Control of Hybrid Machines with 2-DOF for Trajectory Tracking Problems

F. X. Wu, W. J. Zhang, Q, Li, P. R. Ouyang, Z. X. Zhou

Abstract—**There are two types of drivers in production machine systems: constant velocity (CV) motor and servo-motor. If a system contains two drivers or more, among which some are of the CV motor while the other are the servo-motor, the system has the so-called hybrid driver architecture and is called hybrid machine for short. The hybrid system has the advantage of high payload and application flexibility. In this paper, we propose a control algorithm and show that the controlled hybrid machine is stable. A simulation is performed to show verify the proposed controller. The CV motor has the velocity fluctuation due to the change of its workload. The common approach to attenuate the velocity fluctuation is via a flywheel which is attached on the shaft of the CV motor. We show that this can further improve the tracking performance of the hybrid system. A five-bar linkage with two degrees of freedom is used for illustration throughout the paper.**

Index Terms— **controller, hybrid machine, trajectory tracking problems.**

I. Introduction

There are two types of drivers in production machine systems: constant velocity (CV) motor and servo-motor. If a system contains two or more drivers, among which some are of the CV motor while the other are the servo-motor, the system has the so-called hybrid driver architecture and is called hybrid machine for short. The hybrid system has the advantage of high payload and application flexibility, because the CV motor can undertake a high constant-part workload while the servo-motor can be real-time regulated to meet the change of a task [1-5].

It is known that the CV motor has the velocity fluctuation when it carries the time-varying workload. Such fluctuation will propagate to the end-effector or executive component of the hybrid machine. Control of the hybrid machine is then a challenge. Previous published studies on the hybrid machine usually substituted the CV motor by the servo-motor with the constant velocity profile [1-5]. This treatment will inevitably bring in errors in modeling of the dynamics of the hybrid machine system and control of the system.

This paper reports a study that considers the CV motor as it is and develops a control method based on the general idea that the disturbance on the trajectory of the end-effector due to the velocity fluctuation in the CV motor can be compensated by the controller for the servomotor. A five-bar hybrid mechanism with 2-DOF is taken as an example in this study. The next section (Section 2) outlines the dynamic model of the five-bar hybrid mechanism including its electric motors. Section 3 presents the proposed control method with its stability analysis. Section 4 gives simulation results. Section 5 is a conclusion.

II. The Dynamic Model of a Five-Bar Hybrid Machanism with 2-DOF

A. The Dynamic Model of the Five-Bar Mechanism with 2-DOF

As suggested in [9,10], a planar mechanism can be viewed to consist of free systems to which constraints are applied. In the planar five-bar mechanism (Fig. 1), the free system is two open-chain serial links, each of which contains two links, and the constraints are two independent scleronomic holonomic constraint equations as follows:

$$\phi(\boldsymbol{q}') = \begin{bmatrix} \phi_1(\boldsymbol{q}') \\ \phi_2(\boldsymbol{q}') \end{bmatrix} = \begin{bmatrix} L_1\cos(q_1) + L_3\cos(q_1+q_3) - \\ L_2\cos(q_2) - L_4\cos(q_2+q_4) - L_5 \\ L_1\sin(q_1) + L_3\sin(q_1+q_3) - \\ L_2\sin(q_2) - L_4\sin(q_2+q_4) \end{bmatrix} = 0 \tag{1}$$

where $\boldsymbol{q}' = [q_1 \quad q_2 \quad q_3 \quad q_3]^T$ is the vector of the generalized coordinates of the free system. Let r_i and δ_i denote, for link i, the location of the center of mass, m_i and L_i denote the mass and the length, respectively, and J_i the moment of inertia with respect to the centroid. Applying the Lagrangian

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method leads to the dynamic model of the free system as follows:

$$D'(q')\ddot{q}' + C'(q',\dot{q}')\dot{q}' + g'(q') = B\tau \quad (2)$$

where $D'(q')$ is the inertia matrix defined as follows:

$$D'(q') = \begin{bmatrix} d'_{11} & 0 & d'_{13} & 0 \\ 0 & d'_{22} & 0 & d'_{24} \\ d'_{31} & 0 & d'_{33} & 0 \\ 0 & d'_{42} & 0 & d'_{44} \end{bmatrix} \quad (3)$$

where

$$d'_{11} = m_1 r_1^2 + m_3 (L_1^2 + r_3^2 + 2L_1 r_3 \cos(q_3 + \delta_3)) + J_1 + J_3,$$
$$d'_{13} = d'_{31} = m_3 (r_3^2 + L_1 r_3 \cos(q_3 + \delta_3)) + J_3,$$
$$d'_{22} = m_2 r_2^2 + m_4 (L_2^2 + r_4^2 + 2L_2 r_4 \cos(q_4 + \delta_4)) + J_2 + J_4,$$
$$d'_{24} = d'_{42} = m_4 (r_4^2 + L_2 r_4 \cos(q_4 + \delta_4)) + J_4,$$
$$d'_{33} = m_3 r_3^2 + J_3,$$
$$d'_{44} = m_4 r_4^2 + J_4,$$

and $C'(q',\dot{q}')\dot{q}'$ contains the centrifugal and Coriolis terms, and it is defined as follows:

$$C'(q',\dot{q}') = \begin{bmatrix} h_1\dot{q}_3 & 0 & h_1(\dot{q}_1 + \dot{q}_3) & 0 \\ 0 & h_2\dot{q}_4 & 0 & h_2(\dot{q}_2 + \dot{q}_4) \\ -h_1\dot{q}_1 & 0 & 0 & 0 \\ 0 & -h_2\dot{q}_2 & 0 & 0 \end{bmatrix} \quad (4)$$

where $h_1 = -m_3 L_1 r_3 \sin(q_3 + \delta_3)$, $h_2 = -m_4 L_2 r_4 \sin(q_4 + \delta_4)$. $g'(q')$ is the gravity vector, and it is defined as follows:

$$g'(q') = g\begin{bmatrix} m_1 r_1 \cos(q_1 + \delta_1) + m_3 (L_1 \cos(q_1) + r_3 \cos(q_1 + q_3 + \delta_3)) \\ m_2 r_2 \cos(q_2 + \delta_2) + m_4 (L_2 \cos(q_2) + r_4 \cos(q_2 + q_4 + \delta_4)) \\ m_3 r_3 \cos(q_1 + q_3 + \delta_3) \\ m_4 r_4 \cos(q_2 + q_4 + \delta_4) \end{bmatrix} \quad (5)$$

where $g = 9.81 m/s^2$ is the gravitational acceleration constant. $B\tau$ is the input torque. Noticing that the actuated joints are Joint 1 and 2, respectively, we have:

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \ \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (6)$$

where τ_1 and τ_2 are the torques applied to Joint 1 and 2, respectively. The following mapping can be obtained:

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} q' = \alpha(q'). \quad (7)$$

The dynamic model of the 2-DOF five-far mechanism is then given as follows [9,10]:

$$\begin{cases} D(q')\ddot{q} + C(q',\dot{q}')\dot{q} + g(q') = \tau \\ \dot{q}' = \rho(q')\dot{q} \\ q' = \sigma(q) \end{cases} \quad (8)$$

where

$$D(q') = \rho^T(q') D'(q') \rho(q') \quad (9)$$

$$C(q',\dot{q}') = \rho^T(q') C'(q',\dot{q}') \rho(q') + \rho^T(q') D'(q',\dot{q}') \dot{\rho}(q',\dot{q}') \quad (10)$$

$$g(q') = \rho^T(q') g'(q') \quad (11)$$

where

$$\rho(q') = \Psi_{q'}^{-1}(q') \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (12)$$

where $\Psi_{q'}(q')$ is the Jacobian matrix of the following function $\Psi(q') = \begin{bmatrix} \phi(q') \\ \alpha(q') \end{bmatrix}$ with respect to the vector q'. The mapping $\dot{\rho}(q',\dot{q}')$ is the derivative of $\rho(q')$ with respect to time *t*. For the particular five-bar mechanism under consideration, $q' = \sigma(q)$ is expressed by

$$q_4 = \tan^{-1}(\sqrt{A^2 + B^2 - C^2}/C) + \tan^{-1}(A/B) - q_2 \quad (13)$$

where $A = 2L_4\lambda$, $B = 2L_4\mu$, and $C = L_3^2 - L_4^2 - \lambda^2 - \mu^2$, and $\lambda = L_2\cos(q_2) - L_1\cos(q_1) + L_5$,

$$q_3 = \tan^{-1}((\mu + L_4\sin(q_2 + q_4))/(\lambda + L_4\sin(q_2 + q_4))) - q_1 \quad (14)$$

B. The Dynamic model of Electric Motors

Based on a Newtonian kinetics law, sum of torques acting upon the motor shaft gives [12,13]

$$t_d = ki = t_L + bw + j\frac{dw}{dt} \quad (15)$$

where t_d is the magnetic motor torque, b the viscous damping coefficient, j the moment of inertia of the motor, t_L the load torque, and k the motor constant. Rearranging equation (15) leads to:

$$t_d - t_L = bw + j\frac{dw}{dt} \qquad (16)$$

When $t_d - t_L = bw$ the motor is at the steady state and runs at a constant speed. For the CV motor, during the operating process, the load torque t_L varies periodically, so the speed fluctuation will be present in the motor.

C. The Dynamic model of Hybrid Machine

As shown in Fig.1, the CV motor drives the actuated joint 1 while the servomotor drives the actuated joint 2. In addition, Link 1 and Link 2 are directly mounted to the motor shafts. Integration of (8) and (15) leads to the dynamic model of the hybrid machine as follows:

$$\begin{cases} \overline{\boldsymbol{D}}(\boldsymbol{q}')\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}',\dot{\boldsymbol{q}}')\dot{\boldsymbol{q}} + \boldsymbol{B}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}') = \boldsymbol{KI} \\ \dot{\boldsymbol{q}}' = \rho(\boldsymbol{q}')\dot{\boldsymbol{q}} \\ \boldsymbol{q}' = \sigma(\boldsymbol{q}) \end{cases} \qquad (17)$$

where $\overline{\boldsymbol{D}}(\boldsymbol{q}') = (\boldsymbol{D}(\boldsymbol{q}') + \boldsymbol{J})$, $\boldsymbol{J} = diag[j_1 \quad j_2]$, $\boldsymbol{B} = diag[b_1 \quad b_2]$, $\boldsymbol{K} = diag[k_1 \quad k_2]$, $\boldsymbol{I} = [i_1 \quad i_2]$, and $j_i, b_i, k_i, i_i (i = 1,2)$ are the moment of inertia, the viscous damping coefficient, the motor constant, and the armature current of the *i-th* motor, respectively.

III. CONTOLLER DESIGN AND ANALYSIS

Notice that the dynamic model as derived has a coupling relation between two motor variables, q_1 and q_2. Therefore the speed fluctuation in the CV motor should affect the motion q_2. It is then possible that the controller for the servomotor may offset the effect due to the speed fluctuation in the CV motor. Thus, the control problem can be stated as: Given the operating speed of the CV motor, a (rad/s), and the desired trajectory q_{2d}, $\dot{q}_{2d}$, $\ddot{q}_{2d}$, determine a control law for the servomotor to follow the desired trajectory with the consideration of the speed fluctuation in the constant speed motor. Furthermore, we assume the desired q_{2d}, $\dot{q}_{2d}$, $\ddot{q}_{2d}$ are bounded and notice that i_1 is a constant.

A control method proposed by Slotine and Li [15] is adapted to the control problem here. Let $\boldsymbol{q}_d = [q_{1d} \quad q_{2d}]^T$, $\dot{\boldsymbol{q}}_d = [\dot{q}_{1d} \quad \dot{q}_{2d}]^T$. Further we have $\dot{q}_{1d} = a$ and $q_{1d} = at$ (for simplicity). Also let $\boldsymbol{e} = \boldsymbol{q} - \boldsymbol{q}_d$, $\dot{\boldsymbol{q}}_r = \dot{\boldsymbol{q}}_d - \boldsymbol{\Lambda e}$, and

$$\boldsymbol{s} = \dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_r = \dot{\boldsymbol{e}} + \boldsymbol{\Lambda e} = [s_1 \quad s_2]^T \qquad (18)$$

where the diagonal matrix $\boldsymbol{\Lambda} = diag[0 \quad \lambda]$, and λ is a positive constant number.

The dynamic model of the hybrid mechanism can be written in terms of $\boldsymbol{s}$ as follows:

$$\begin{aligned} &\overline{\boldsymbol{D}}(\boldsymbol{q}')\dot{\boldsymbol{s}} + \boldsymbol{C}(\boldsymbol{q}',\dot{\boldsymbol{q}}')\boldsymbol{s} + \boldsymbol{Bs} \\ &= \boldsymbol{KI} - \boldsymbol{g}(\boldsymbol{q}') - \boldsymbol{B}\dot{\boldsymbol{q}} - \overline{\boldsymbol{D}}(\boldsymbol{q}')\ddot{\boldsymbol{q}}_r - \boldsymbol{C}(\boldsymbol{q}',\dot{\boldsymbol{q}}')\dot{\boldsymbol{q}}_r + \boldsymbol{Bs} \\ &= \begin{bmatrix} k_1 i_1 - g_1(\boldsymbol{q}') - b_1\dot{q}_1 + b_1 s_1 - \overline{d}_{11}(\boldsymbol{q}')\ddot{\boldsymbol{q}}_{r1} \\ -\overline{d}_{12}(\boldsymbol{q}')\ddot{\boldsymbol{q}}_{r1} - c_{11}(\boldsymbol{q}',\dot{\boldsymbol{q}}')\dot{\boldsymbol{q}}_{r1} - c_{12}(\boldsymbol{q}',\dot{\boldsymbol{q}}')\dot{\boldsymbol{q}}_{r2} \\ k_2 i_2 - g_2(\boldsymbol{q}') - b_2\dot{q}_2 + b_2 s_2 - \overline{d}_{21}(\boldsymbol{q}')\ddot{\boldsymbol{q}}_{r1} \\ -\overline{d}_{22}(\boldsymbol{q}')\ddot{\boldsymbol{q}}_{r1} - c_{21}(\boldsymbol{q}',\dot{\boldsymbol{q}}')\dot{\boldsymbol{q}}_{r1} - c_{22}(\boldsymbol{q}',\dot{\boldsymbol{q}}')\dot{\boldsymbol{q}}_{r2} \end{bmatrix} \end{aligned} \qquad (19)$$

Let

$$\begin{aligned} f_1 &= g_1(\boldsymbol{q}') + \overline{d}_{11}(\boldsymbol{q}')\ddot{\boldsymbol{q}}_{r1} + \overline{d}_{12}(\boldsymbol{q}')\ddot{\boldsymbol{q}}_{r2} \\ &+ c_{11}(\boldsymbol{q}',\dot{\boldsymbol{q}}')\dot{\boldsymbol{q}}_{r1} + c_{12}(\boldsymbol{q}',\dot{\boldsymbol{q}}')\dot{\boldsymbol{q}}_{r2} \end{aligned} \qquad (20)$$

$$\begin{aligned} f_2 &= g_2(\boldsymbol{q}') + \overline{d}_{21}(\boldsymbol{q}')\ddot{\boldsymbol{q}}_{r1} + \overline{d}_{22}(\boldsymbol{q}')\ddot{\boldsymbol{q}}_{r2} \\ &+ c_{21}(\boldsymbol{q}',\dot{\boldsymbol{q}}')\dot{\boldsymbol{q}}_{r1} + c_{22}(\boldsymbol{q}',\dot{\boldsymbol{q}}')\dot{\boldsymbol{q}}_{r2} \end{aligned} \qquad (21)$$

Assume the armature current in the CV motor is

$$i_1 = b_1 a / k_1 \qquad (22)$$

and let

$$i_2 = (b_2\dot{q}_{2r} + f_2 - k_d s_2 + i_r)/k_2 \qquad (23)$$

Then, we have

$$\overline{\boldsymbol{D}}(\boldsymbol{q}')\dot{\boldsymbol{s}} + \boldsymbol{C}(\boldsymbol{q}',\dot{\boldsymbol{q}}')\boldsymbol{s} + \overline{\boldsymbol{B}}\boldsymbol{s} = \begin{bmatrix} -f_1 \\ i_r \end{bmatrix} \qquad (24)$$

where the matrix $\overline{\boldsymbol{B}} = diag[b_1 \quad b_2 + k_d]$, $k_d > 0$ is a design parameter, and i_r is a design signal.

Remark 1. The matrix $\boldsymbol{\Lambda}$ in (18) is a non-negative definite matrix while its counterparts in [14, 15, 16] are the positive definite matrices.

The stability analysis of the proposed controller is presented as follows. The dynamic model of a five-bar linkage should have the following properties:

Lemma 1[9,10]: 1) The mass matrix $\overline{\boldsymbol{D}}(\boldsymbol{q}')$ is symmetric and positive definite. 2) By a suitable arrangement of elements of the matrix $\boldsymbol{C}(\boldsymbol{q}',\dot{\boldsymbol{q}}')$, $\frac{1}{2}\dot{\overline{\boldsymbol{D}}}(\boldsymbol{q}') - \boldsymbol{C}(\boldsymbol{q}',\dot{\boldsymbol{q}}')$ is skew.

Proof. Since the matrix $\boldsymbol{D}(\boldsymbol{q}')$ and the matrix $\boldsymbol{J}$ are symmetric and positive definite, the matrix $\overline{\boldsymbol{D}}(\boldsymbol{q}')$ is also symmetric and positive definite. The elements of the matrix $\boldsymbol{C}(\boldsymbol{q}',\dot{\boldsymbol{q}}')$ are made up of the partial derivates of the matrix $\boldsymbol{D}(\boldsymbol{q}')$, so $\frac{1}{2}\dot{\overline{\boldsymbol{D}}}(\boldsymbol{q}') - \boldsymbol{C}(\boldsymbol{q}',\dot{\boldsymbol{q}}')$ is also skew.

Theorem 1. If (i) the current in the CV motor is given by equation (22), (ii) the design signal i_r in equation (23) is defined as follows:

$$i_r = \begin{cases} -|s_1 f_1| / s_2, & \text{if } s_2 \neq 0 \\ 0, & \text{if } s_2 = 0 \end{cases} \quad (25)$$

and (iii) the control law is given by equations (23) and (25), then the trajectory tracking errors $\dot{e}_1(t)$, $e_2(t)$, and $\dot{e}_2(t)$ will exponentially converge to zero when time t tends to infinite large, and $e_1(t)$ is bounded for any time $t \geq 0$.

Proof. Consider the following energy term as the Lypunove function

$$V = \tfrac{1}{2} \boldsymbol{s}^T \overline{\boldsymbol{D}}(\boldsymbol{q}') \boldsymbol{s} \quad (26)$$

We can find along the solution of equation (24):

$$\begin{aligned} \dot{V} &= \boldsymbol{s}^T \overline{\boldsymbol{D}}(\boldsymbol{q}') \dot{\boldsymbol{s}} + \tfrac{1}{2} \boldsymbol{s}^T \dot{\overline{\boldsymbol{D}}}(\boldsymbol{q}') \boldsymbol{s} \\ &= \boldsymbol{s}^T (\tfrac{1}{2} \dot{\overline{\boldsymbol{D}}}(\boldsymbol{q}') - \boldsymbol{C}(\boldsymbol{q}', \dot{\boldsymbol{q}}') - \overline{\boldsymbol{B}}) \boldsymbol{s} + \boldsymbol{s}^T \begin{bmatrix} -f_1 \\ i_r \end{bmatrix} \\ &= -\boldsymbol{s}^T \overline{\boldsymbol{B}} \boldsymbol{s} - s_1 f_1 + s_2 i_r \end{aligned} \quad (27)$$

where the identical relation $\boldsymbol{s}^T (\tfrac{1}{2} \dot{\overline{\boldsymbol{D}}}(\boldsymbol{q}') - \boldsymbol{C}(\boldsymbol{q}', \dot{\boldsymbol{q}}')) \boldsymbol{s} = 0$ is used to derive equation (27). Furthermore, using equation (25) we can verify

$$-s_1 f_1 + s_2 i_r = -s_1 f_1 - |s_1 f_1| \leq 0 \quad (28)$$

Substituting equation (28) into equation (27) yield

$$\dot{V} \leq -\boldsymbol{s}^T \overline{\boldsymbol{B}} \boldsymbol{s}$$

According to the Lyapunov's stability theorem, $\boldsymbol{s}(t)$ will exponentially tends to zero when t tends to infinite large. From the standard stable filter theory, this then implies that errors $\dot{e}_1(t)$, $e_2(t)$, and $\dot{e}_2(t)$ will exponentially converge to zero when time t tends to infinite large. Furthermore, since $\dot{e}_1(t)$ exponentially converges to zero, $e_1(t)$ will be normally bounded. This then concludes the proof.

Remark 2. The control law given by equations (22), (23) and (25) is discontinuous and needs to be smoothed at the implementation level. For this, we replace equation (25) by

$$i_r = -\frac{|s_1 f_1| s_2}{\delta + s_2^2} \quad (29)$$

where $\delta > 0$ is the boundary layer thickness. Such a smoothing method [14] generally leads to the conclusion that trajectory tracking error is globally uniformly ultimately bounded. In fact, the simulation results in the next section confirm the boundedness with this method.

IV. SIMULATION VERIFICATION

The parameters of the five-bar mechanism and two motors are listed in Table I and Table II, respectively. Noticing that the desired trajectory in Link 1 should be a line, while the desired trajectory in Link 2 can be any curve. Assume the following trajectories:

$$q_{1d} = at, \; q_{2d} = 2\pi(6\frac{t^5}{t_f^5} - 15\frac{t^4}{t_f^4} + 10\frac{t^3}{t_f^3}) \quad (30)$$

where $t_f = 4(s)$, and a =15 (rad/s). The simulation starts from $t_0 = 0(s)$ and stops at $t_f = 4(s)$.

In the first simulation, the controller parameters are assumed to be $\Lambda = diag[0 \;\; 15]$, $k_d = 30$. Fig. 2 depicts the error curves and torques. Specifically, Fig. 2a, b show that the maximum of the angular displacement error and the maximum of the angular velocity error of Link 1 are, respectively, 0.08 and 0.25, and they look bounded. Fig. 2c, d show that the errors of the angular displacement and the angular velocity of Link 2 are very small, specifically 10^{-4} and 10^{-3}, respectively. Theoretically, the velocity error of Link 1 is related to the boundary layer thickness $\delta > 0$. The less the boundary layer thickness the less the velocity error of Link 1 is. However, in implementation, reduction of the boundary layer thickness can be hindered due to the computational problem. Fig. 2e, f depict the curves of the torques (where the torques are the product of the motor constant k_i and the armature current i_i, i=1,2). Fig. 3a,d depict the curves of the desired (solid line) and simulated (dashed line) trajectories, respectively. The results in these figures are excellent except for the one shown in Fig. 3b, which is expected, as this is associated with the speed fluctuation in the CV motor.

In the second simulation, a flywheel (9 kg•m^2) is attached to the shaft of the CV motor because the flywheel is notably a means of reduction of the speed fluctuation in the CV motor. The controller parameters are the same as those in the first simulation. The simulation results are depicted in Fig. 4. Comparing the simulation results in Fig. 4 with those in Fig. 2, it is observed that after a flywheel is attached, the overall tracking performance of the hybrid machine is improved significantly. Fig. 4a, b show that the angular displacement error and the angular velocity error of Link 1 are reduced to the order of 10^{-2}. However, there is no significant improvement in the angular displacement and velocity of the servomotor; see Fig. 4c,d. Fig. 4e,f show the torques in the CV motor and the servomotor, respectively, which do not increase with the inclusion of the flywheel in the system, compared to the results of the torques depicted in Fig. 2e,f. A further simulation can show the following trend; i.e., with the increase of the inertia of flywheel, (1) the angular displacement error and the angular velocity error of Link 1 continue to reduce, (2) the angular displacement error and the angular velocity error of Link 2 keep the orders of 10^{-4} and 10^{-3}, respectively, and (3) the control energy remains the same.

V. CONCLUSION

The proposed controller for the five-bar hybrid mechanism with two DOF appears effective. In this case, the CV motor is not controllable while the servomotor is controllable. The proposed controller is adapted from the one proposed by Slotine and Li in [15,16] and is asymptotically convergent. The inclusion of a flywheel on the shaft of the CV motor has a significantly positive effect on the control performance of the hybrid machine system.

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