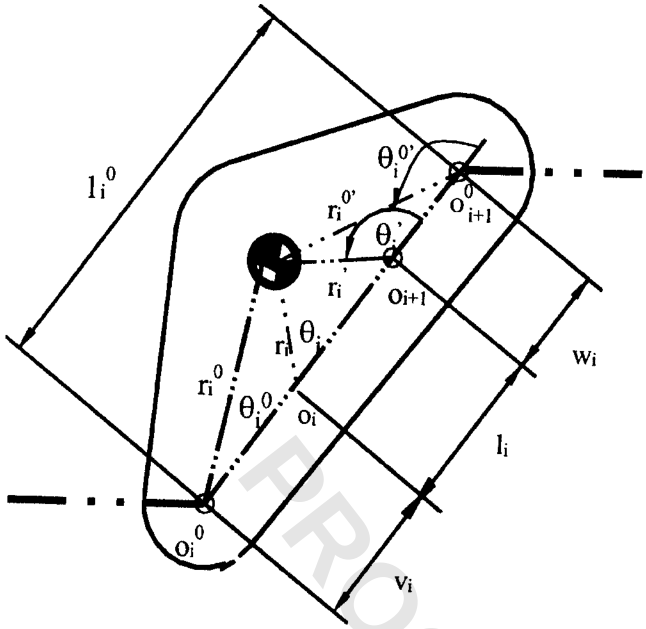

Fig. 2 Two-step kinematic parameter adjustment in the AKP method

AKP method. When the kinematic parameter changes, the joint motion will change as the kinematic parameter is in the inverse kinematic relation. This implies that the controller at the joint level must replan the motion, which results in a trajectory. The same control algorithm can readily be applied to the trajectory, which makes the AKP method only viable for the RTC mechanism. It is noted, however, that when the RTC mechanism has only one servomotor, e.g., a four-bar link mechanism, the AKP method is not applicable because the shape of the trajectory of the end effector is determined exclusively by the kinematic parameter. So the AKP method is applicable to the RTC mechanism (both planar and spatial and both closed-loop and open-loop) which has more than one degree-of-freedom (DOF).

The next section presents the design equation for computing the adjustment of the kinematic parameters for the AKP method. It is to be noted that not all the kinematic parameters need to be adjusted to satisfy the condition equations.

3 The Design Equation of the AKP Method

We consider two situations: (1) the off-line situation (the mass center of a link is off the link axis), and (2) the in-line situation (the mass center of a link is on the link axis). In the off-line situation, assume that the mass centers of the links are arbitrarily distributed, i.e., $\theta_i \neq 0$ for $i=1$ to 4, as shown in Fig. 2. Let l_i^0 and (r_i^0, θ_i^0) represent the length and the mass center of link i, respectively, where the superscript "0" indicates the initial parameter prior to the adjustment of the pivot o_i^0. To satisfy the condition Eqs. (13)–(15), adjustment of two pivots is necessary. The first step is to adjust a pivot from o_i^0 to o_i so that the angle relationship between link i and link $i+1$ can be satisfied. The second step is to adjust o_{i+1}^0 to o_{i+1} so that all Eqs. (13)–(15) can be satisfied. Now, let l_i and (r_i, θ_i) represent the new length and the new mass center of link i, respectively, and let v_i and w_i represent the amount of adjustment of two pivots o_i and o_{i+1}, respectively.

From Fig. 2, the following equations can be obtained:

$$l_i = l_i^0 - v_i - w_i \tag{16}$$

$$r_i^0 \cos\theta_i^0 = v_i + r_i \cos\theta_i \tag{17}$$

$$r_i^0 \sin\theta_i^0 = r_i \sin\theta_i \tag{18}$$

for $i=1,2,3,4$.

Equation (18) can be rewritten as

$$r_i = r_i^0 \sin\theta_i^0 / \sin\theta_i \tag{19}$$

Substituting Eq. (19) into Eq. (17) yields

$$v_i = r_i^0 \sin(\theta_i - \theta_i^0)/\sin\theta_i \tag{20}$$

There are various possibilities of adjusting pivots on links in order to satisfy the condition equation. As link 2 has relationships with all other three moving links, it is convenient to use link 2 as a reference. In the case all the links are changeable, we first adjust l_2 and obtain the mass distribution from Eq. (19) and (20), then use the following equations to determine the adjustment of l_1, l_3, and l_4, respectively. Therefore, we only consider a case where l_2 is unchanged. In that case, we will need to change l_1, l_3, and l_4, respectively. The amounts of adjustments of l_1, l_3, and l_4 are, respectively calculated as follows:

3.1 For link 1. Pivot O_1 on link 1 is adjusted using the following equation derived from Eqs. (20) and (13):

$$v_1 = r_1^0 \sin(\tilde{\theta}_2' - \theta_1^0)/\sin\tilde{\theta}_2' \tag{21}$$

To determine the amount of adjustment of pivot O_2 on link 1, i.e., w_1, substituting Eq. (21) and Eqs. (16)–(18) into Eq. (13) yields

$$w_1 = l_1^0 - \frac{r_1^0(m_2\tilde{r}_2' \sin(\tilde{\theta}_2' - \theta_1^0) + m_1 l_2 \sin\theta_1^0)}{m_2\tilde{r}_2' \sin\tilde{\theta}_2'} \tag{22}$$

3.2 For link 3. Substituting Eq. (14) into Eq. (20) yields

$$v_3 = r_3^0 \sin(\tilde{\theta}_2 - \theta_3^0)/\sin\tilde{\theta}_2 \tag{23}$$

Substituting Eq. (23) and Eqs. (16)–(18) into Eq. (14) yields

$$w_3 = l_3^0 - \frac{r_3^0(m_2\tilde{r}_2 \sin(\tilde{\theta}_2 - \theta_3^0) - m_3 l_2 \sin\theta_3^0)}{m_2\tilde{r}_2 \sin\tilde{\theta}_2} \tag{24}$$

3.3 For link 4. Substituting Eq. (15) into Eq. (20) yields

$$v_4 = r_4^0 \sin(\hat{\theta}_2 - \theta_4^0)/\sin\hat{\theta}_2 \tag{25}$$

Substituting Eq. (25) and Eqs. (16)–(18) into Eq. (15) yields

$$w_4 = l_4^0 - \frac{r_4^0(m_2\hat{r}_2 \sin(\hat{\theta}_2 - \theta_4^0) - m_4 l_2 \sin\theta_4^0)}{m_2\hat{r}_2 \sin\hat{\theta}_2} \tag{26}$$

In the in-line situation, adjusting only one pivot for each link is sufficient. In this case, since the conditions $\theta_i^0=0$ or π, and $\theta_i=0$ or π hold, Eq. (18) thus becomes an identical one. Furthermore, if we ignore the pivot mass, $m_{i,j}$, we can obtain the following condition equations for force balancing, i.e.:

$$l_2 m_1 r_1 = l_1 m_2 r_2' \text{ and } \theta_1 = \theta_2' \tag{27}$$

$$l_2 m_3 r_3 = l_3 m_2 r_2 \text{ and } \theta_3 = \theta_2 + \pi \tag{28}$$

$$l_3 m_4 r_4 = l_4 m_3 r_3' \text{ and } \theta_4 = \theta_3' \tag{29}$$

Substituting Eqs. (16) and (17) into Eqs. (27)–(29) yields

$$v_1 = \frac{m_2 r_2' l_1^0 \cos\theta_1 - m_1 r_1^0 l_2 \cos\theta_1^0}{m_2 r_2' \cos\theta_1 - m_1 l_2} \tag{30}$$

$$v_3 = \frac{m_2 r_2 l_3^0 \cos\theta_3 - m_3 r_3^0 l_2 \cos\theta_3^0}{m_2 r_2 \cos\theta_3 - m_3 l_2} \tag{31}$$

$$v_4 = \frac{k l_4^0 \cos\theta_4 - m_4 r_4^0 \cos\theta_4^0}{k \cos\theta_4 - m_4} \tag{32}$$

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Table 1 Parameters for different mechanisms (in-line)

Parameter	Unbalanced	CW	AKP
l_1 (m)	0.200	0.200	0.098
l_2 (m)	0.300	0.300	0.280
l_3 (m)	0.400	0.400	0.245
l_4 (m)	0.300	0.300	0.101
l_5 (m)	0.300	0.300	0.300
r_1 (m)	0.050	0.176	0.052
r_2 (m)	0.150	0.051	0.130
r_3 (m)	0.080	0.080	0.076
r_4 (m)	0.100	0.103	0.099
m_1 (kg)	0.250	0.591	0.250
m_2 (kg)	0.250	0.445	0.250
m_3 (kg)	0.375	0.375	0.375
m_4 (kg)	0.500	0.874	0.500
I_1 (kg m^2)	0.004	0.013	0.004
I_2 (kg m^2)	0.010	0.023	0.010
I_3 (kg m^2)	0.020	0.020	0.020
I_4 (kg m^2)	0.020	0.047	0.020
θ_1 (rad)	0	π	π
θ_2 (rad)	0	π	0
θ_3 (rad)	0	0	π
θ_4 (rad)	0	π	π

with $w_i=0$, $k=\sqrt{(m_2r_2/l_2)^2+m_3^2+(2m_2r_2m_3\cos\theta_2/l_2)}$.

Note that v_1, v_3, and v_4 are the amount of adjustment on links 1, 3, and 4, respectively.

4 Comparison of the CW and AKP Methods

Two examples are used here to show the effectiveness of the AKP method, as opposed to the CW method, in terms of the reduction of the joint forces and the driving torques.

4.1 Example 1 (in-line case). The kinematic parameters of the original unbalanced mechanism are listed in Table 1. Using the CW method and the AKP method, the kinematic parameters of the force balanced mechanism are computed and listed in the same table. In the CW case, assume that link 3 is unchanged and the other three links are subject to additional masses. In the AKP case, assume that all the movable links are subject to pivot adjustments. For simplicity, the pivot mass, $m_{i,j}$, is neglected.

The mechanism is supposed to fulfill the following task: the end effector moves from point A (0.3, 0.2) to point B (0.2, 0.3), and subsequently to point C (0.1, 0.2). The time durations are 1.0 and 2.0 s for low speed motions and 0.1 and 0.2 s for high speed motions, respectively. The unit of the coordinates is meter for all examples. Furthermore, for each segment of the trajectories, (i) the velocities of the end effector at these three points are zero, and (ii) the accelerations of the end effector at the initial and final points are zero. The trajectories at the two servomotors can be determined based on the inverse kinematics and the motion planning method, referring to Ref. [11]. The joint forces in the five pivots can be calculated using the software called "Working Model 2D" [12].

Figure 3 shows the joint forces in the two servomotors at low speeds. It can be seen from Fig. 3 that the force fluctuations in the two servomotors, respectively, for both the force balanced mechanisms are small (the AKP method is better than the CW method).

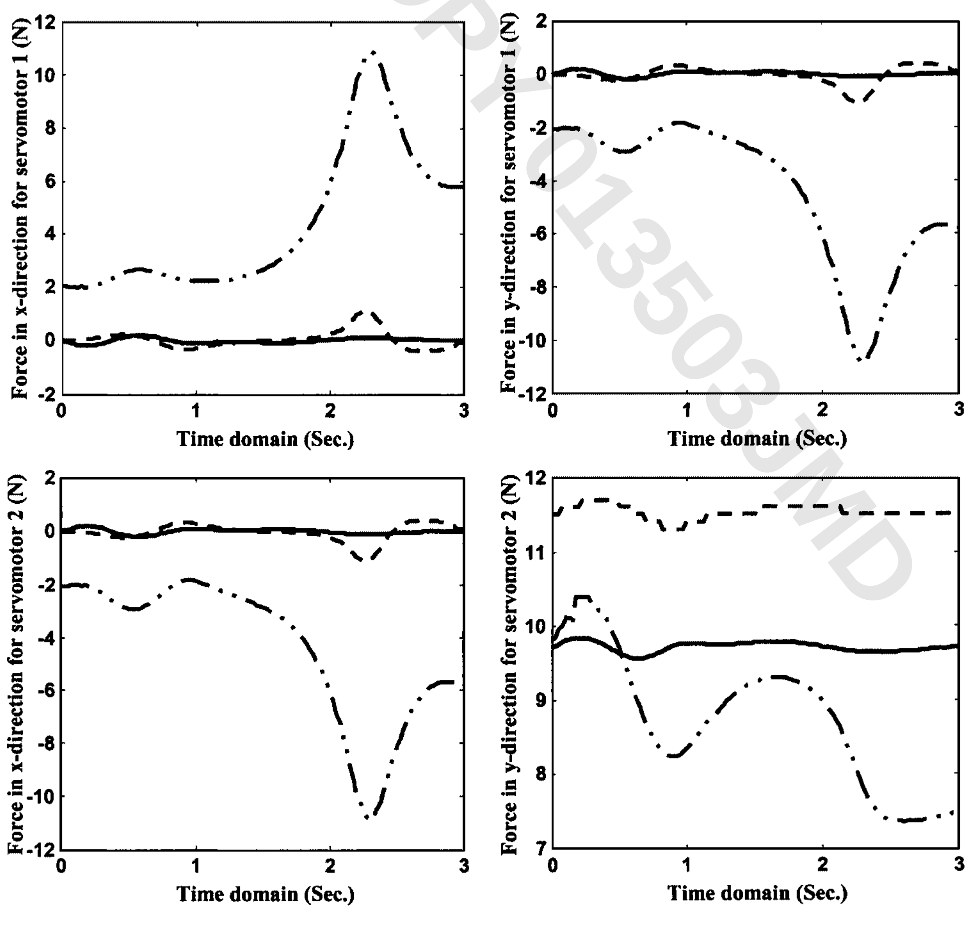

Fig. 3 The joint forces in two servomotors at low speeds for in-line case

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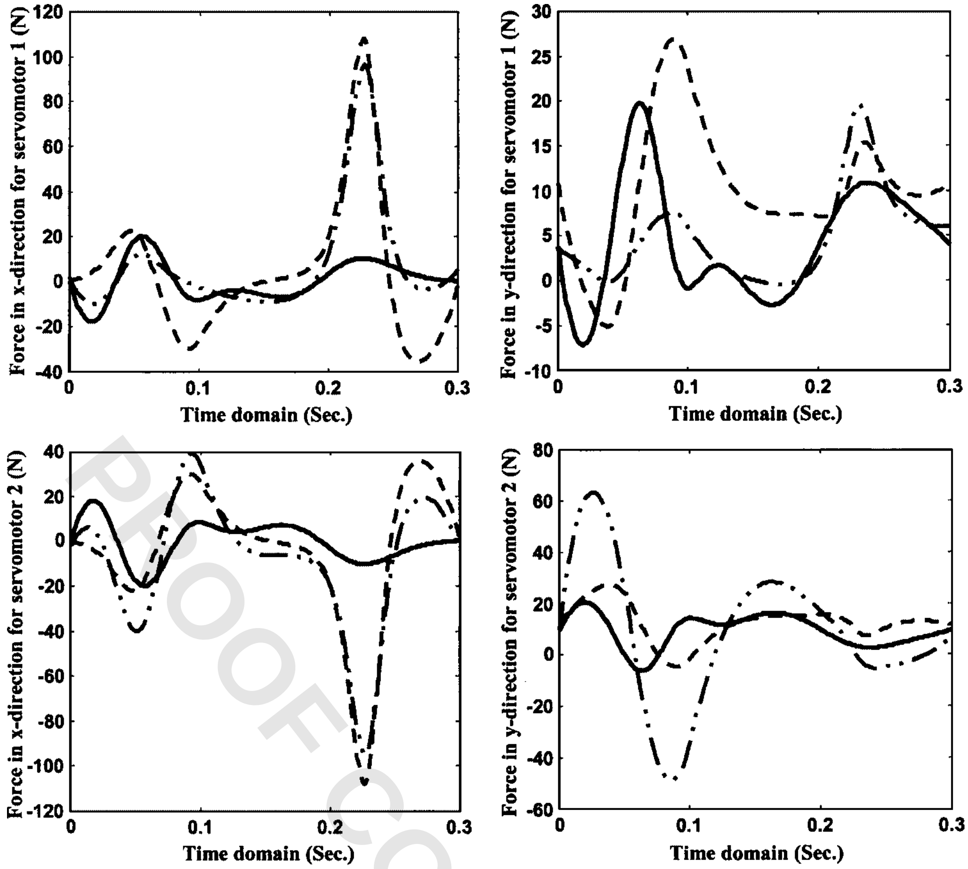

Fig. 4 The joint forces in two servomotors at high speeds for in-line case

In the *x* direction, the joint force in the two servomotors, respectively, for both force balanced mechanism, are very close (the AKP method appears slightly better than the CW method). Nevertheless, the force balanced mechanism is better than the unbalanced mechanism in terms of the joint force in the *x* direction. In the *y* direction, the joint forces using the CW method are significantly larger than those using the AKP method and even larger than those with the unbalanced mechanism. This phenomenon agrees on the known finding in literature, i.e., the CW method may increase the joint forces. It is interesting to observe that the joint force fluctuation in the both directions are small using both the CW and the AKP methods, while the joint forces of the unbalanced mechanism vary considerably, especially in the *x* direction. The reason for this can be explained. After the shaking force is cancelled, the mass center of the system is stationary during operation. Furthermore, the contribution of the inertia to the joint forces is small at low speeds. Therefore, the force fluctuation in the two servomotors is small for the balanced mechanism. On the other hand, for the unbalanced mechanism, the mass center of the mechanism changes with the system configurations. Therefore, the force fluctuation in the two servomotors is very large.

Figure 4 shows the results when the mechanism runs at high speeds (about 50 rpm). It is observed a totally different performance, as opposed to the low speed motion. First, the force fluctuation is very large for all three cases. Second, the CW method generates the worst performance in the *x* direction, though the total forces in the two servomotors in the *x* direction are still maintained at zero, the sharp force fluctuation exhibits. Nevertheless, the AKP method remains to produce the best performance. It should be noted that, for the force balanced mechanism, the total forces at the two servomotors should be zero in the *x* direction and should be equal to the total weight of the mechanism in the *y* direction. This observation is confirmed with the results shown in Figs. 3 and 4.

The earlier results are expected. When the mechanism runs at high speeds, the inertia forces becomes the dominant term in the dynamics. Since the CW method adopts the adding mass approach for force balancing purpose, its inertia force takes more contribution in the system dynamics. While with the AKP method, the total mass of the system is unchanged; therefore the inertia force does not differ significantly from the unbalanced mechanism.

Figures 5(*a*) and 5(*b*) show the driving torques in two servomotors at low speeds and high speeds, respectively. It can be seen that the AKP method requires less torque to drive the servomotors to finish the desired tracking than the CW method.

To further illustrate the effectiveness of the AKP method, Table 2 lists the minimum and maximum joint forces in the *x* axis and the *y* axis of each joint for the cases of applying the AKP method and the CW method, respectively. It is observed that the joint forces generated by the AKP method have a smaller fluctuation range than those by the CW method. The reduction of the forces in the *x* direction is more remarkable by using the AKP method than by using the CW method.

4.2 Example 2 (off-line case). In this example, a mechanism with the mass center off line of the kinematic axis is studied. The kinematic parameters of the original mechanism without force balancing and the modified mechanisms using the CW method and the AKP method are listed in Table 3, respectively. In the redesign of the mechanism, link 2 is assumed to be unchanged.

In this example, the mechanism is supposed to execute the following task: the end effector moves from point A (0.30, 0.15) to point C (0.10, 0.20). In this case, the two servomotors are moving from (52.14°, −29.08°) to (168.55°, 108.17°) for the AKP method and from (74.51°, −37.38°) to (149.25°, 68.83°) for the CW method, respectively. The time duration between any two posi-

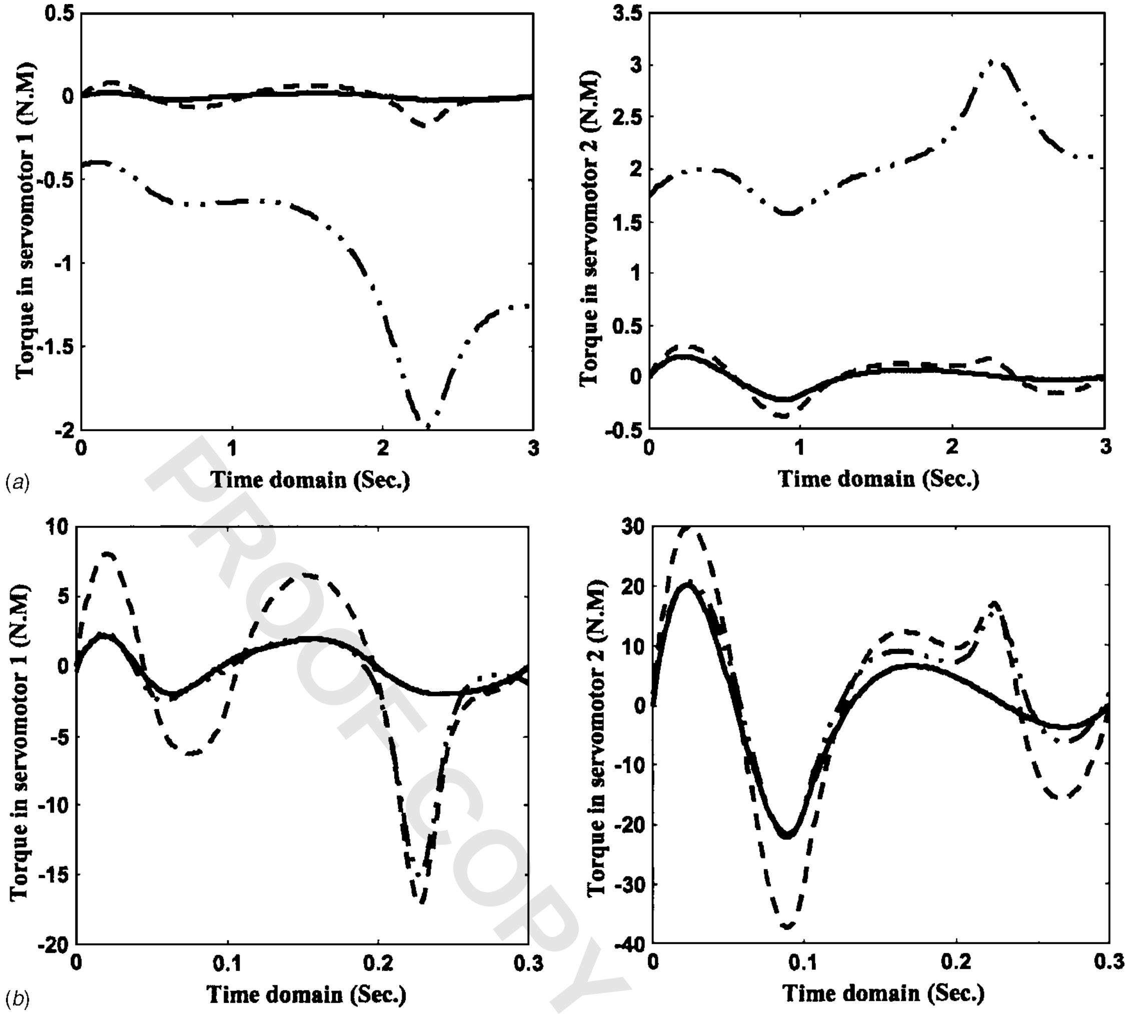

Fig. 5 The driving torques to drive two servomotors for in-line case: (a) for low speed and (b) for high speed

tions is 2 s. Furthermore, for the trajectory, the velocities and the accelerations of the end effector at the initial and final tracking positions are zero.

Figure 6 shows the total forces in the two servomotors for the balanced mechanisms using the AKP method and the CW method. From this figure, it can be seen that the maximum joint force of the AKP method is only about 60% of the maximum joint force of the CW method. Figure 7 shows the driving torques for the two servomotors to track the desired trajectory. It is shown that the driving torque is less than 30% for the AKP method than that for the CW method. This means that the AKP method for force balancing is more energy effective than the CW method.

Table 2 Joint forces using different force balancing methods

Pivot No.			1	2	3	4	5
Low Speed	AKP	Min-x	−0.205	−0.208	−0.233	−0.514	−0.204
		Max-x	0.204	0.228	0.164	0.444	0.205
		Min-y	3.640	−1.470	0.930	4.320	9.560
		Max-y	3.930	−1.200	1.320	5.300	9.840
	CW	Min-x	−0.405	−1.210	−1.130	−1.140	−1.130
		Max-x	1.130	0.338	0.387	0.487	0.405
		Min-y	10.701	−5.350	−0.869	2.430	11.302
		Max-y	11.102	−4.870	−0.595	3.420	11.701
High Speed	AKP	Min-x	−17.813	−20.632	−22.283	−51.154	−20.281
		Max-x	20.281	19.910	14.242	38.910	17.813
		Min-y	−7.072	−16.452	−19.005	−42.473	−6.212
		Max-y	19.691	8.583	17.073	47.121	20.552
	CW	Min-x	−35.492	−115.534	−108.233	−109.602	−108.006
		Max-x	108.006	29.691	33.982	42.663	35.492
		Min-y	5.042	−27.812	−12.402	−42.114	−4.491
		Max-y	26.901	15.913	12.191	45.263	27.440

Table 3 Parameters for different mechanisms (off-line)

Parameter	Unbalanced	CW	AKP
l_1 (m)	0.150	0.150	0.087
l_2 (m)	0.260	0.260	0.260
l_3 (m)	0.260	0.260	0.208
l_4 (m)	0.140	0.140	0.088
l_5 (m)	0.300	0.300	0.300
r_1 (m)	0.075	0.087	0.100
r_2 (m)	0.150	0.150	0.150
r_3 (m)	0.085	0.100	0.120
r_4 (m)	0.115	0.142	0.135
m_1 (kg)	0.5	1.0	0.5
m_2 (kg)	1.0	1.0	1.0
m_3 (kg)	1.0	1.5	1.0
m_4 (kg)	1.0	2.0	1.0
I_1 (kg m^2)	0.01	0.03	0.01
I_2 (kg m^2)	0.05	0.05	0.05
I_3 (kg m^2)	0.04	0.06	0.04
I_4 (kg m^2)	0.02	0.04	0.02
θ_1 (deg)	90.00	150.00	150.00
θ_2 (deg)	30.00	30.00	30.00
θ_3 (deg)	225.00	210.00	210.00
θ_4 (deg)	192.83	190.89	190.89

From the force profiles shown in these figures, it can be seen that, in order to perform the same motion task, the AKP method needs the least amount of forces at both low speeds and high speeds among all three design cases. The CW method demands the highest forces. The AKP method is thus demonstrated to be better than the CW method in terms of the joint force reduction, the joint force fluctuation smoothing, and the driving torque reduction. Furthermore, the RTC mechanism balanced using the AKP method can also obtain better tracking performance compared with the mechanism balanced using the CW method [13].

5 Conclusions and Discussion

A method for force balancing, called AKP, was proposed and presented in this paper. The key idea in the AKP method is that when a system is not in a force balancing status, change of the kinematic parameters can make the system balanced. This idea is not viable to the non-RTC mechanism, as the path or the trajectory of this kind of mechanism is exclusively determined by the kinematic parameter. This idea, however, suits the RTC mechanism with more than one degree of freedom, because for this kind of mechanism, the trajectory of the end effector of the mechanism can be changed by the controllers of the RTC motors or servomotors. As such, when the kinematic parameter is changed for the force balancing purpose, the effect on the trajectory due to the change of the kinematic parameter can be "compensated" by the controllers of the RTC motors. With the AKP method, both situations for the mass scheme for individual components, i.e., in-line mass center and off-line mass center, can be coped with. For the in-line mass center situation, the adjustment of one pivot on each of the relevant links is needed, and for the off-line mass center, two pivots on each of the relevant links need to be adjusted. The formulas for calculating the amount of adjustment are given. It is noted that for a specific RTC mechanism with more than one DOF, the minimum number of links that need to be adjusted de-

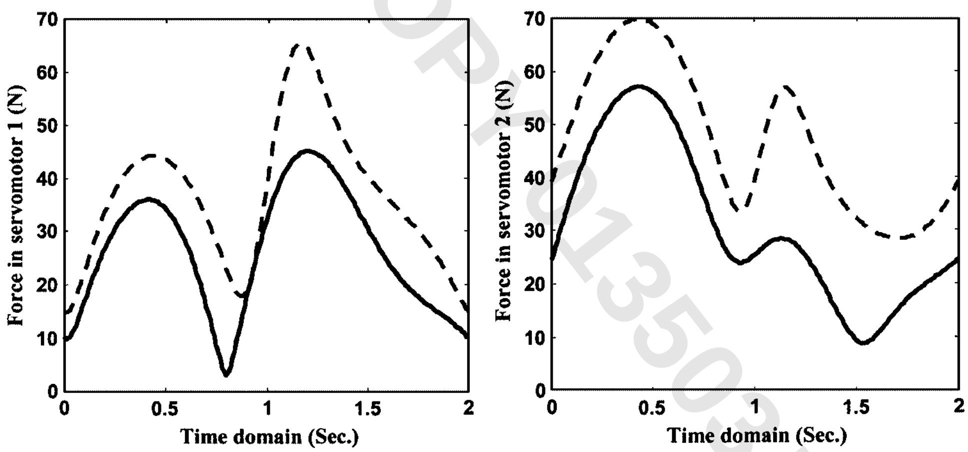

Fig. 6 The total forces in two servomotors for the off-line case

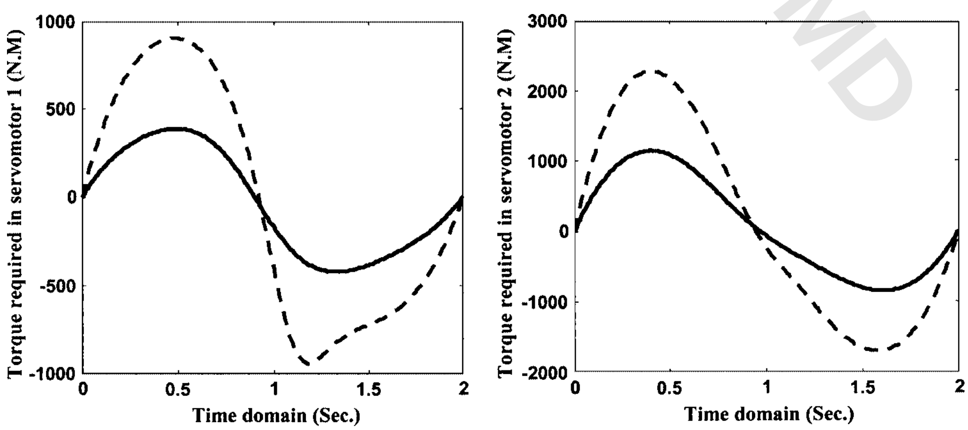

Fig. 7 The driving torques in two servomotors for the off-line case

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pends on a particular configuration of the mechanism, which is similar to the CW method. For instance, for a five-bar linkage with 2-DOF, the minimum number of links that need to be adjusted is 3.

Two different configurations for three cases: the force unbalanced mechanism, the force balanced mechanism using the AKP method and the CW method, respectively, were compared. Through simulation, it has been shown that the AKP method is better than the CW method in terms of (1) the reduction of the peak value of the joint force, (2) the smoothing of the fluctuation of the joint force at both low and high speeds, and (3) the reduction of the driving torque in the servomotors.

It should be noted that the workspace of the mechanism depends on the lengths of the links. The proposed AKP method for force balancing will change the workspace, so some regions of the original workspace may not be reachable. Therefore, it is not possible that the workspace after applying the AKP method could completely replicate the original workspace. However, usually a workspace is characterized by a set of features that are required to achieve. It is possible that the workspace after applying the AKP method for achieving force balancing maintains to possess that set of features of the original workspace. The AKP method for this feature is considered as a future work.

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